Abstract

In the paper, the dynamic model of a gearbox system inserted in a car transmission system is taken into account and a simple control strategy for controlling the transmitted torque during a gear shift operation is presented. All the main components of the gearbox system have been modelled in details by using the graphical modelling technique named Power-Oriented Graph. Simulation results show the usefulness of the presented model and the effectiveness of the proposed control strategy.

1 Introduction

The availability of a good dynamic model for the gearbox and for the whole transmission system is an important element for designing good strategies for controlling the dynamic behavior of a car during all the critical working situations. A simplified representation of a car transmission system is shown in Fig. 1. It is composed by: the engine, the clutch, the torsional dumper-spring, the gearbox, the differential, the axles and the wheels. The clutch and the gearbox are “Variable Dynamic Dimension Systems” (VDDS): the dynamic models of these systems change their dimension during the normal functioning. A detailed dynamic model of the clutch has been presented in (Zanasi et al., 2001b), while a first simplified model of the complete transmission system has been given in (Zanasi et al., 2001a). In this paper, a more detailed dynamic model of the gearbox system is given. This model can be used to accurately simulate the transmission system in these conditions: fast starts and gear shift operations.

A schematic representation of a six gearbox system is shown in Fig. 2. All the components of the system (toothed-wheels, shafts, synchronizers, etc.) are modelled, at high level of detail, by using a graphical modelling technique named Power-Oriented Graph.
Figure 1: Simplified representation of a car transmission system.

Figure 2: Schematic representation of a six gearbox system.

Graph (POG), see Section 1.1. In particular, the model describes in detail the synchronizer, an important element that acts during the gear shift operations modulating the coulomb friction between two coupled mechanical elements. Due to the presence of this coulomb friction, the simulation of the synchronizer is critical. In the paper, a particularly efficient state space transformation is proposed to speed-up and improve the simulation of this critical element.

The paper is organized as follows: the basic concepts of the POG graphical technique are reported in Section 1.1. The dynamic models of all the components of the gearbox system are described in Section 2. The identification of the model parameters is done in Section 3. A simple control strategy for controlling the transmitted torque during a gear shift operation and simulation results are presented in Section 4.

1.1 Power-Oriented Graphs: basic concepts

The “Power-Oriented Graphs” are “signal flow graphs” combined with a particular “modular” structure essentially based on the two blocks shown in Fig. 3. The basic characteristic of this modular structure is the direct correspondence between pairs of system variables and real power flows: the product of the two variables involved in each dashed line of the graph has the physical meaning of “power flowing through the section”. The two basic blocks shown in Fig. 3 are named “elaboration block” and
“connection block”. There is no restriction on the choice of variables $x$ and $y$, other than the fact that the inner product $\langle x, y \rangle = x^T y$ must have the physical meaning of a “power”. The elaboration and connection blocks are suitable for representing both scalar and vectorial systems. In the vectorial case, $G(s)$ and $K$ are matrices: $G(s)$ is always square, $K$ can also be rectangular. While the elaboration block can store and dissipate energy (i.e. springs, masses and dampers), the connection block can only “transform” the energy, that is, transform the system variables from one type of energy-field to another (i.e. any type of gear reduction). For a more detailed description of the POG graphical technique, please refer to (Zanasi, 1991) and (R. Zanasi, 1994).

2 Dynamic Model of the Gearbox System

The Power-Oriented Graph model of the gearbox system of Fig. 2 is shown in Fig. 4: $C_p$, $\omega_p$, $J_p$ and $b_p$ are, respectively, the external torque, the velocity, the inertial moment and the angular viscous friction coefficient of the primary shaft; $C_s$, $\omega_s$, $J_s$ and $b_s$ are the analogous variables for the secondary shaft; $\omega_r = [\omega_1, \omega_2, \ldots, \omega_0]$ is the angular velocity vector of the idle toothed-wheels; $\theta_p = [\theta_1, \theta_2, 0, \ldots, 0]$ and $\theta_s = [0, 0, \theta_3, \ldots, \theta_0]$ are the angular displacement vectors of the primary and secondary toothed-wheels. The matrices $J_r$, $B_r$, $K_p$, $K_s$, $Y_p$, $Y_s$, $G_p$ and $G_s$ which are present in the POG model of Fig. 4 are defined as follows:

$$
J_r = \text{diag}([J_1, J_2, J_3, J_4, J_5, J_6]), \quad K_p = \text{diag}([0, 0, K_3, K_3, K_3, K_0]), \\
B_r = \text{diag}([b_1, b_2, b_3, b_4, b_5, b_6]), \quad K_s = \text{diag}([K_1, K_2, 0, 0, 0, 0]), \\
Y_p = \text{diag}([\tau_1, 1, 1, 1, 1]), \quad G_p = \text{diag}([f_1(\theta_1), f_2(\theta_2), 0, 0, 0, 0]), \\
Y_s = \text{diag}([1, 1, \tau_3, \tau_4, \tau_5, \tau_6]), \quad G_s = \text{diag}([0, 0, f_3(\theta_3), f_4(\theta_4), f_5(\theta_5), f_6(\theta_6)])
$$

Parameters $J_i$ and $b_i$, for $i \in \{1, 2, \ldots, 6\}$, are the inertial momenta and the viscous friction coefficients of the idle toothed-wheels connecting the primary and the secondary shafts. Parameter $K_i$ is the maximum amplitude of the coulomb friction torque present between the $i$-th idle toothed-wheel and the shaft on which it is mounted. The nonlinear function $f_i(\theta_i)$ is defined in the last part of Fig. 3. The transformation ratios $\tau_i$ are defined as $\tau_i = r_{pi}/r_{pi}$ where $r_{pi}$ and $r_{si}$ are the medium radii of the $i$-th toothed-wheels mounted, respectively, on the primary and secondary shafts; $\theta_i$ is the relative angular displacement of the $i$-th couple of toothed-wheels; $h_i$ and $g_i$ are, respectively, the angular stiffness and the angular semi-amplitude of the backlash of the toothed-wheels. From the POG dynamic model of Fig. 4 one can directly obtain the following system
of differential equations describing the inertial part of the gearbox system:

\[
\begin{bmatrix}
J_p & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & J_1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & J_2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & J_3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & J_4 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & J_5 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & J_6 \\
\end{bmatrix}
\begin{bmatrix}
\dot{\omega}_p \\
\dot{\omega}_1 \\
\dot{\omega}_2 \\
\dot{\omega}_3 \\
\dot{\omega}_4 \\
\dot{\omega}_5 \\
\dot{\omega}_6 \\
\end{bmatrix}
= 
\begin{bmatrix}
C_{mp} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & C_{m1} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & C_{m2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & C_{m3} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & C_{m4} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & C_{m5} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & C_{m6} \\
\end{bmatrix}
\begin{bmatrix}
0 \\
K_1 \\
0 \\
-\omega_3 \\
0 \\
0 \\
-\omega_6 \\
\end{bmatrix}
\begin{bmatrix}
\omega_1 - \omega_p \\
\omega_2 - \omega_1 \\
\omega_3 - \omega_2 \\
\omega_4 - \omega_3 \\
\omega_5 - \omega_4 \\
\omega_6 - \omega_5 \\
\end{bmatrix}
\]

where \( J = \text{diag}([J_p, J_1, J_2, J_3, J_4, J_5, J_6]) \), \( \vec{\omega} = [\omega_p, \omega_1, \omega_2, \ldots, \omega_6, \omega_6] \) and \( K = K_p + K_s \). The torque vector \( C \) and matrix \( D \) are defined as follows:

\[
C = 
\begin{bmatrix}
C_{mp} & C_{m1} & C_{m2} & C_{m3} & C_{m4} & C_{m5} & C_{m6} \\
\end{bmatrix}
= 
\begin{bmatrix}
C_p - \sum_{i=1}^{n} f_i(\theta_i) - b_p \omega_p \\
\tau_1 f_1(\theta_1) - b_1 \omega_1 \\
\tau_2 f_2(\theta_2) - b_2 \omega_2 \\
-\tau_3 f_3(\theta_3) - b_3 \omega_3 \\
-\tau_4 f_4(\theta_4) - b_4 \omega_4 \\
-\tau_5 f_5(\theta_5) - b_5 \omega_5 \\
-\tau_6 f_6(\theta_6) - b_6 \omega_6 \\
\end{bmatrix}
\]

\[
D = 
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \\
1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
\end{bmatrix}
\]
where the displacements $\theta_i$ are the solutions of the differential equations: $\dot{\theta}_i = \omega_p - \tau_i \omega_i$, for $i \in \{1, 2\}$, and $\dot{\theta}_i = \omega_i - \tau_i \omega_i$, for $i \in \{3, 4, 5, 6\}$. The dynamic model of the gearbox system can be rewritten in a compact form as:

$$J \ddot{\omega} = C - D^T K \text{sgn}(D \omega)$$  \hspace{1cm} (1)

Note that, due to the presence of the discontinuous term $K \text{sgn}()$, system (1) is a "Variable Dynamic Dimension System".

### 2.1 State Space Transformation

The dynamic model presented in the previous section can be usefully used for simulating the gear shift operations and in particular the action of the synchronizers. During the engagement of the $j$-th gear this element can be easily simulated by increasing the coulomb friction coefficient $K_j$ and decreasing the other coefficients $K_i$, for $i \neq j$: after a transient, the $j$-th idle toothed-wheel becomes locked on the corresponding shaft. During the gear shift operation the terms $K_i \text{sgn}(\omega_i - \omega_{p_0})$ can switch at a very high frequency and can strongly slow down the efficiency and the precision of the simulation algorithm. To cope with this problem we propose to use the following congruent state space transformation $\dot{\omega} = T\dot{z}$, where $\dot{z}$ is the new state vector and $T = \begin{bmatrix} \mathbf{T}_1 & \mathbf{T}_2 \end{bmatrix}$ is defined as follows:

$$T = \begin{bmatrix}
1 & 0 & \frac{\Delta_1}{\Delta_1} & \frac{\Delta_2}{\Delta_1} & \frac{\Delta_3}{\Delta_1} & \frac{\Delta_4}{\Delta_1} & \frac{\Delta_5}{\Delta_1} & \frac{\Delta_6}{\Delta_1} & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\Delta_1}{\Delta_3} & \frac{\Delta_1}{\Delta_3} \\
0 & 1 & -\frac{\Delta_1}{\Delta_2} & 0 & 0 & 0 & 0 & 0 & -\frac{\Delta_1}{\Delta_3} & \frac{\Delta_1}{\Delta_3} \\
1 & 0 & \frac{\Delta_3}{\Delta_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & \frac{\Delta_1}{\Delta_2} & \frac{\Delta_3}{\Delta_1} & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & \frac{\Delta_1}{\Delta_2} & \frac{\Delta_3}{\Delta_1} & -\frac{\Delta_1}{\Delta_2} & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\Delta_1}{\Delta_2} & -\frac{\Delta_1}{\Delta_2}
\end{bmatrix}$$

where $\Delta_1 = J_p + J_3 + J_4 + J_5 + J_6$ and $\Delta_2 = J_4 + J_5 + J_2$. The physical meaning of the components of the new state vector $\dot{z}$ becomes evident by inverting relation $\dot{\omega} = T\dot{z}$:

$$\dot{z} = T^{-1} \dot{\omega} = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_8 \end{bmatrix} = \begin{bmatrix} \frac{\Delta_1}{\Delta_1} & \frac{\Delta_1}{\Delta_2} & \frac{\Delta_1}{\Delta_2} & \frac{\Delta_1}{\Delta_2} \\
\omega_p - \omega_3 \\ \omega_p - \omega_4 \\ \omega_p - \omega_5 \\ \omega_p - \omega_6 \\ \omega_p - \omega_7 \\ \omega_p - \omega_8 \end{bmatrix}$$

where $\Delta_1 = J_p + J_3 + J_4 + J_5 + J_6$, and $\Delta_2 = J_4 + J_5 + J_2$. The two variables $z_1$ and $z_2$ are mean weighted velocities describing the Main Dynamics of the system. The other variables $z_3, z_4, \ldots, z_8$, (the Relative Dynamics) are the relative velocities of the toothed-wheels with respect to the primary and secondary shafts. By using the transformation $\dot{\omega} = T\dot{z}$, system (1) simplifies as follows:

$$J_T \ddot{z} = C_T - D_T^T K \text{sgn}(D_T z)$$  \hspace{1cm} (2)
where matrices \( J_T = T'JT = \text{diag}[J_{T1}, J_{T2}], D_T = DT \) and vector \( C_T = T'C \) have the following structure:

\[
J_T = \begin{bmatrix}
\Delta_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \Delta_2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{J_1(J_1 - J_2)}{\Delta_1} & \frac{J_2 J_1}{\Delta_1} & -\frac{J_2 J_2}{\Delta_1} & -\frac{J_1 J_2}{\Delta_1} & 0 & 0 \\
0 & 0 & \frac{J_1(J_1 + J_3)}{\Delta_1} & \frac{J_3 J_1}{\Delta_1} & -\frac{J_2 J_3}{\Delta_1} & -\frac{J_1 J_3}{\Delta_1} & 0 & 0 \\
0 & 0 & \frac{J_1(J_1 + J_4)}{\Delta_1} & \frac{J_4 J_1}{\Delta_1} & -\frac{J_2 J_4}{\Delta_1} & -\frac{J_1 J_4}{\Delta_1} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{J_1(J_2 + J_3)}{\Delta_2} & \frac{J_2 J_3}{\Delta_2} \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{J_1(J_2 + J_4)}{\Delta_2} & \frac{J_2 J_4}{\Delta_2} \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{J_1(J_3 + J_4)}{\Delta_2} & \frac{J_3 J_4}{\Delta_2} \\
\end{bmatrix}
\]

and

\[
D_T = \begin{bmatrix} 0 & D_{T2} \end{bmatrix}, \quad C_T = \begin{bmatrix} C_{T1} \\
C_{T2} \\
\vdots \\
C_{T8} \end{bmatrix}
\]

Note that \( D_T z = [z_7, z_8, z_3, z_4, z_5, z_6]^T \). By inverting matrix \( J_T \), system (2) can be rewritten as \( \dot{z} = J_T^{-1}[C_T - D_T K \text{sgn}(D_T z)] \), that is in the following form:

\[
\begin{bmatrix}
\dot{z}_1 \\
\dot{z}_2 \\
\dot{z}_3 \\
\dot{z}_4 \\
\dot{z}_5 \\
\dot{z}_6 \\
\dot{z}_7 \\
\dot{z}_8 \\
\end{bmatrix} =
\begin{bmatrix}
J_{T1}^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & J_{T2}^{-1} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
C_{T1} - K_3 \text{sgn}(z_3) \\
C_{T2} - K_4 \text{sgn}(z_4) \\
C_{T3} - K_5 \text{sgn}(z_5) \\
C_{T4} - K_6 \text{sgn}(z_6) \\
C_{T5} - K_7 \text{sgn}(z_7) \\
C_{T6} - K_8 \text{sgn}(z_8) \\
\end{bmatrix}
\]

(3)

The proposed state space transformation decomposes the original system in two independent parallel systems: the variables \( z_1 \) and \( z_2 \) do not influence and are not influenced by the variables \( z_3, z_4 \ldots z_8 \). A POG graphical representation of system (3) is shown in Fig. 5. The second part of system (3) is a six order Variable Dynamic Dimension System. To correctly simulate this subsystem, a proper simulation algorithm has been designed. The basic structure of this algorithm is the following, see (Zanasi et al., 2001b):

\[
\text{loop} \quad (z_j = 0)_{j=3,4,5,6,7,8} \\
\tau_j = J_T^{-1} [C_{T1} - K_B \text{sgn}(z_{i+1})] + C_{Tj} \\
\text{if} |\tau_j| < K_Tj \text{then} \\
(\dot{z}_j)_{k+1} = 0 \\
\text{else} \\
(\dot{z}_j)_{k+1} = J_{Tc} [C_{T1} - K_Ti \text{sgn}(z_i)] \\
\text{end if} \\
\text{end loop}
\]
where $J_{T_{j}}$ is the matrix obtained by selecting the rows and the columns of matrix $J^{-1}_{T_{j}}$ corresponding to the sliding variables $z_{j} = 0$; $J_{T_{1}}$ is the sub-matrix of matrix $J^{-1}_{T_{2}}$ obtained by selecting the rows corresponding to the sliding variables $z_{j} = 0$ and the columns corresponding the other variables $z_{j} \neq 0$; $J_{T_{c}}$ is the sub-matrix of matrix $J^{-1}_{T_{2}}$ obtained by selecting the rows corresponding to the variables $z_{j} \neq 0$; $K_{i_{1}} = K_{i}$ for $i = 3, \ldots, 6$, $K_{T_{1}} = K_{1}$ and $K_{T_{8}} = K_{2}$.

### 3 Simulation results

The simulation results reported in this section have been obtained by using the dynamic model of the gearbox system presented in the previous section and the dynamic model of the car transmission system given in (Zanasi et al., 2001a). The parameters used in simulation have been identified on the basis of real experimental data. The simulation results obtained during a start and a gear shift operation are shown in Fig. 6 and Fig. 7. In particular, in Fig. 6 the real angular velocities $\omega_{cr}$, $\omega_{pr}$ and $\omega_{sr}$ of the engine, primary and secondary shafts, are compared with the corresponding angular velocities $\omega_{e}$, $\omega_{p}$ and $\omega_{s}$ obtained in simulation. In Fig. 7 are reported the time behaviors of the coulomb friction coefficients $K_{1}$ and $K_{2}$ (divided by a factor 10), and the torques $C_{p}$ and $C_{s}$ acting on the primary and secondary shafts. In Fig. 6 and 7 the gear shift operation is evidenced by dashed vertical lines and is divided in three phases:

**Phase I** The engine and the transmitted torques decrease to zero and the clutch is opened. This phase starts when the driver asks for a gear shift operation. During this phase the control system automatically generates the reference signal $\omega_{oby} = \tau_{2} \omega_{s}$ where $\tau_{2}$ is the transformation ratio of the new gear. This phase ends when the engine torque is zero and the clutch is completely opened.

**Phase II** The actual gear is disengaged and the new gear is selected and inserted. The synchronizer is active during this phase. The new gear starts to be inserted only when the primary shaft angular velocity $\omega_{p}$ reaches the value $\omega_{oby}$.

**Phase III** The clutch is gradually closed and the transmitted torque increases. This phase ends when clutch is completely closed, that is when the engine angular velocity $\omega_{e}$ becomes equal to the primary shaft angular velocity $\omega_{p}$.

For the driver comfort, it is important that in phases I and III the movement of the clutch is coordinated with the position of the accelerator pedal that gives torque at the
transmission system. In fact, if in phase I the clutch is opened before the transmitted torque is zero, the engine angular velocity \( \omega_e \) can increase too much. On the contrary, if the transmitted torque decreases too fast when the clutch is still closed, an oscillation can appear in the system. Similar behaviors can happen in phase III when the clutch is going to be closed and the engine torque starts to be increased. So, for improving the driver comfort, it is evident that during phases I and III the control action have to be particularly precise.

4 Gear Shift Control

For increasing both the speed/accuracy of the gear shift operation and the comfort of the driver, it is important to properly control the gear shift operation. The simulation results reported in this section have been obtained by using a gear shift control strategy based on the parallel control of the engine torque \( C_m \), see (Pettersson and Nielsen, 2000), and transmitted torque \( C_p \), see (Haj-Fraj and Pfeiffer, 1999). A block scheme of the control strategy used for controlling the gear shift operations is shown in Fig. 8.

This control scheme is based on the feedback of the torque \( C_p \) transmitted to the primary shaft. The main goal of the control is to generate a reference \( r_c \) for the clutch position that permits in each instant to transmit through the clutch a torque \( C_p \) equal to the difference between the torque \( C_m \) generated by the engine and an estimation \( b_c \) of the engine internal losses. The controller is composed by the sum of a proportional term \( FB \) and a feed forward term \( FF \):

\[
 r_c = \frac{K_p(C_p + b_c - C_r)}{FB} + \frac{f_F(C_r - b_c)}{FF}
\]
where $C_r$ is the torque reference for the engine and $f_T(r)$ is the inverse of the “position - transmitted torque” characteristics of the clutch. The simulation parameters are the same given in Section 3. The obtained simulation results are shown in Fig. 9: during the gear shift operation the time behaviors of the angular velocities $\omega_r$ and $\omega_p$ are much smoother than the ones shown in Fig. 6. These simulation results show the improvements that can obtained in a gear shift operation by using a torque sensor.

5 Conclusions

The paper addressed the problems of modelling and controlling a gearbox system. All the main components of the system have been considered: the inertia and the friction of the primary and secondary shafts; the inertia, the friction, the stiffness and the backlash of the internal toothed-wheels; the presence of the synchronizers. The Power-Oriented Graphs technique has been used both for modelling the system and for describing the “power” flowing through the transmission system. The parameters of the obtained model has been identified on the basis of experimental data. Finally, the tuned model has been used for testing a new control strategy for the gear shift operation mainly based on the feedback control of the transmitted torque. The performance of the proposed control strategy has been tested in simulation.
Figure 9: Time behaviors of the angular velocities $\omega_r$, $\omega_{obj}$ and $\omega_p$ obtained during a controlled gear shift operation. The variables are normalized.

References


