The POG Technique for Modeling Planetary Gears and Hybrid Automotive Systems

UNIVERSITÀ DEGLI STUDI
DI MODENA E REGGIO EMILIA

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Outline

1) The Power-Oriented Graphs technique (intro)
2) POG modeling of a *Planetary Gear*
   a) system transformation and system reduction
   b) different dynamic models
3) POG modeling of an *Hybrid Automotive System*
4) Simulations and control
5) Conclusions

Power-Oriented Graphs (**POG**)

The Power-Oriented Graphs are "block diagrams" obtained by using a "modular" structure essentially based on the following two blocks:

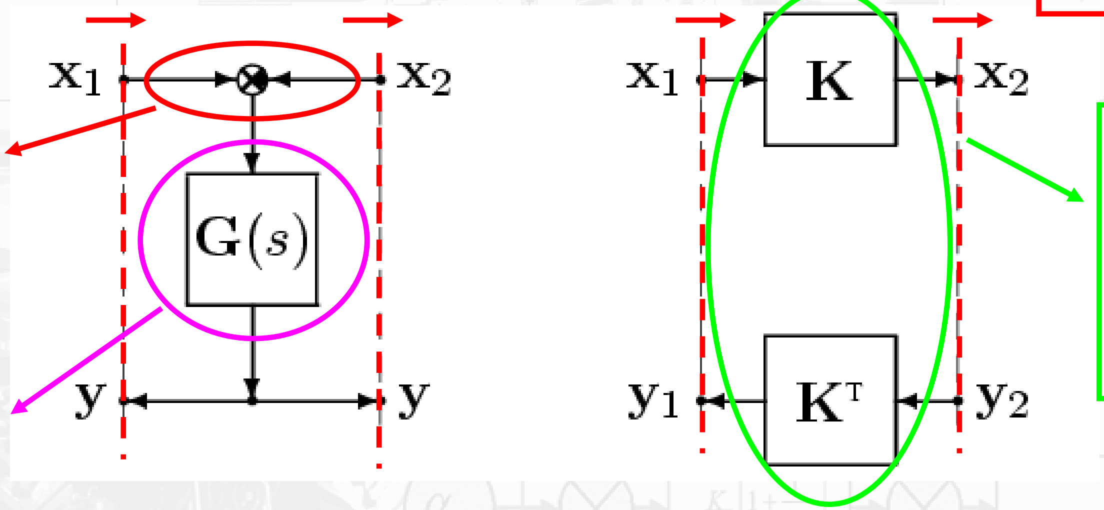
Positive power flows

3-Port Junctions:

- 0-Junction;
- 1-Junction;

1-Port Elements:

- Capacitor C;
- Inertia I;
- Resistor R;



Elaboration block **Connection block**

2-Port Elements :

- Transformers TR;
- Girators GY;
- Modulated TR;
- Modulated GY;

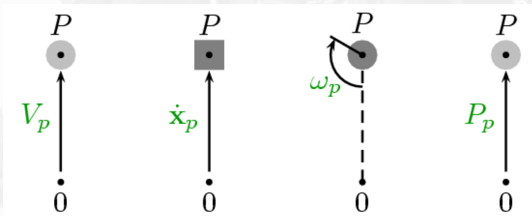
- POG maintains a direct correspondence between pairs of system variables and real power flows: the product of the two variables involved in each dashed line of the graph has the physical meaning of ``power flowing through that section''.
- The Elaboration block can store and dissipate/generate energy.
- The Connection block can only "transform" the energy.

POG dynamic modeling: electrical example

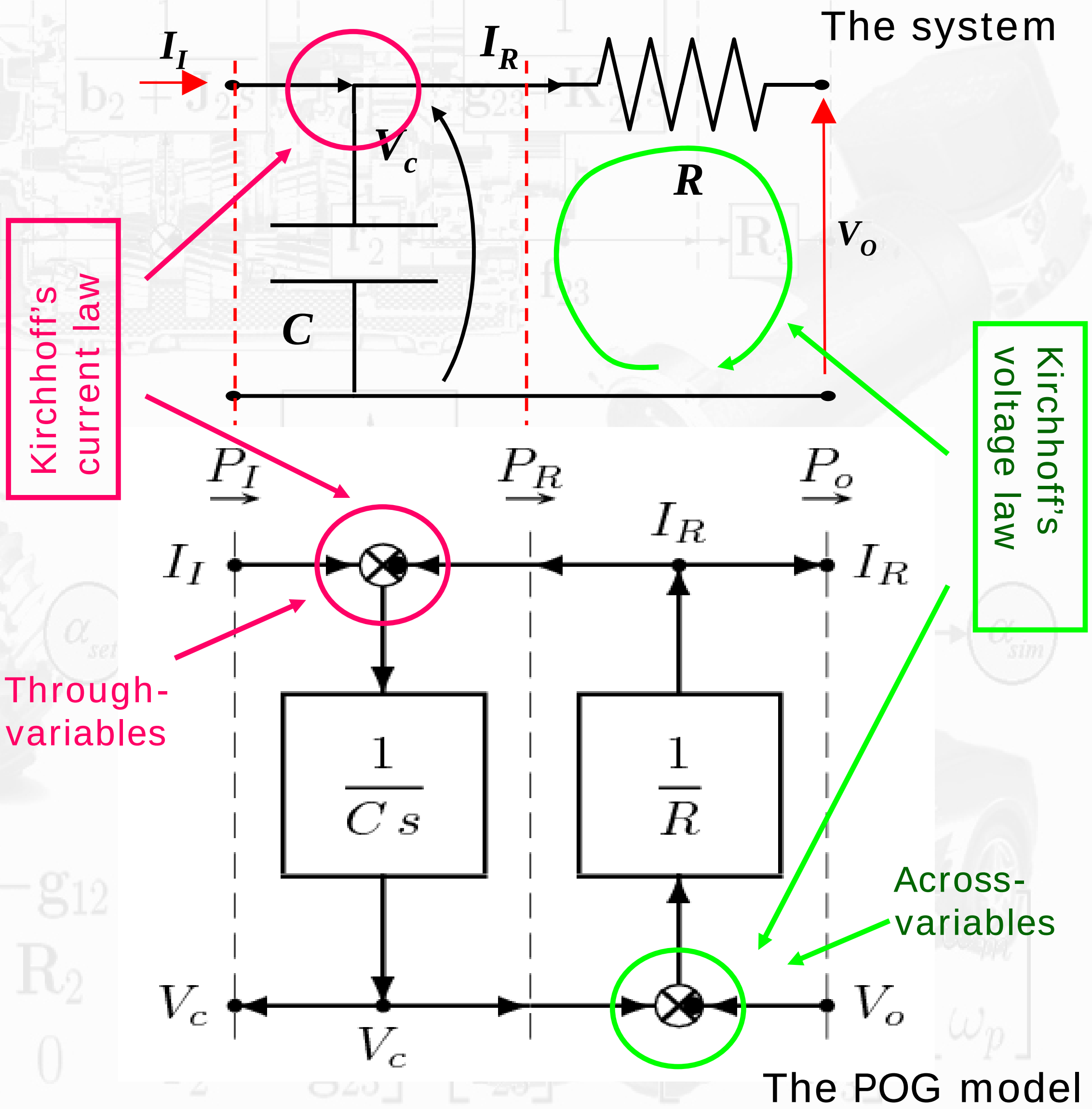
Across and through variables:

	POG Variables	
Domains:	*Across:* v_e	*Through:* v_f
Electrical	Voltage V	Current I
Mec. Tra.	Velocity v	Force F
Mec. Rot.	Ang. Vel. ω	Torque τ
Hydraulic	Pressure P	Flow rate Q

Across variables are defined between two points:



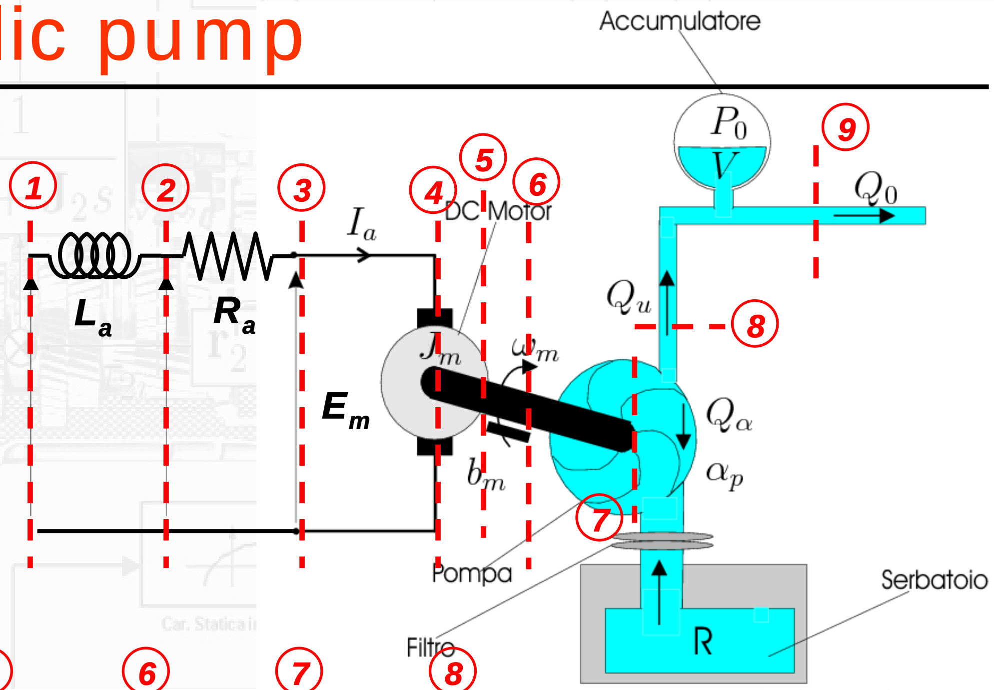
Through variables are defined in a single point:



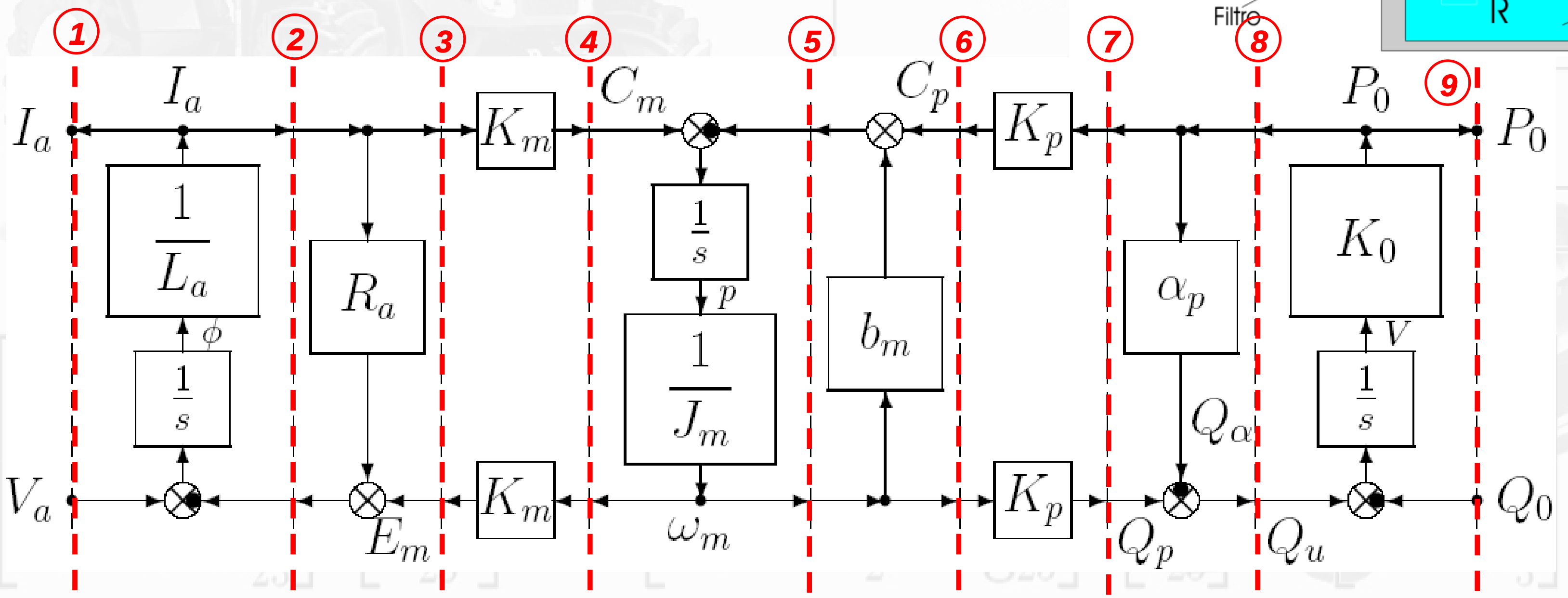
Example of POG modeling: DC electric motor with an hydraulic pump

A DC motor connected to an hydraulic pump:

There is a direct correspondence between the POG blocks and the physical elements …

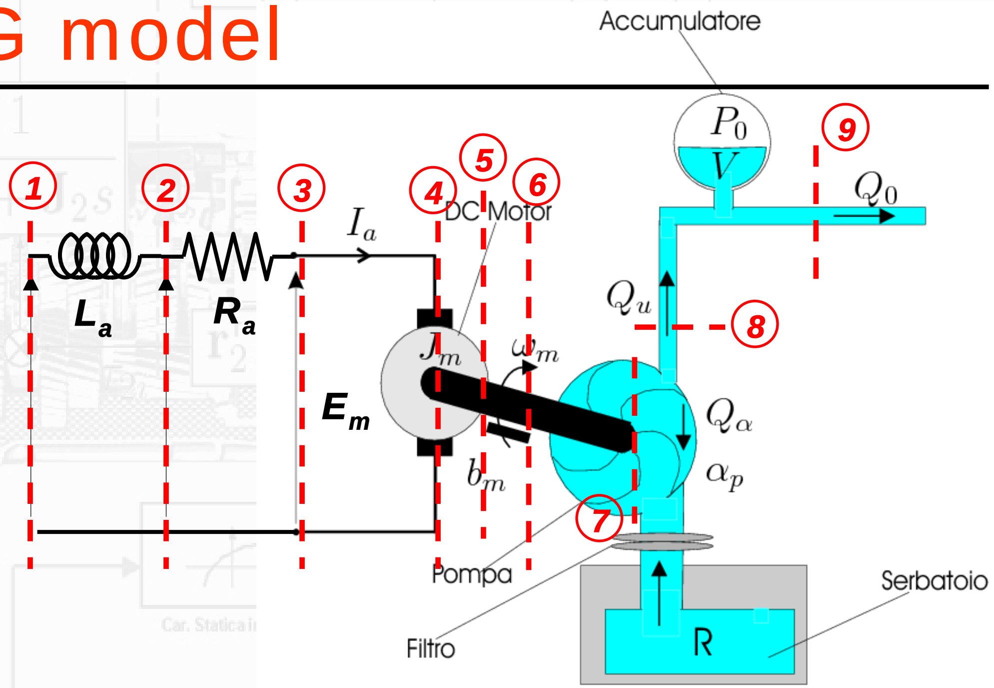


The POG model:



POG models can be directly inserted in Simulink.

DC electric motor with an hydraulic pump: comparison with the BG model



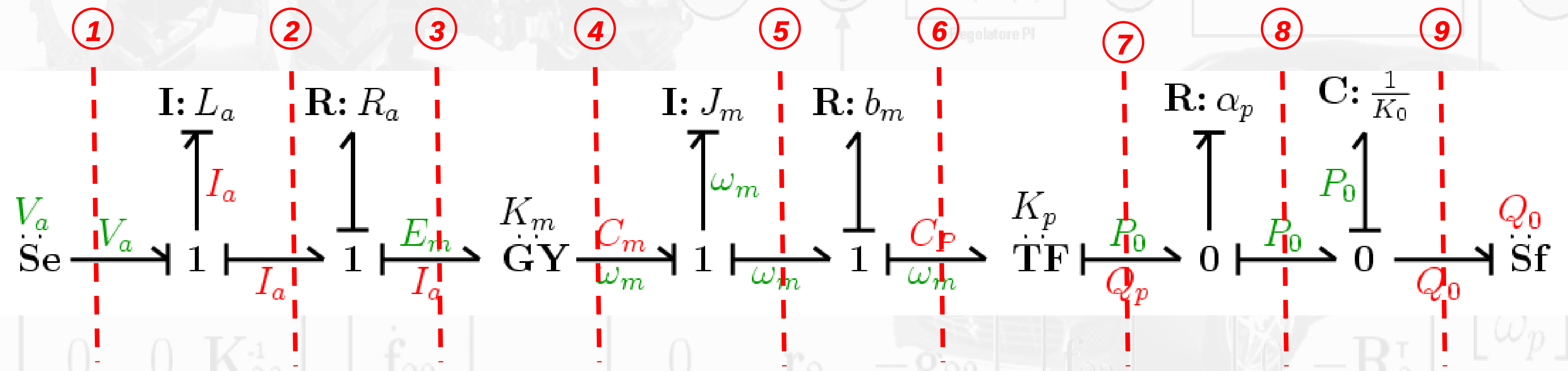
A DC motor connected to an hydraulic pump:

+ The graphical representation is more flexible and more compact.

- Not easy for the beginners.

- Not direct use in Simulink.

The Bond Graph model:



Introduction
Power-Oriented Graphs - LTI Systems

The POG state space description of the DC motor with hydraulic pump:

$$\underbrace{\begin{bmatrix} L_a & 0 & 0 \\ 0 & \cancel{J_m} & 0 \\ 0 & 0 & \frac{1}{K_0} \end{bmatrix}}_{\mathbf{L}} \underbrace{\begin{bmatrix} \dot{I}_a \\ \dot{\omega}_m \\ \dot{P}_0 \end{bmatrix}}_{\dot{\mathbf{x}}} = \underbrace{\begin{bmatrix} -R_a & -K_m & 0 \\ K_m & -b_m & -K_p \\ 0 & K_p & -\alpha_p \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} I_a \\ \omega_m \\ P_0 \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix}}_{\mathbf{B}} \underbrace{\begin{bmatrix} V_a \\ Q_0 \end{bmatrix}}_{\mathbf{u}} \qquad \mathbf{y} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{C}} \mathbf{x}$$

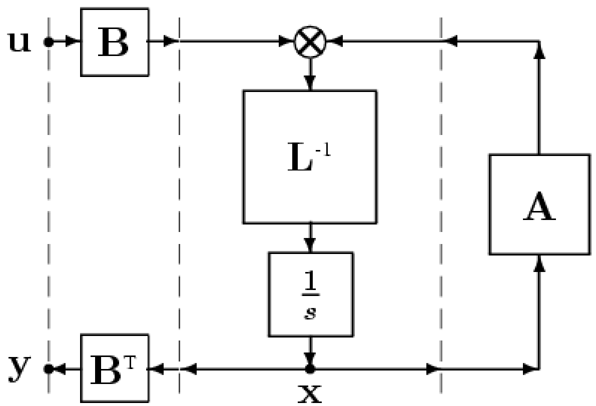
- Direct correspondence between POG and state space descriptions:

Energy matrix **Power matrix**

$$\begin{cases} \mathbf{L}\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \\ \mathbf{y} = \mathbf{B}^{\mathrm{T}}\mathbf{x} \end{cases}$$

Stored Energy: $E_s = \frac{1}{2}\mathbf{x}^{\mathrm{T}}\mathbf{L}\,\mathbf{x}$

Dissipating Power: $P_d = \mathbf{x}^{\mathrm{T}}\mathbf{A}\,\mathbf{x}$

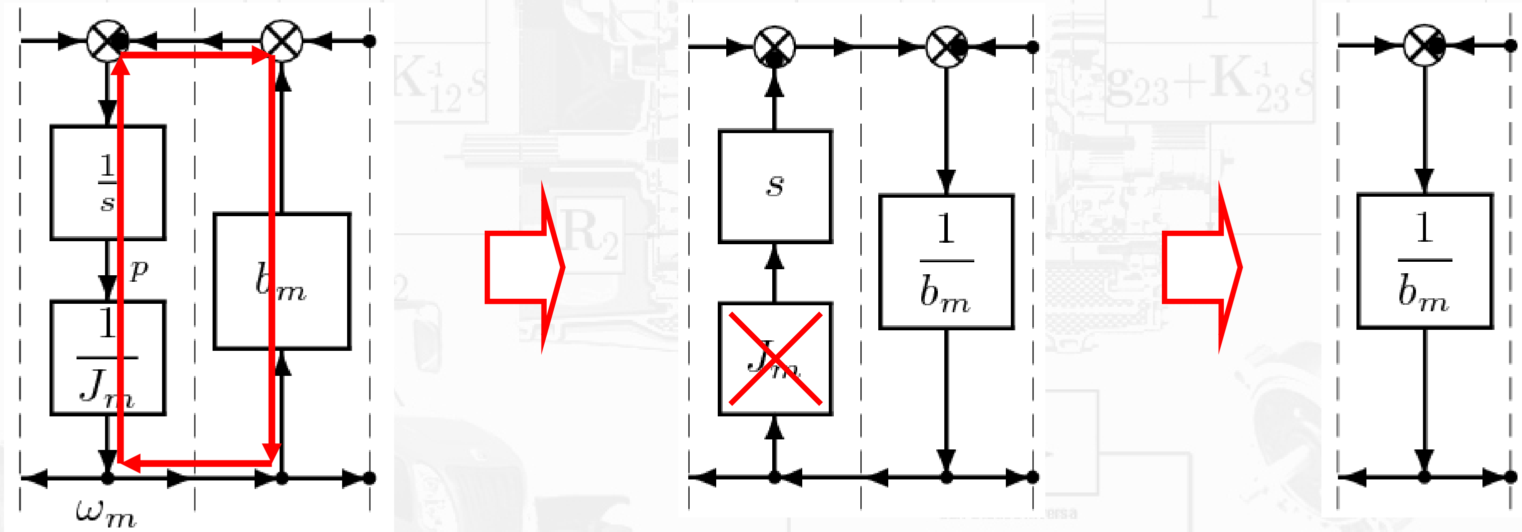


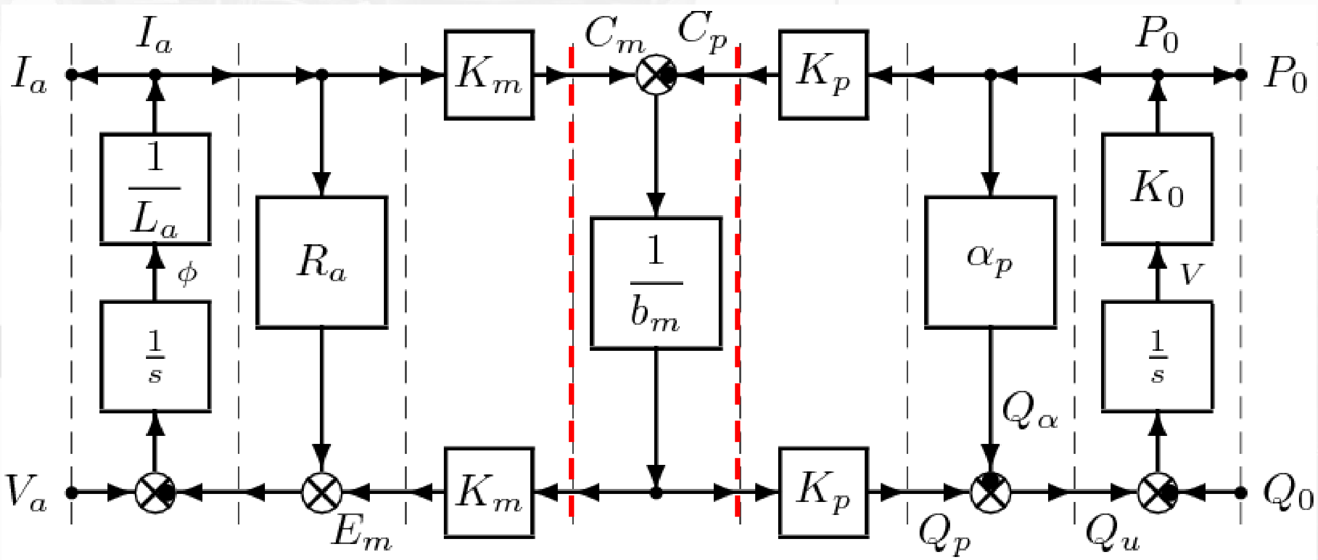
- With POG it easy to obtain reduced models of the system

Which is the "reduced model" when J_m->0 ?

Two possible solutions:
1) graphically inverting a path ...;
2) using a congruent transformation

POG modeling reduction: graphically inverting a path





POG reduced model !

POG modeling reduction: using a "congruent" transformation

When an eigenvalue of matrix ***L*** goes to zero (or to infinity), the system degenerates towards a lower dynamic dimension system. The "reduced system" can be obtained by using a congruent transformation $x=\mathbf{T}z$ where ***T*** is a rectangular matrix:

congruent transformation

$$\begin{cases} \mathbf{T}^T\mathbf{L}\mathbf{T}\dot{\mathbf{z}} = \mathbf{T}^T\mathbf{A}\mathbf{T}\mathbf{z} + \mathbf{T}^T\mathbf{B}\mathbf{u} \\ \mathbf{y} = \mathbf{B}^T\mathbf{T}\mathbf{z} \end{cases} \Leftrightarrow \begin{cases} \overline{\mathbf{L}}\dot{\mathbf{z}} = \overline{\mathbf{A}}\mathbf{z} + \overline{\mathbf{B}}\mathbf{u} \\ \mathbf{y} = \overline{\mathbf{B}}^T\mathbf{z} \end{cases}$$

Transformed system

When a parameter goes to zero a static relation between state variable arises:

$$J_m = 0 \quad \Rightarrow \quad K_m I_a - b_m \omega_m - K_p P_0 = 0 \quad \Rightarrow \quad \omega_m = \frac{K_m}{b_m} I_a - \frac{K_p}{b_m} P_0$$

A <u>rectangular</u> state space transformation can be easily obtained:

$$\underbrace{\begin{bmatrix} I_a \\ \omega_m \\ P_0 \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} 1 & 0 \\ \frac{K_m}{b_m} & -\frac{K_p}{b_m} \\ 0 & 1 \end{bmatrix}}_{\mathbf{T}} \underbrace{\begin{bmatrix} I_a \\ P_0 \end{bmatrix}}_{\mathbf{z}}$$

Rectangular matrix

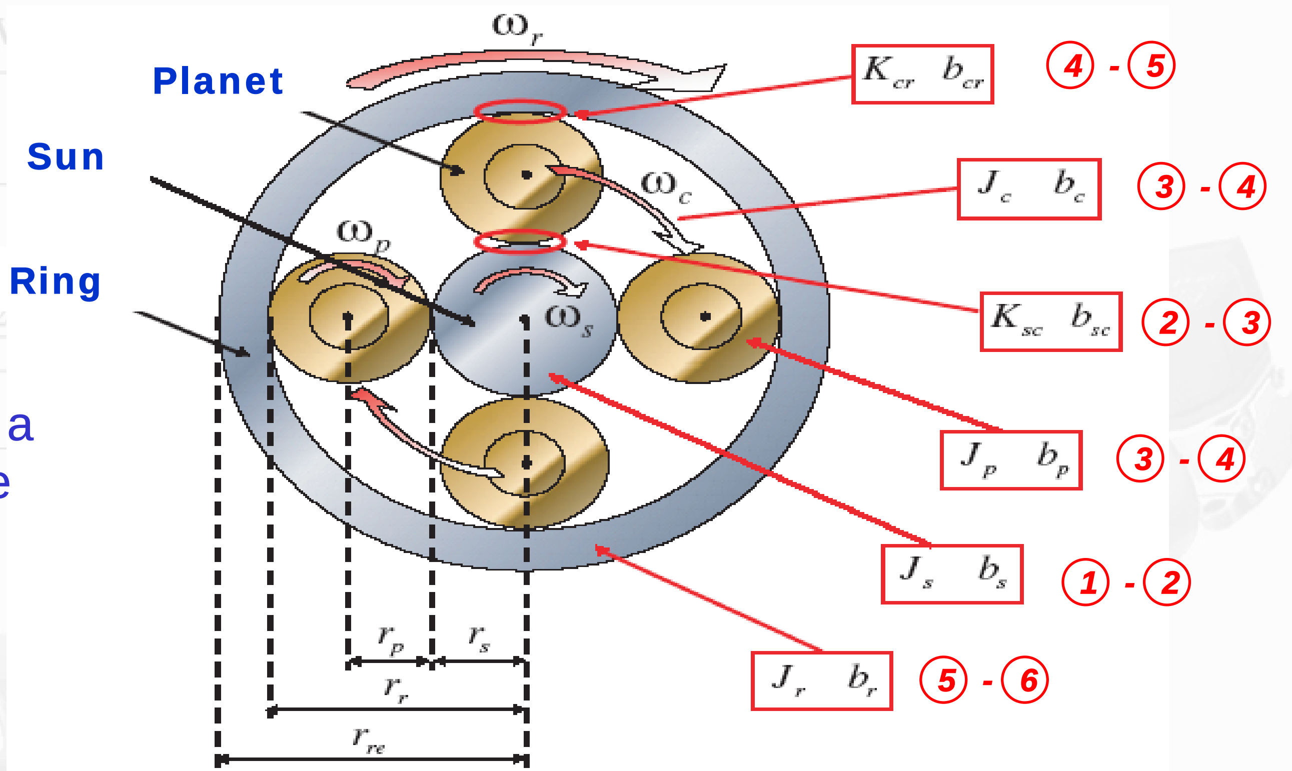
Old state vector

New state vector

Applying the congruent transformation <u>one directly obtains the reduced system</u>:

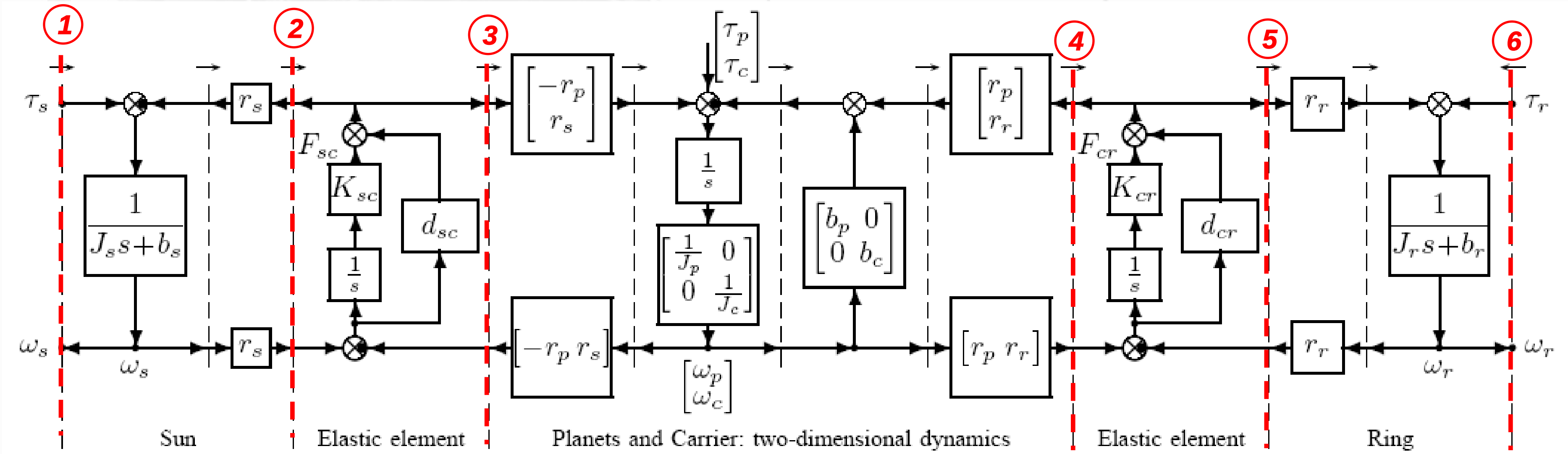
$$\begin{bmatrix} L_a & 0 \\ 0 & \frac{1}{K_0} \end{bmatrix} \begin{bmatrix} \dot{I}_a \\ \dot{P}_0 \end{bmatrix} = \begin{bmatrix} -R_a - \frac{K_m^2}{b_m} & \frac{K_m K_p}{b_m} \\ \frac{K_m K_p}{b_m} & -\alpha_p - \frac{K_p^2}{b_m} \end{bmatrix} \begin{bmatrix} I_a \\ P_0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} V_a \\ Q_0 \end{bmatrix}$$

Modeling a Planetary Gear



The POG blocks have a direct correspondence with the physical elements.

The POG model:



Planetary Gear: state space model

From the POG block scheme directly follows the dynamic state space model of the system (n=6):

$$\overline{\mathbf{L}}\,\dot{\overline{\mathbf{x}}} = -\overline{\mathbf{A}}\,\overline{\mathbf{x}} + \overline{\mathbf{B}}\,\mathbf{u},$$
$$\mathbf{y} = \overline{\mathbf{B}}^{\mathrm{T}}\overline{\mathbf{x}}$$

State vector:

$$\overline{\mathbf{x}} = \begin{bmatrix} \omega_s \\ F_{sc} \\ \omega_p \\ \omega_c \\ F_{cr} \\ \omega_r \end{bmatrix}$$

Energy matrix:

$$\overline{\mathbf{L}} = \begin{bmatrix} J_s & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{K_{sc}} & 0 & 0 & 0 & 0 \\ 0 & 0 & J_p & 0 & 0 & 0 \\ 0 & 0 & 0 & J_c & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{K_{cr}} & 0 \\ 0 & 0 & 0 & 0 & 0 & J_r \end{bmatrix}$$

Input matrix:

$$\overline{\mathbf{B}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Power matrix:

$$\overline{\mathbf{A}} = -\begin{bmatrix} -b_s - r_s^2 d_{sc} & -r_s & -r_s d_{sc} r_p & r_s^2 d_{sc} & 0 & 0 \\ r_s & 0 & r_p & -r_s & 0 & 0 \\ -r_s d_{sc} r_p & -r_p & -b_p - d_{sc} r_p^2 - d_{cr} r_p^2 & r_s d_{sc} r_p - d_{cr} r_p r_r & -r_p & d_{cr} r_p r_r \\ r_s^2 d_{sc} & r_s & r_s d_{sc} r_p - d_{cr} r_p r_r & -b_c - r_s^2 d_{sc} - d_{cr} r_r^2 & -r_r & d_{cr} r_r^2 \\ 0 & 0 & r_p & r_r & 0 & -r_r \\ 0 & 0 & d_{cr} r_p r_r & d_{cr} r_r^2 & r_r & -b_r - d_{cr} r_r^2 \end{bmatrix}$$

Is this model too complex? $\Rightarrow$ POG technique provides the reduced models when the stiffness tend to infinity or the inertias tend to zero.

Planetary gear: reduced inertial model

When the stiffness coefficients tend to infinity $K_{cr} \to \infty$, $K_{sc} \to \infty$ a static relation between the state variables appears:

$$\mathbf{x}_2 = \begin{bmatrix} \omega_p \\ \omega_r \end{bmatrix} = -\mathbf{A}_{32}^{-1}\,\mathbf{A}_{31}\,\mathbf{x}_1 = \begin{bmatrix} -\frac{r_s}{r_p} & \frac{r_s}{r_p} \\ -\frac{r_s}{r_r} & 1+\frac{r_s}{r_r} \end{bmatrix} \begin{bmatrix} \omega_s \\ \omega_c \end{bmatrix}$$

Applying a *rectangular and congruent state space transformation* $\mathbf{x} = \mathbf{T}_2\,\mathbf{x}_1$ where:

$$\mathbf{T}_2 = \begin{bmatrix} \mathbf{I}_2 \\ -\mathbf{A}_{32}^{-1}\,\mathbf{A}_{31} \\ 0 \end{bmatrix}$$

one obtains the transformed and reduced system:

$$\mathbf{L}_r\,\dot{\mathbf{x}}_1 = -\mathbf{A}_r\,\mathbf{x}_1 + \mathbf{B}_r\,\mathbf{u}$$

$$\mathbf{y} = \mathbf{B}_r^{\mathrm{T}}\mathbf{x}_1$$

Matrices $\mathbf{L}_r = \mathbf{T}_2^{\mathrm{T}}\mathbf{L}\,\mathbf{T}_2$, $\mathbf{A}_r = \mathbf{T}_2^{\mathrm{T}}\mathbf{A}\,\mathbf{T}_2$ and $\mathbf{B}_r = \mathbf{T}_2^{\mathrm{T}}\,\mathbf{B}$ have the following structure:

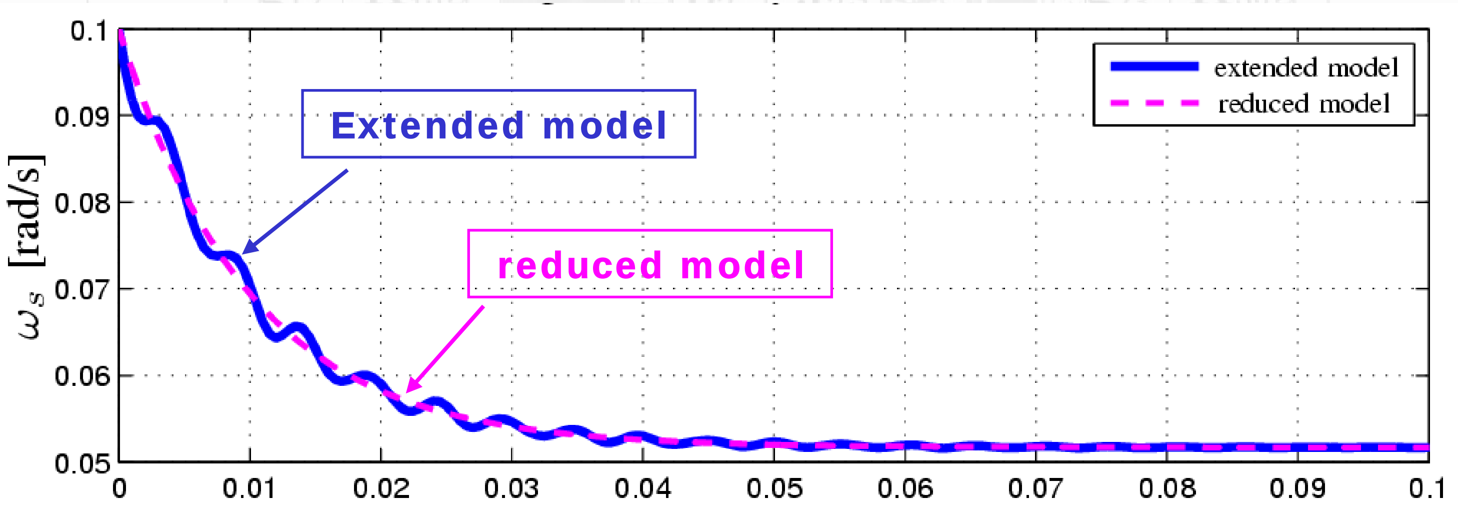
$$\mathbf{L}_r = \begin{bmatrix} J_s+\frac{r_s^2}{r_p^2}J_p+\frac{r_s^2}{r_r^2}J_r & -\frac{r_s^2}{r_p^2}J_p-\frac{r_s}{r_r}\left(1+\frac{r_s}{r_r}\right)J_r \\ -\frac{r_s^2}{r_p^2}J_p-\frac{r_s}{r_r}\left(1+\frac{r_s}{r_r}\right)J_r & J_c+\frac{r_s^2}{r_p^2}J_p+\left(1+\frac{r_s}{r_r}\right)^2 J_r \end{bmatrix}$$

$$\mathbf{A}_r = \begin{bmatrix} b_s+\frac{r_s^2}{r_p^2}b_p+\frac{r_s^2}{r_r^2}b_r & -\frac{r_s^2}{r_p^2}b_p-\frac{r_s}{r_r}\left(1+\frac{r_s}{r_r}\right)b_r \\ -\frac{r_s^2}{r_p^2}b_p-\frac{r_s}{r_r}\left(1+\frac{r_s}{r_r}\right)b_r & b_c+\frac{r_s^2}{r_p^2}b_p+\left(1+\frac{r_s}{r_r}\right)^2 b_r \end{bmatrix}$$

$$\mathbf{B}_r = \begin{bmatrix} 1 & 0 & -\frac{r_s}{r_r} \\ 0 & 1 & 1+\frac{r_s}{r_r} \end{bmatrix} \qquad \mathbf{x}_1 = \begin{bmatrix} \omega_s \\ \omega_c \end{bmatrix} \qquad \mathbf{y} = \begin{bmatrix} \omega_s \\ \omega_c \\ \omega_r \end{bmatrix}$$

Reduced inertial model: simulation results

Comparison between the extended and reduced models:



Parameters used in simulation (SI): $[J_s,\ b_s,\ r_s] = [0.049,\ 4.946,\ 0.102]$

$[J_r,\ b_r,\ r_r] = [2.180,\ 218.02,\ 0.248]$ $[J_p,\ b_p,\ r_p] = [0.081,\ 8.123,\ 0.073]$

$[J_c,\ b_c] = [0.929,\ 92.89]$ $[K_{sc},\ d_{sc}] = [K_{cr},\ d_{cr}] = [10^7,\ 10]$ $[\tau_s,\ \tau_c,\ \tau_r] = [0,\ 4,\ 0]$

$[\omega_s(0),\ \omega_c(0),\ \omega_r(0)] = [0.1,\ -0.2,\ -0.323]$

Note: the use of a "rectangular" matrix for transforming and reducing a dynamical system is a "specific characteristics" of the POG technique.

Planetary gear: reduced elastic model

When the inertias coefficients tend to zero $J_s = J_c = J_r = 0$ a static relation between the state and input variables appears in the system:

$$\mathbf{z}_2 = -\mathbf{A}_{22}^{-1}\mathbf{A}_{21}\,\mathbf{z}_1 - \mathbf{A}_{22}^{-1}\mathbf{B}_2\,\mathbf{u}$$

where

$$\mathbf{z}_1 = \begin{bmatrix} F_{sc} \\ \omega_p \\ F_{cr} \end{bmatrix} \qquad \mathbf{z}_2 = \begin{bmatrix} \omega_s \\ \omega_c \\ \omega_r \end{bmatrix} \qquad \mathbf{B}_2 = \begin{bmatrix} 1\,0\,0 \\ 0\,1\,0 \\ 0\,0\,1 \end{bmatrix}$$

$$\mathbf{A}_{22} = \begin{bmatrix} -b_s - r_s^2 d_{sc} & r_s^2 d_{sc} & 0 \\ r_s^2 d_{sc} & -b_c - r_s^2 d_{sc} - r_r^2 d_{cr} & r_r^2 d_{cr} \\ 0 & r_r^2 d_{cr} & -b_r - r_r^2 d_{cr} \end{bmatrix} \qquad \mathbf{A}_{21} = \begin{bmatrix} -r_s & -r_s d_{sc} r_p & 0 \\ r_s & r_s d_{sc} r_p - r_p r_r d_{cr} & -r_r \\ 0 & r_p r_r d_{cr} & r_r \end{bmatrix}$$

Applying a congruent transformation and inverting the system one obtains the POG *reduced elastic model*:

$$\mathbf{z} = \begin{bmatrix} \mathbf{I}_2 \\ -\mathbf{A}_{22}^{-1}\mathbf{A}_{21} \end{bmatrix} \mathbf{z}_1 + \begin{bmatrix} 0 \\ -\mathbf{A}_{22}^{-1}\mathbf{B}_2 \end{bmatrix} \mathbf{u}$$

$$\underbrace{\begin{bmatrix} \frac{1}{K_{sc}} & 0 & 0 \\ 0 & J_p & 0 \\ 0 & 0 & \frac{1}{K_{cr}} \end{bmatrix}}_{\tilde{\mathbf{L}}_e} \underbrace{\begin{bmatrix} \dot{F}_{sc} \\ \dot{\omega}_p \\ \dot{F}_{cr} \end{bmatrix}}_{\dot{\tilde{\mathbf{x}}}} = \underbrace{\begin{bmatrix} 0 & r_p & 0 \\ -r_p & -r_p^2 d_{sc} - b_p - r_p^2 d_{cr} & -r_p \\ 0 & r_p & 0 \end{bmatrix}}_{-\tilde{\mathbf{A}}_e} \underbrace{\begin{bmatrix} F_{sc} \\ \omega_p \\ F_{cr} \end{bmatrix}}_{\tilde{\mathbf{x}}} + \underbrace{\begin{bmatrix} r_s & -r_s & 0 \\ -r_s d_{sc} r_p & r_s d_{sc} r_p - r_r d_{cr} r_p & r_r d_{cr} r_p \\ 0 & r_r & -r_r \end{bmatrix}}_{\tilde{\mathbf{B}}_e} \underbrace{\begin{bmatrix} \omega_s \\ \omega_c \\ \omega_r \end{bmatrix}}_{\tilde{\mathbf{u}}}$$

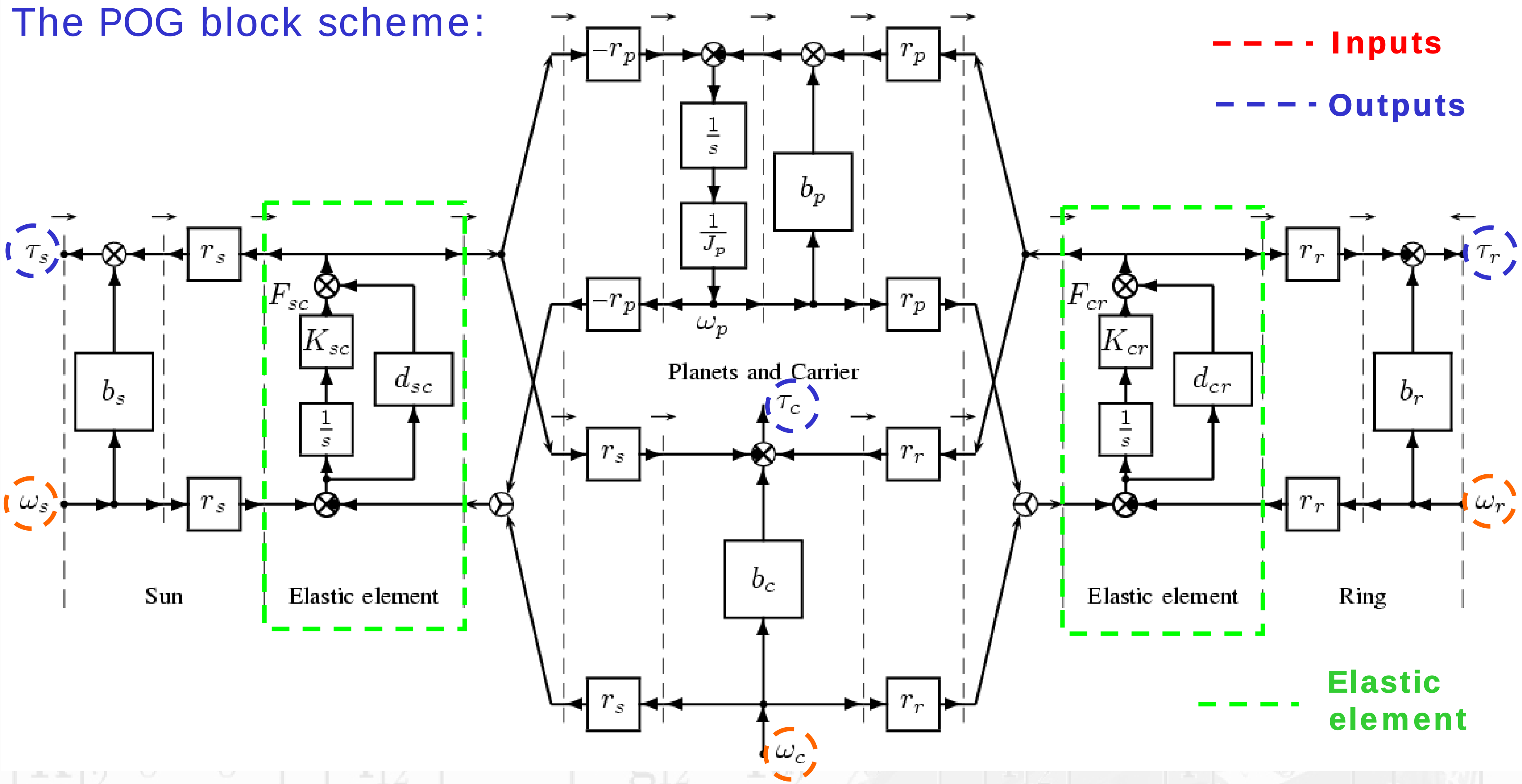
Inputs

Outputs

$$\underbrace{\begin{bmatrix} \tau_s \\ \tau_c \\ \tau_r \end{bmatrix}}_{\tilde{\mathbf{y}}} = \underbrace{\begin{bmatrix} r_s & r_s d_{sc} r_p & 0 \\ -r_s & -r_s d_{sc} r_p + r_r d_{cr} r_p & r_r \\ 0 & -r_r d_{cr} r_p & -r_r \end{bmatrix}}_{\tilde{\mathbf{C}}_e} \underbrace{\begin{bmatrix} F_{sc} \\ \omega_p \\ F_{cr} \end{bmatrix}}_{\tilde{\mathbf{x}}} + \underbrace{\begin{bmatrix} b_s + r_s^2 d_{sc} & -r_s^2 d_{sc} & 0 \\ -r_s^2 d_{sc} & b_c + r_s^2 d_{sc} + r_r^2 d_{cr} & -r_r^2 d_{cr} \\ 0 & -r_r^2 d_{cr} & b_r + r_r^2 d_{cr} \end{bmatrix}}_{\tilde{\mathbf{D}}_e} \underbrace{\begin{bmatrix} \omega_s \\ \omega_c \\ \omega_r \end{bmatrix}}_{\tilde{\mathbf{u}}}$$

Reduced and inverted elastic model

The POG block scheme:



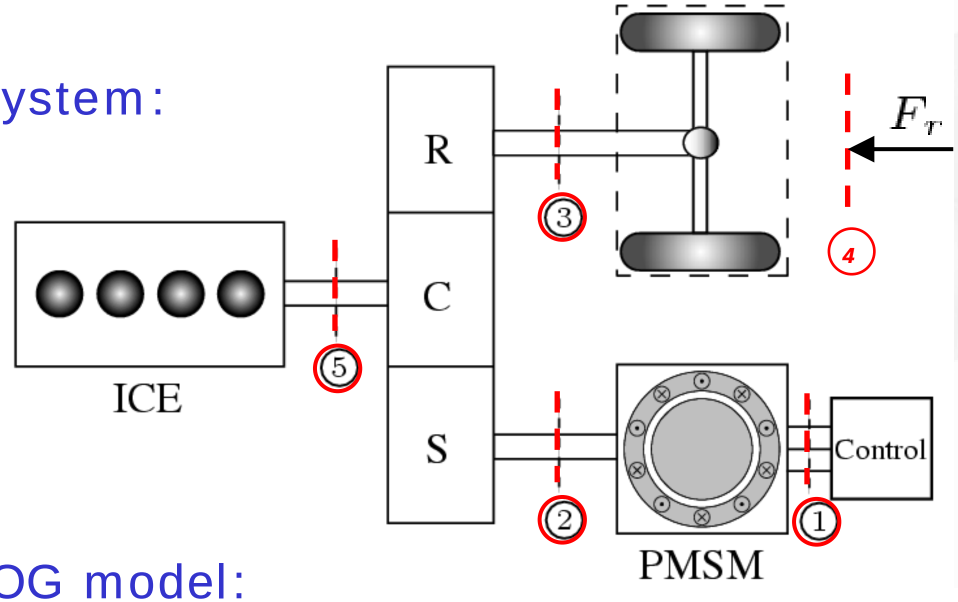
A dissipative reduced system is obtained when $J_s = J_c = J_r = 0$ and $K_{sc} = K_{sc} = \infty$:

$$r_s\,\omega_s - (r_r + r_s)\,\omega_c + r_r\,\omega_r = 0$$

POG model of Hybrid Automotive Systems

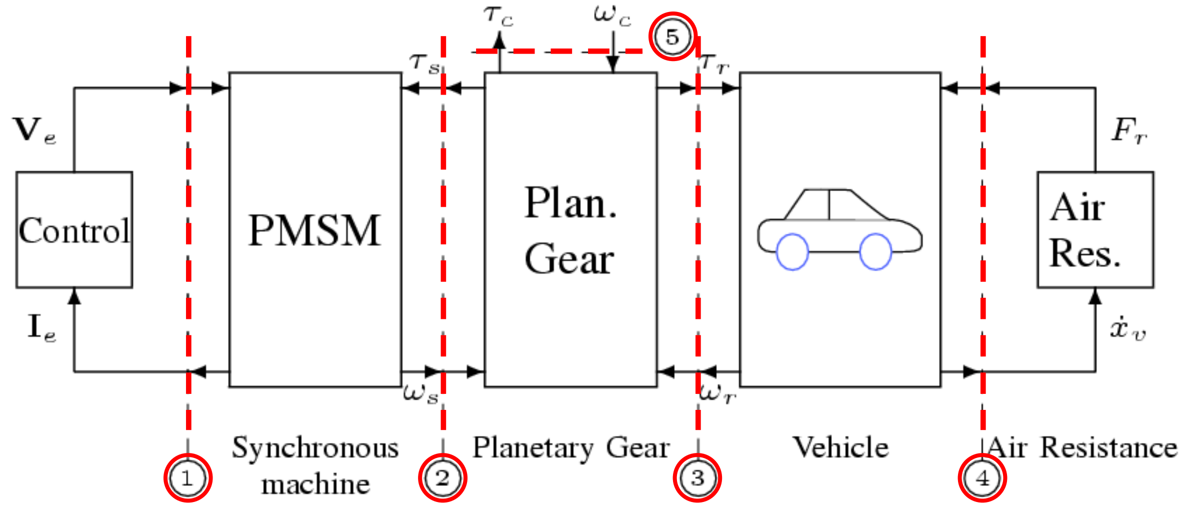
A parallel Hybrid Automotive System:
- ICE
- PMSM
- vehicle

The system:



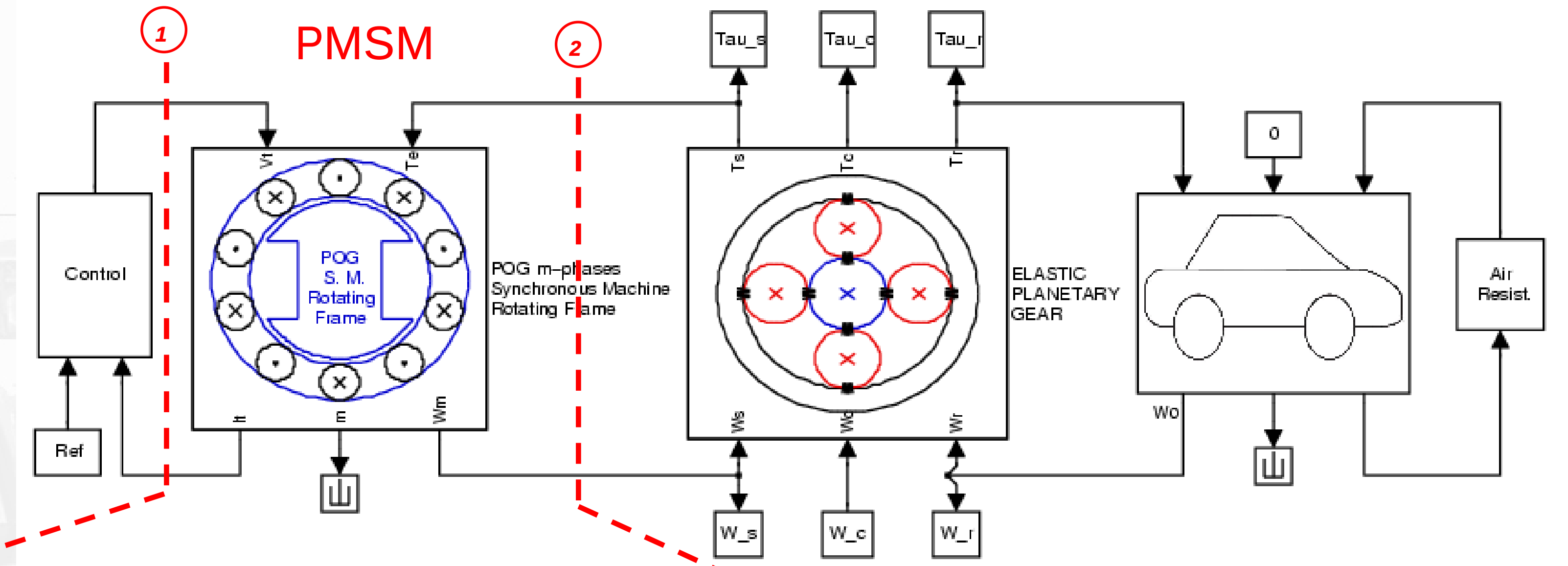
With the POG technique *it is easy to connect the subsystems* because it is based on the use of the "dashed" power sections!

The POG model:

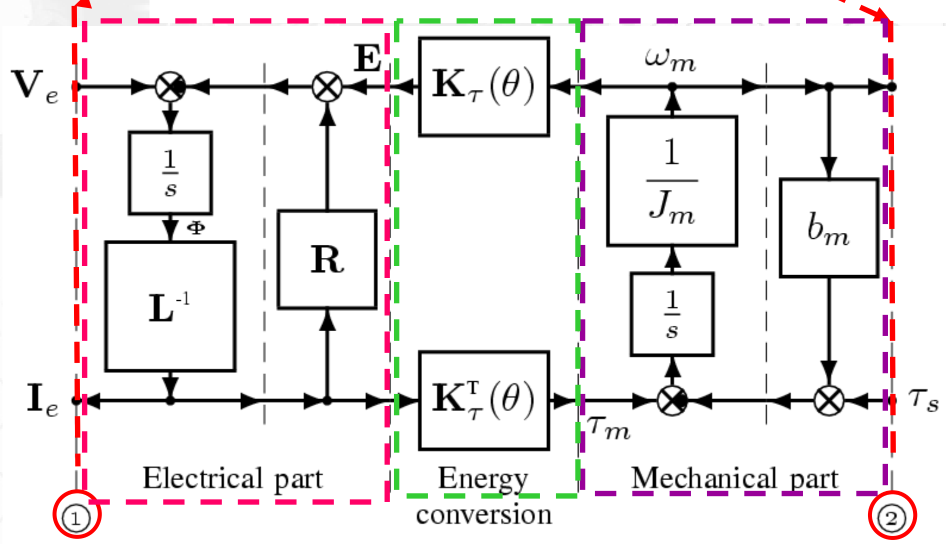


Hybrid Automotive Systems: POG model

The Simulink block scheme:



POG model of PMSM:



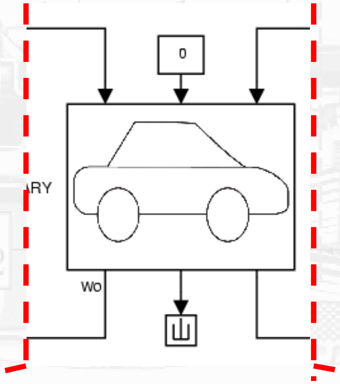
PMSM: Permanent Magnet Syncronous Motor

-- Electrical part

-- Mechanical part

-- Energy conversion

Hybrid Automotive Systems: POG model



POG model of the vehicle:

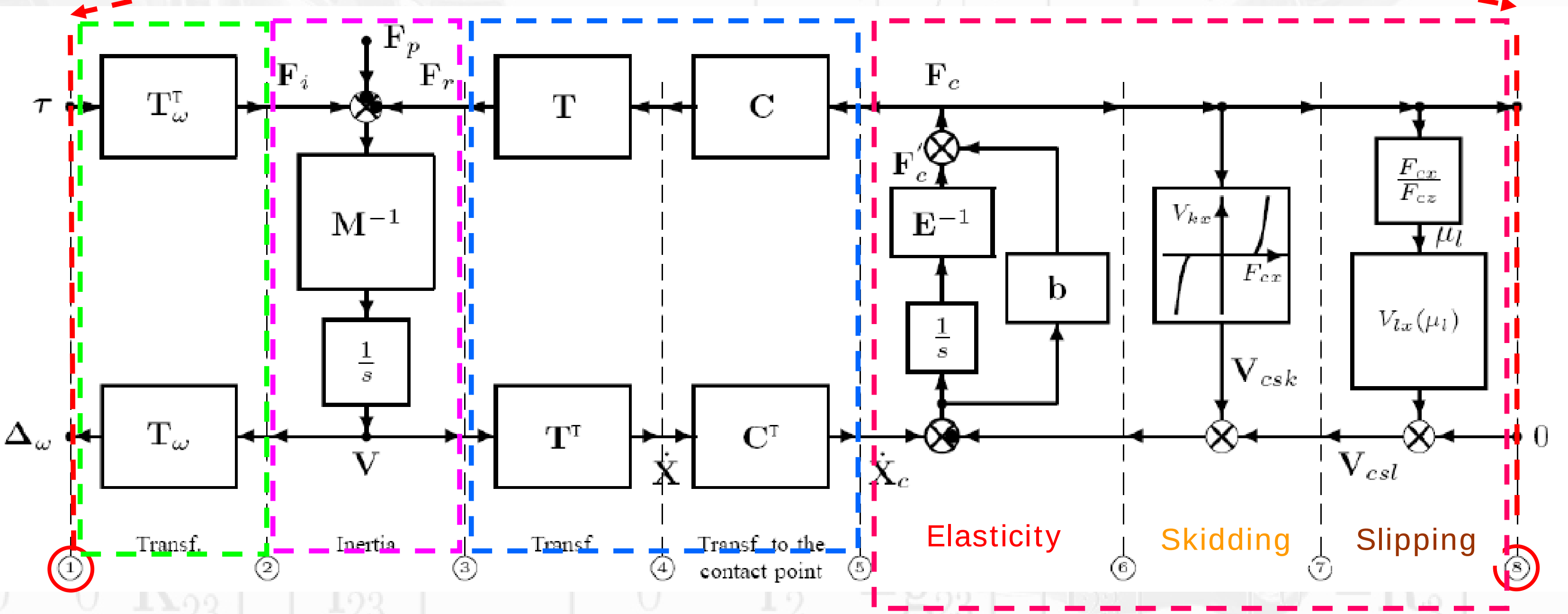
- -- From transmission to center of mass
- -- Mechanical dynamics
- -- From center of mass to road contact

-- Tire-road contact:

Elasticity

Skitting

Slipping



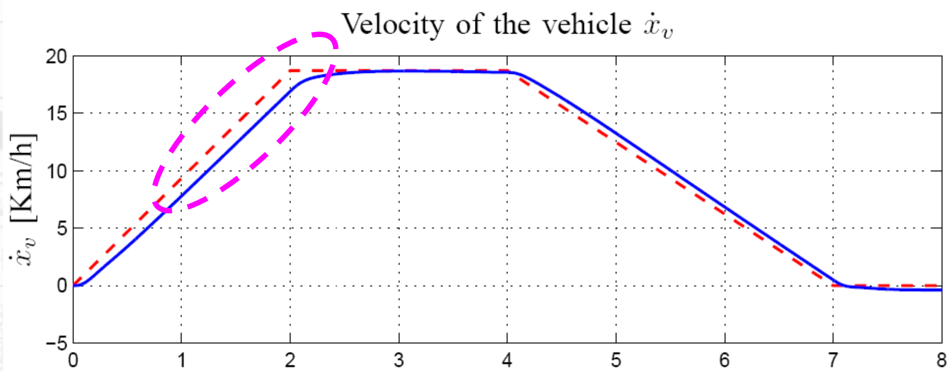
Hybrid Automotive Systems: simulations

"Start and stop" simulation results obtained with the ICE switched off:

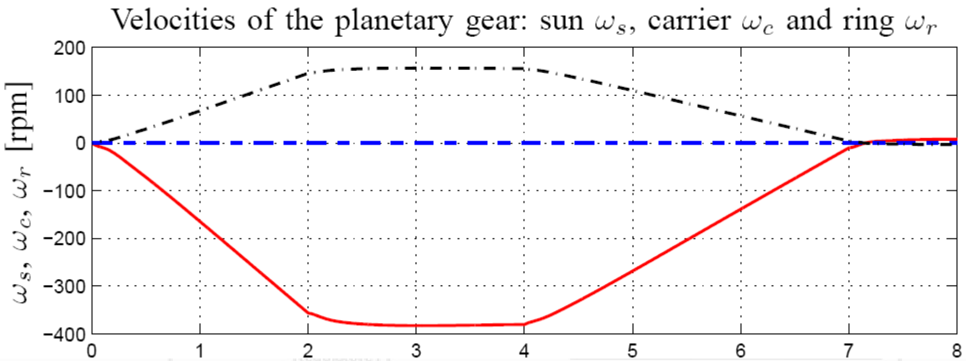
$$\omega_c = 0$$

The small tracking delay is due to the elastic horizontal slipping of the tires on the ground.

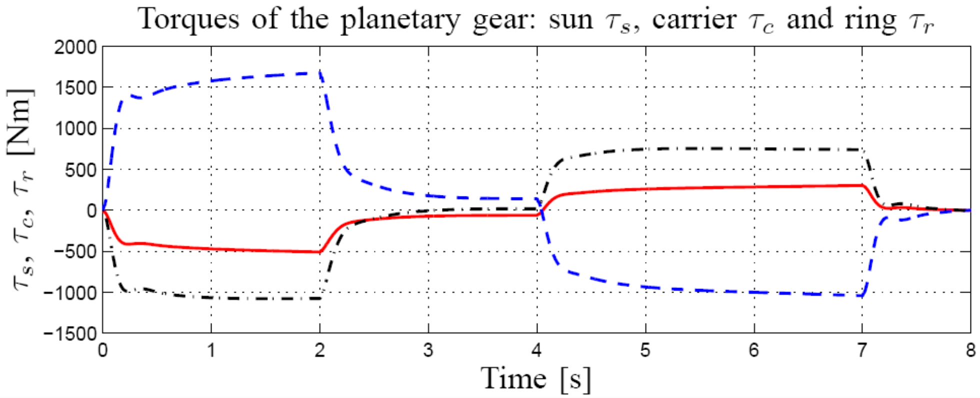
Vehicle velocity



Planetary gear velocities



Planetary gear torques



Hybrid Automotive Systems: simulations

A PMSM multiphase electric motor has been used:

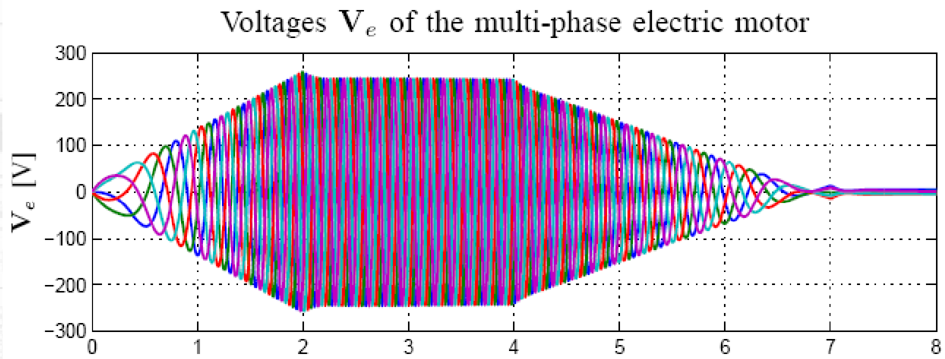
$$m = 5$$

It can be useful to improve the robustness and safety of the system

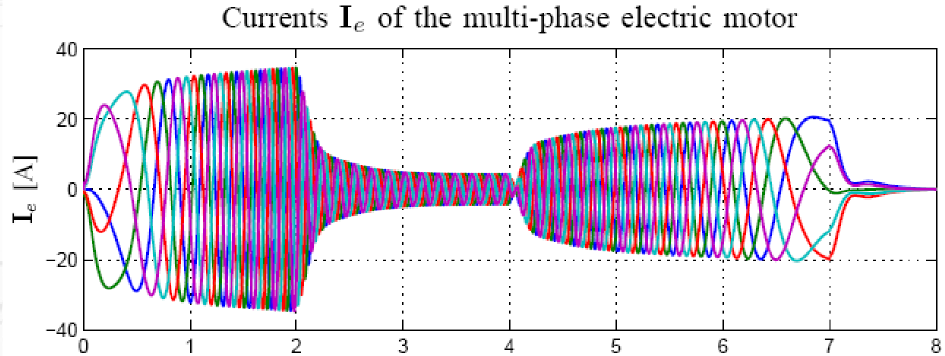
With POG models clearly shows the "power flows" within the system:

- motor
- generator

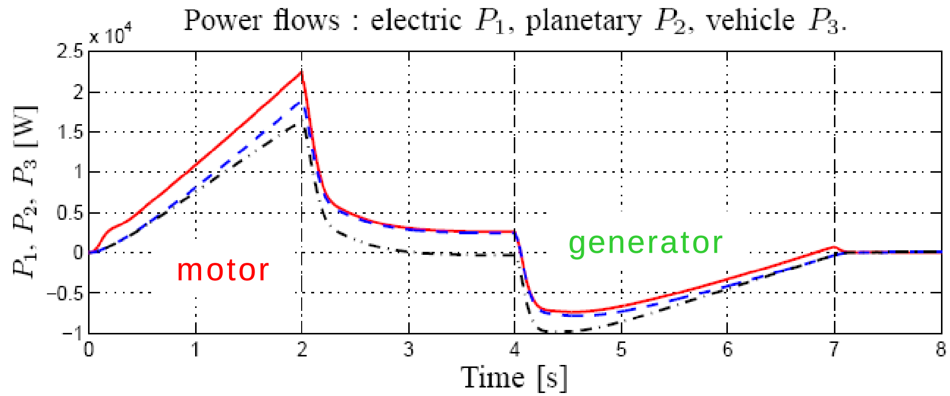
Electric motor voltages



Electric motor currents



Power flows



Conclusions

1) POG modeling technique:
   - a planetary gear has been modeled with different level of details (reduction rules),
   - an hybrid automotive power system (endothermic engine, multi-phase synchronous motor, planetary gear and vehicle dynamics) has been modeled.

2) The POG technique is easy to use, easy to understand and suitable for modeling complex dynamic systems such as Hybrid Autmotive Systems