DYNAMIC MODELING AND CONTROL OF A CAR TRANSMISSION SYSTEM

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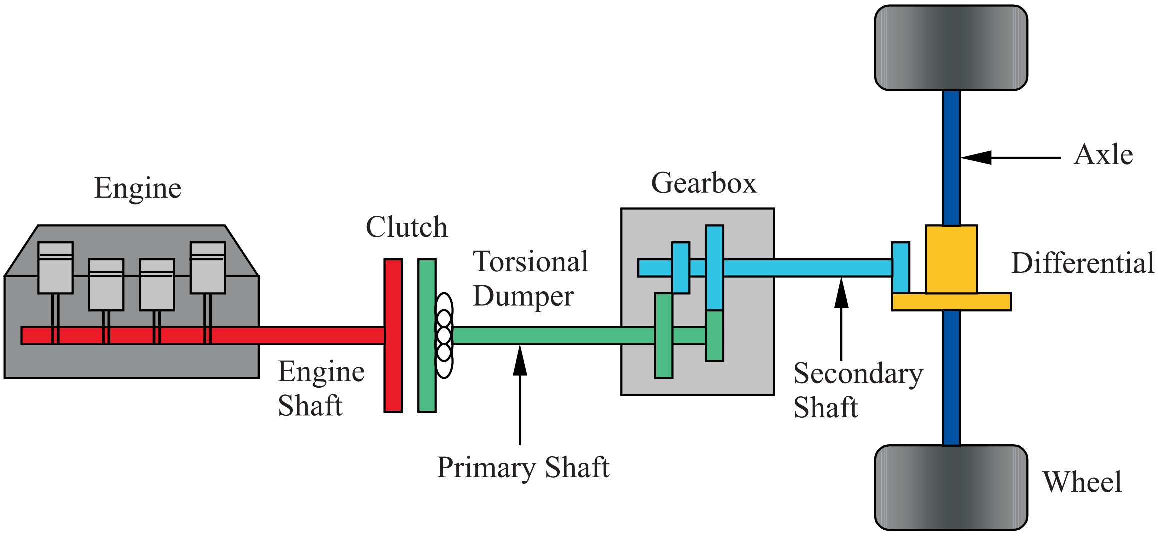
Summary:

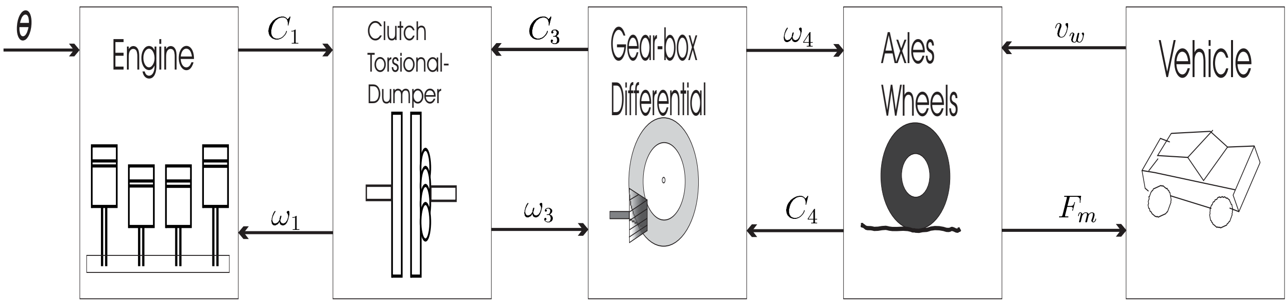
- *Car Transmission System;*
- *POG Modeling Technique;*
- *POG model of the vehicle dynamics;*
- *POG model of the axles and wheels;*
- *POG model of the gear-box and differential;*
- *The clutch with torsional dumper-spring;*
- *Simulation of a variable dynamic dimension system;*
- *Clutch control strategy;*
- *Conclusions.*

Car Transmission System

- Simplified representation of a car transmission system:



- Power-Oriented graphical representation of the transmission system:

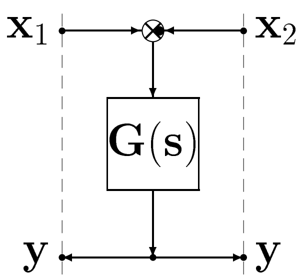


- A good dynamic model of the system can be obtained if the energy variables, whose product has the physical meaning of a power, are kept coupled.

- This is the basic idea of the two graphical modelling techniques: "Bond graphs" and "Power Oriented Graphs".

Power-Oriented Graphs (POG)

- Graphical modelling technique essentially based on two blocks.
- Elaboration block:

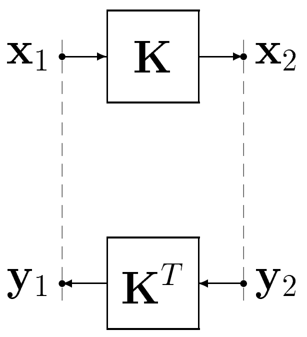


$$\mathbf{G}^{-1}(s) = \mathbf{M}s + \mathbf{R}$$

$\mathbf{P_d} = \mathbf{y}^T \mathbf{R}\, \mathbf{y}$: *Power dissipated*

$\mathbf{E_s} = \frac{1}{2}\mathbf{y}^T \mathbf{M}\, \mathbf{y}$: *Energy Stored*

- Connection Block :



Mono-dimensional case:
$y_1 = k\, y_2\,, \quad x_2 = k\, x_1$
$x_1\, y_1 = x_1\, k\, y_2 = x_2\, y_2$

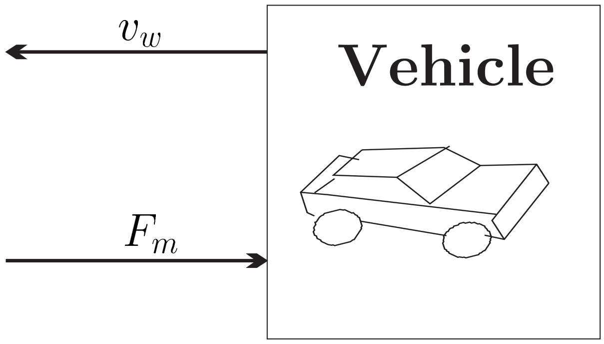
$\mathbf{P_d} = \mathbf{0}$: *Power dissipated*

$\mathbf{E_s} = \mathbf{0}$: *Energy Stored*

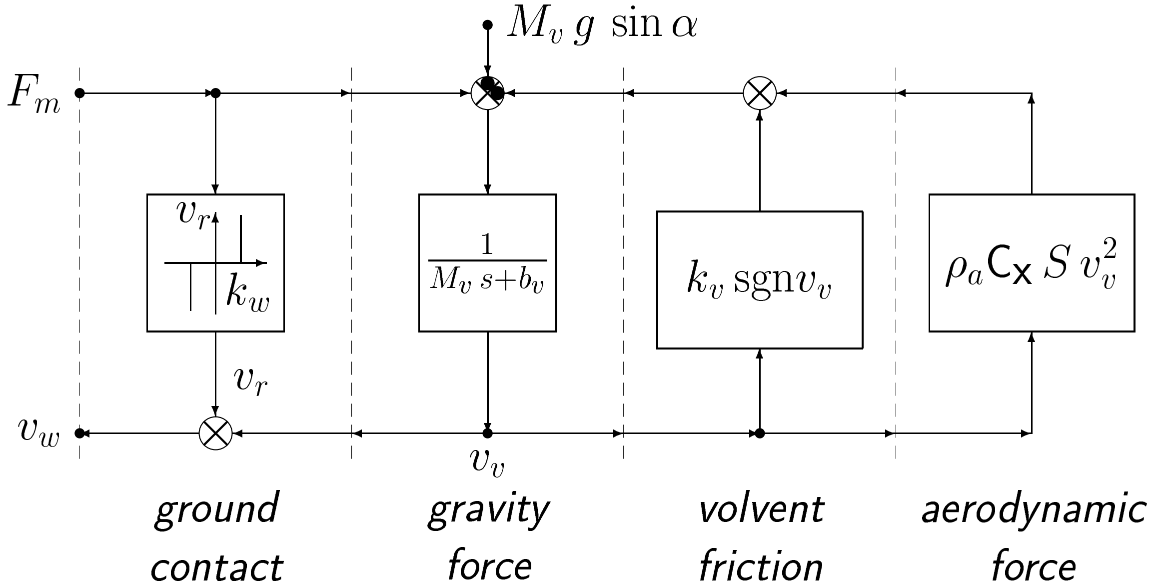
- The product of the two variables involved in each dashed line of the graph has the physical meaning of "power flowing through the section"

POG scheme of the vehicle dynamics

- Block representation



- POG dynamic model



F_m : force acting on the ground;

v_w : wheels velocity at the ground;

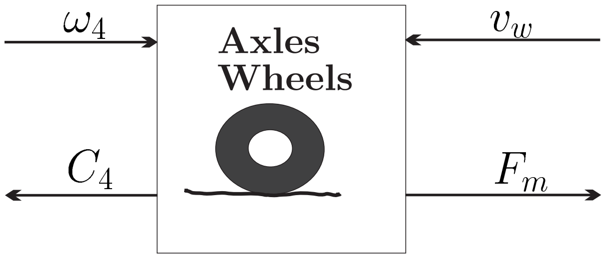
k_v : volvent friction coefficient;

α : is the slope of the ground;

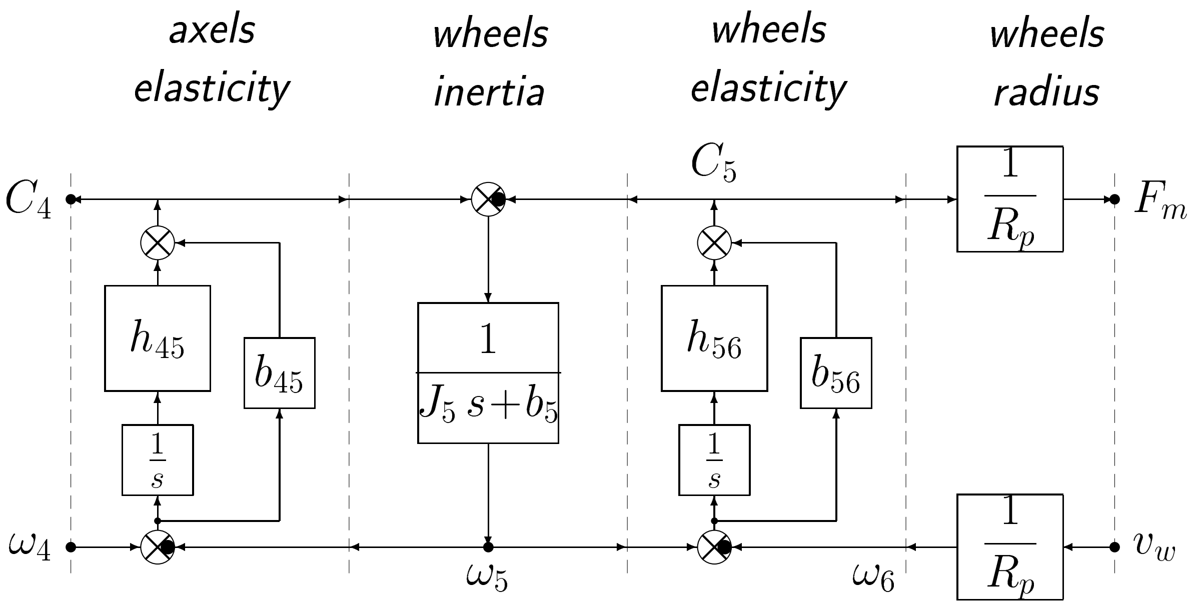
M_v, b_v : mass and friction coefficient of the vehicle

POG scheme of the axles and the wheels

- Block representation



- POG dynamic model



C_4, C_5 : axles and wheels torques;

h_{45}, h_{56} : stiffness coefficients of the axles and wheels;

b_{45}, b_{56} : internal dissipation coefficients of the axles and wheels;

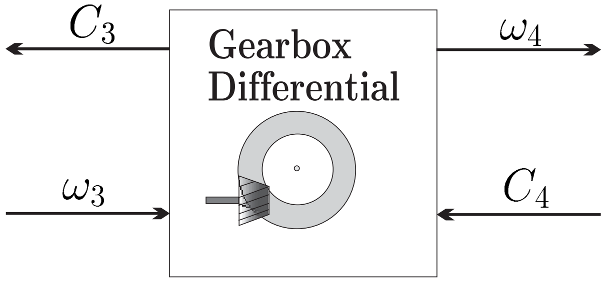
b_5 : viscous friction coefficient of the axles and wheels;

J_5 : inertia momentum of the axles and wheels;

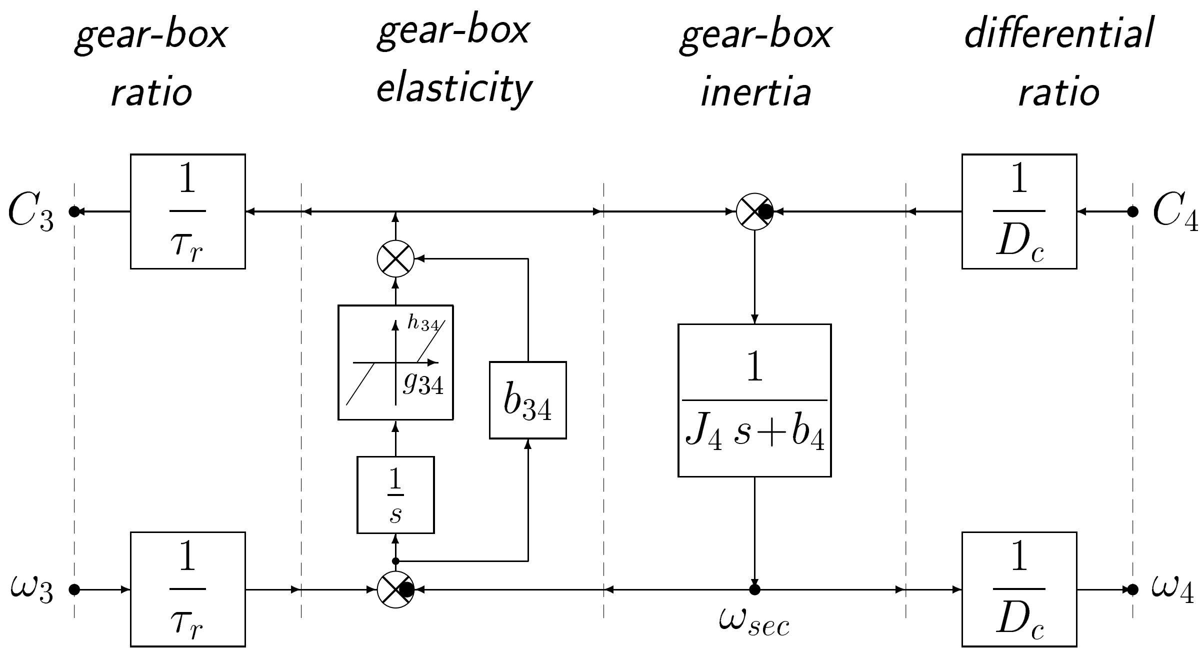
R_p : is the wheel radius.

POG block scheme of the gear-box and the differential

- Block representation



- POG dynamic model:

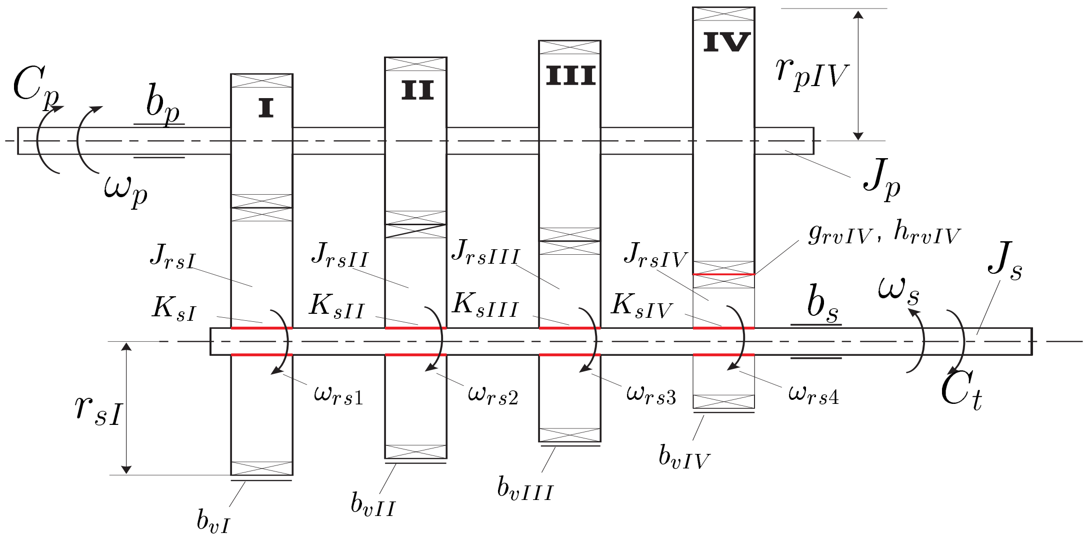


- Transmission ratios:

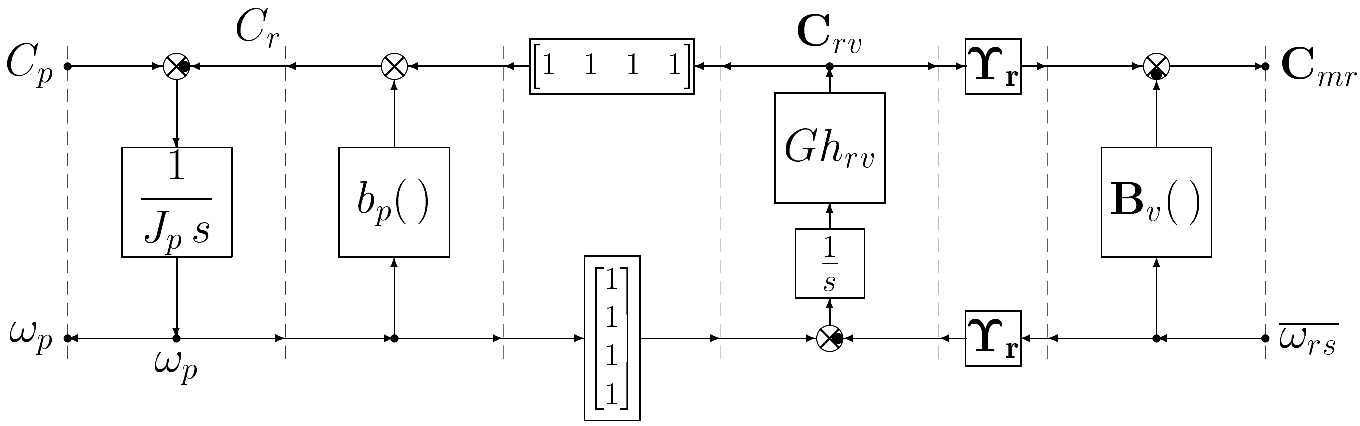
$$\tau_r = \frac{\omega_3}{\omega_{sec}} \qquad D_c = \frac{\omega_{sec}}{\omega_4}$$

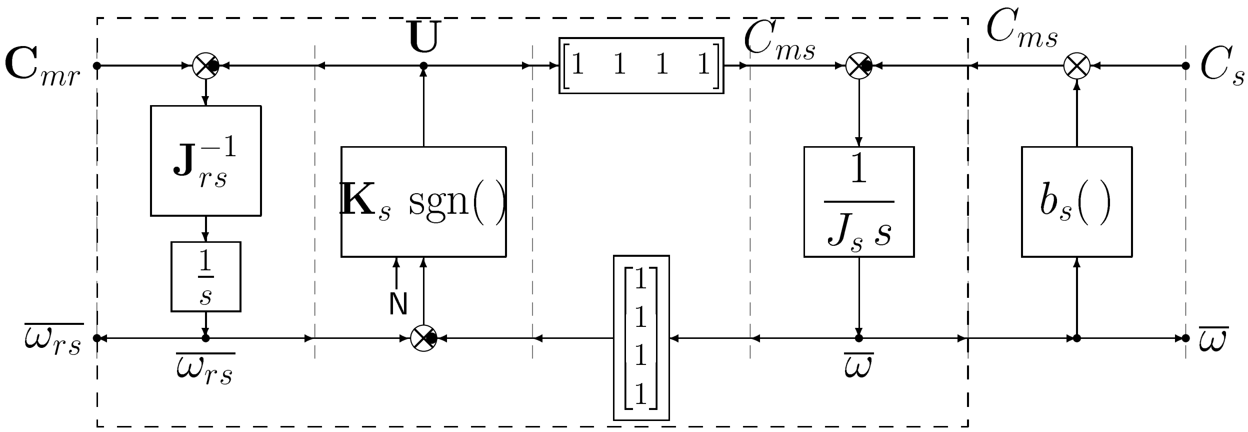
- More detailed models of both the gear-box and the differential can be used.

- A more detailed model of a 4-gear-box:



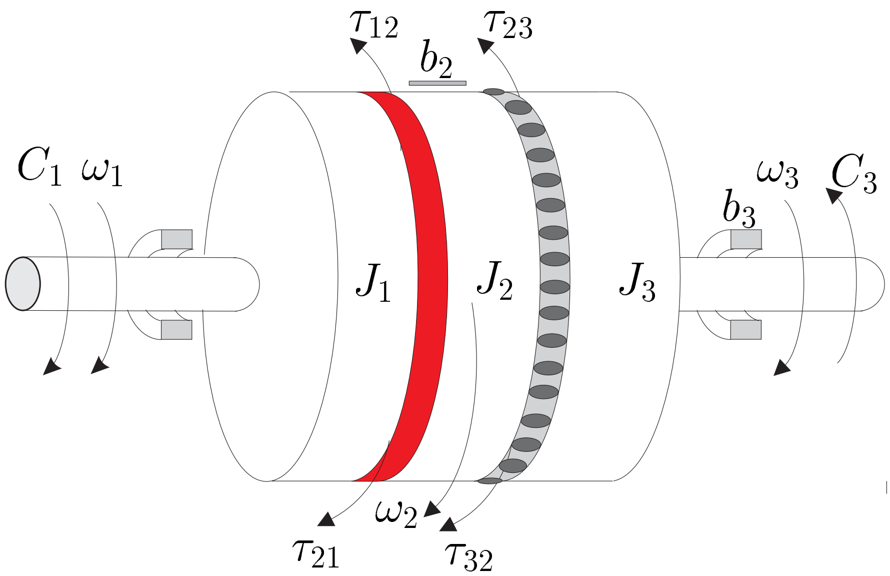
- POG scheme of a 4-gear-box



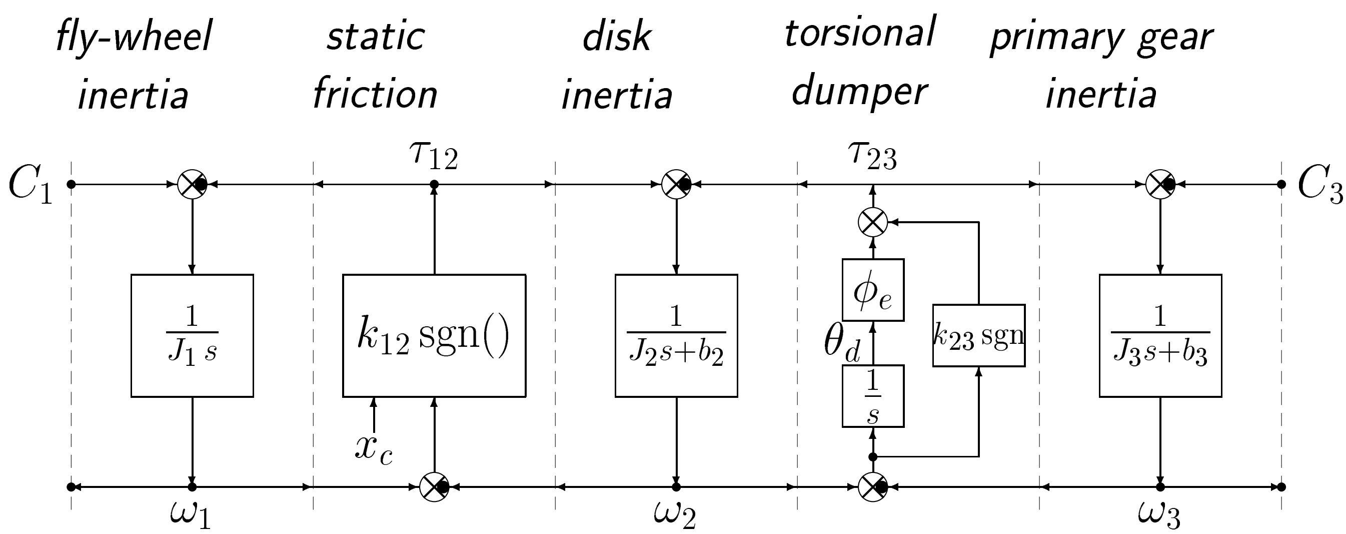


Clutch with torsional dumper-spring

- Schematic representation of the clutch with torsional dumper-spring



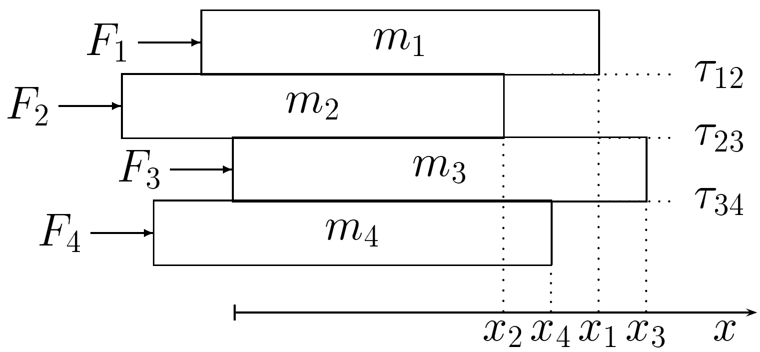
- POG graphical representation of the torsional dumper-spring:



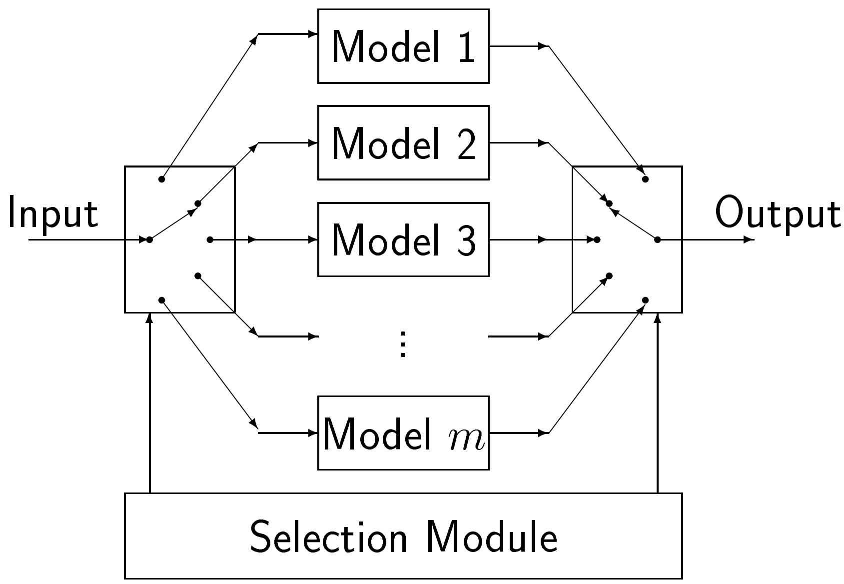
- Due to the presence of the two discontinuous functions $k_{12}\,\mathrm{sgn}()$ and $k_{23}\,\mathrm{sgn}()$, it is a variable dynamic dimension system;
- Simulation problems: a sliding mode arises in the system during the simulation

Variable dynamic dimension systems

- Example:



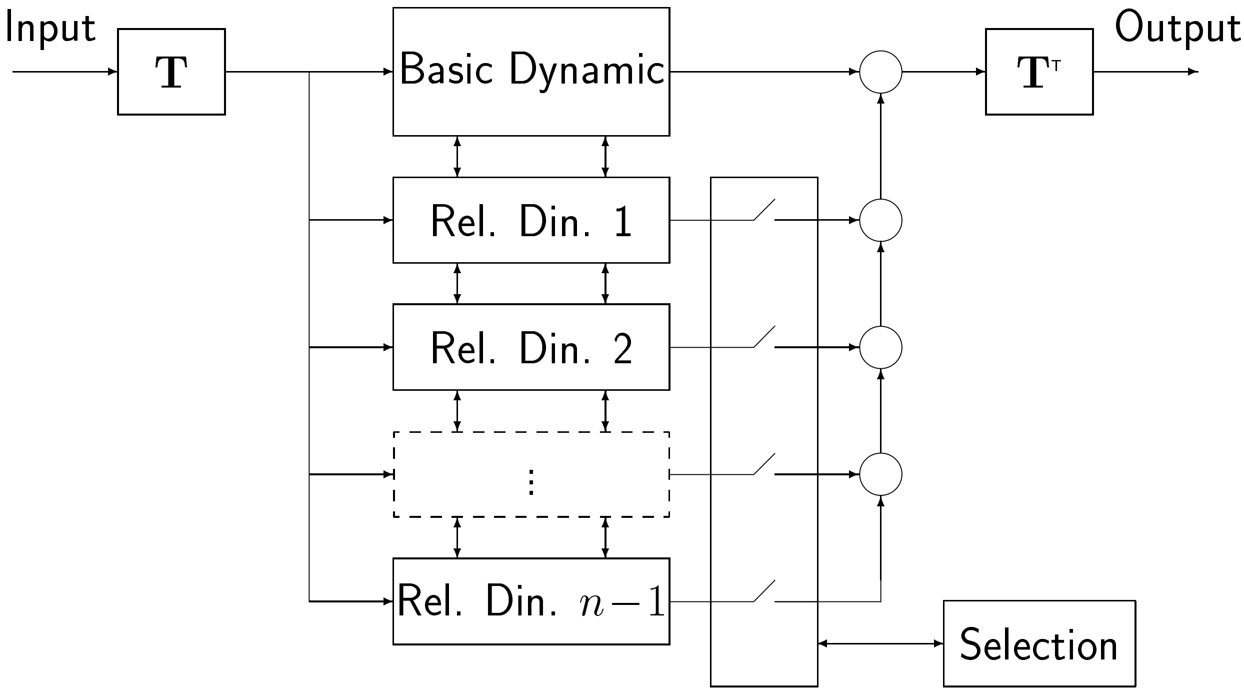
- Typical scheme used for simulating variable dynamic dimension systems:



- The number m of different models increases exponentially with the number n of masses: $m = 2^{n-1}$.
- The selection logic is complex;
- When the system switches from a model to another, the updating of the initial conditions of the new model is required;
- **Heavy simulative structure**.

The proposed simulation scheme

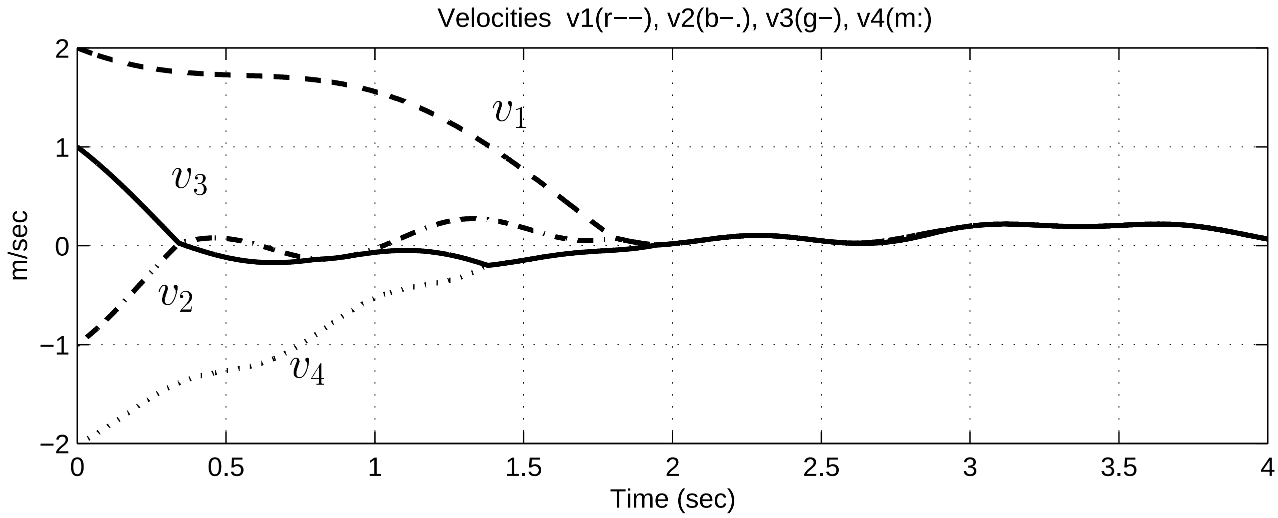
- By using a proper “congruent” state space transformation $\mathbf{T}$ the system can be transformed as follows:



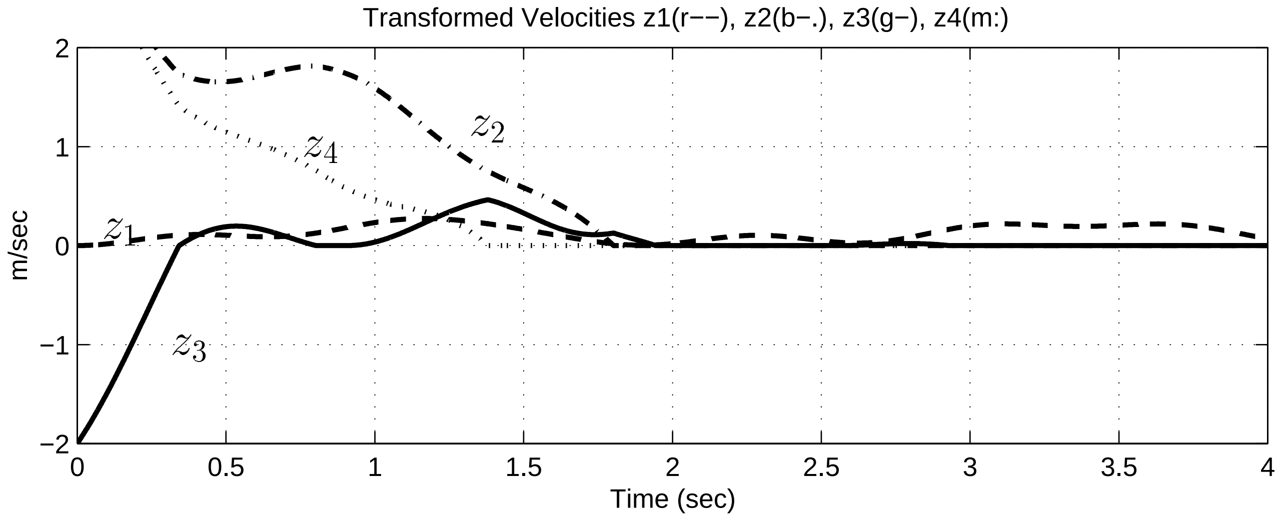
- The “*Main dynamics*” is kept separated from the “*Relative dynamics*”;
- In simulation, when a relative dynamics is zero, it is disconnected;
- Advantages:
  - Only the “transformed dynamic model” is used in simulation;
  - The selection logic is easier;
  - The initial conditions have not to be reset during the simulation;
- A proper simulation algorithm has been designed for the fast and correct simulation of the variable dynamic dimension systems

Example: simulation results

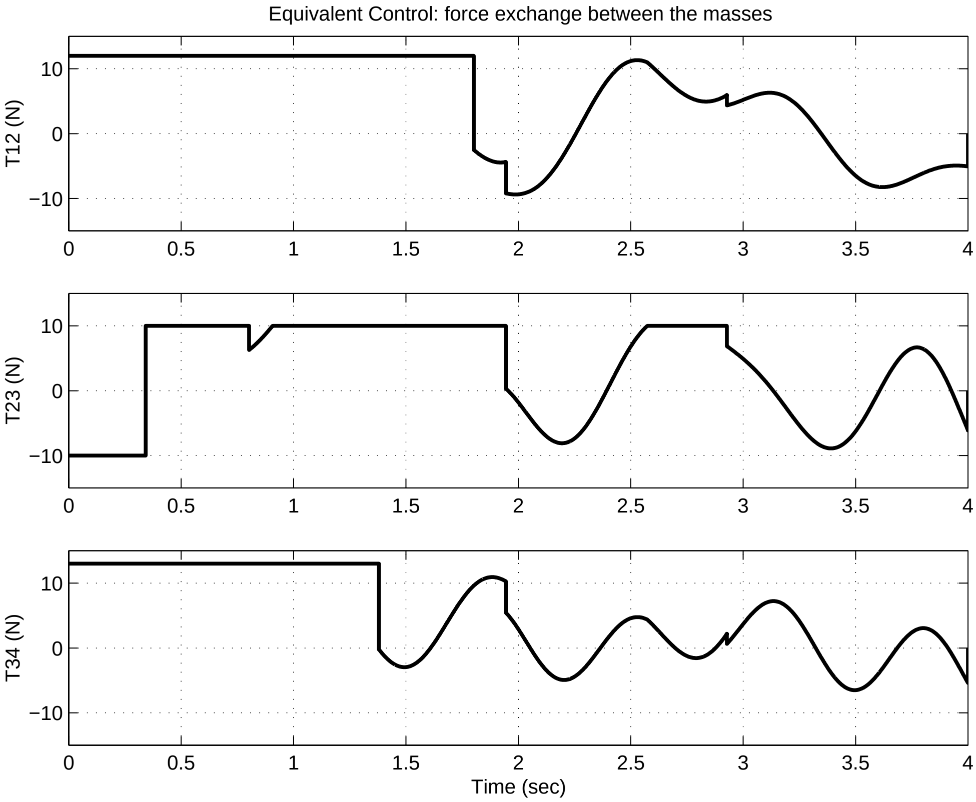
- The parameters used in simulation are: $m_1 = m_2 = m_3 = m_4 = 10$ Kg; $k_{12} = 12$ N, $k_{23} = 10$ N, $k_{34} = 13$ N and initial conditions $v(0) = [2, -1, 1, 2]$ m/s.
- The velocities:



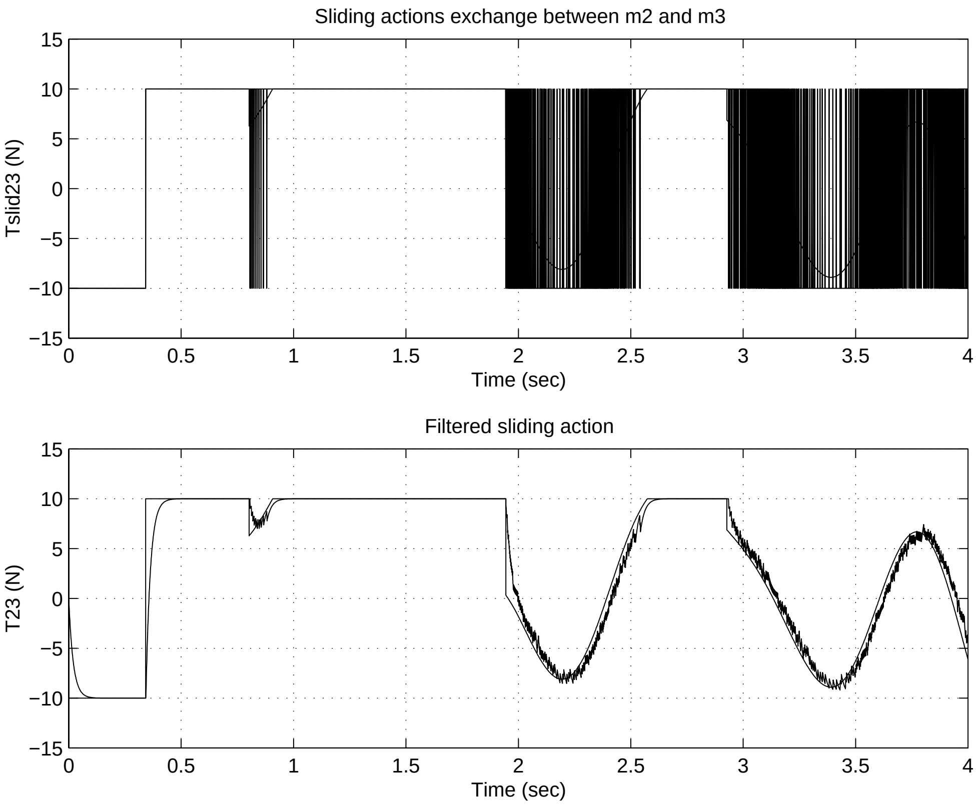
- The forces:



- Equivalent controls $\tau_{i-1,i}(t)$, $i \in \{1, 2, 3, 4\}$.:



- Equivalent control τ_{23} compared with the mean value of the sliding actions:



Clutch with torsional dumper-spring as a variable dynamic dimension system

- Differential equations of the clutch with torsional dumper-spring:

$$\begin{cases} J_1\,\dot{\omega}_1 &= C_1 - k_{12}\,\mathrm{sgn}(\omega_1-\omega_2) \\ J_2\,\dot{\omega}_2 &= -b_2\,\omega_2 + k_{12}\,\mathrm{sgn}(\omega_1-\omega_2) - k_{23}\,\mathrm{sgn}(\omega_2-\omega_3) - \phi_e(\theta_d) \\ J_3\,\dot{\omega}_3 &= -C_3 - b_3\,\omega_3 + k_{23}\,\mathrm{sgn}(\omega_2-\omega_3) + \phi_e(\theta_d) \\ \dot{\theta}_d &= \omega_2 - \omega_3 \end{cases}$$

- By using the following congruent space state transformation ($\Delta = J_1 + J_2 + J_3$):

$$\overline{\omega} = \mathbf{T}\,\mathbf{z},\quad \mathbf{T} = \left[\begin{array}{c|cc} 1 & \frac{J_2+J_3}{\Delta} & \frac{J_3}{\Delta} \\ 1 & -\frac{J_1}{\Delta} & \frac{J_3}{\Delta} \\ 1 & -\frac{J_1}{\Delta} & -\frac{J_1+J_2}{\Delta} \end{array}\right] \quad \leftrightarrow \quad \mathbf{z} = \mathbf{T}^{-1}\overline{\omega} = \begin{bmatrix} \frac{J_1\omega_1+J_2\omega_2+J_3\omega_3}{\Delta} \\ \omega_1-\omega_2 \\ \omega_2-\omega_3 \end{bmatrix}$$

the system simplifies as follows:

$$\begin{cases} \dot{z}_1 = \frac{C_{T1}}{\Delta} \\ \begin{bmatrix} \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} \frac{J_1+J_2}{J_1J_2} & -\frac{1}{J_2} \\ -\frac{1}{J_2} & \frac{J_2+J_3}{J_2J_3} \end{bmatrix} \left(\begin{bmatrix} C_{T2} \\ C_{T3} \end{bmatrix} - \begin{bmatrix} k_{12}\,\mathrm{sgn} z_2 \\ k_{23}\,\mathrm{sgn} z_3 \end{bmatrix} \right) \end{cases}$$

- Main dynamics:

$$\dot{z}_1 = \frac{C_{T1}}{\Delta}$$

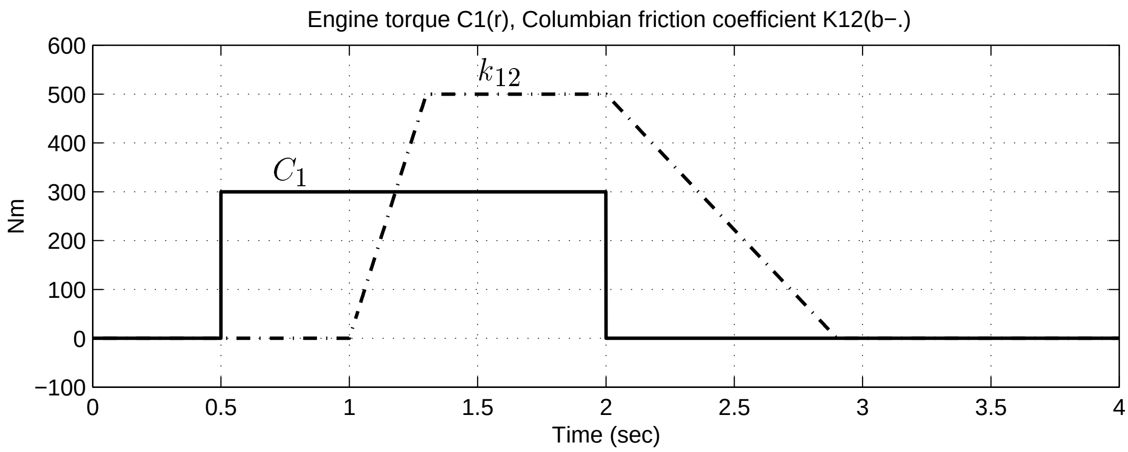
- Relative dynamics:

$$\begin{bmatrix} \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} \frac{J_1+J_2}{J_1J_2} & -\frac{1}{J_2} \\ -\frac{1}{J_2} & \frac{J_2+J_3}{J_2J_3} \end{bmatrix} \left(\begin{bmatrix} C_{T2} \\ C_{T3} \end{bmatrix} - \begin{bmatrix} k_{12}\,\mathrm{sgn} z_2 \\ k_{23}\,\mathrm{sgn} z_3 \end{bmatrix} \right)$$

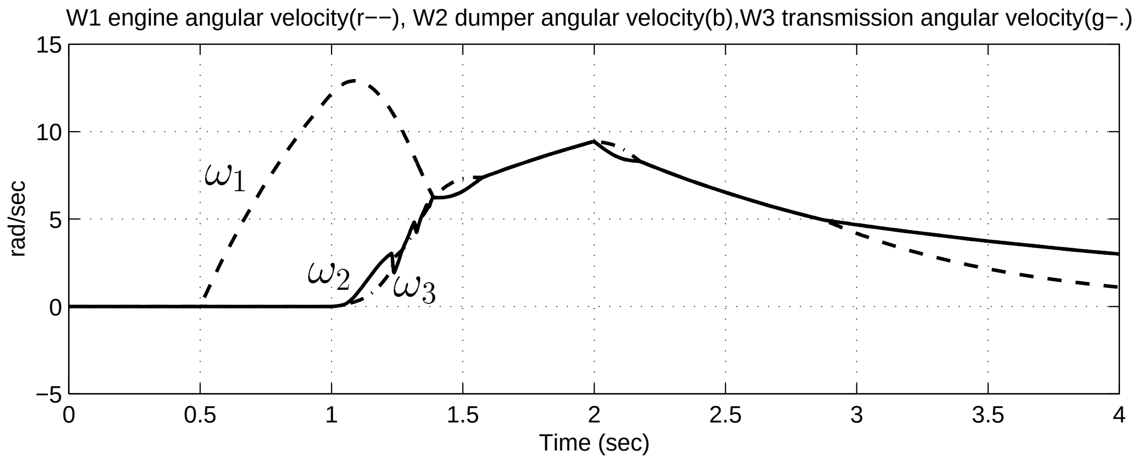
- The simulation algorithm compute the equivalent control of the sliding variables.

Clutch with torsional dumper-spring: simulation results

- The parameters used in simulation are: $J_1 = 9$ Kg m^2, $J_2 = 5$ Kg m^2, $J_3 = 13$ Kg m^2, $b_1 = 12$ N m s/rad, $b_2 = 1$ N m s/rad, $b_3 = 7$ N m s/rad, $k_{23} = 50$ N m and $C_3 = 0$.
- Time behaviours of the coulomb friction k_{12} and torque C_1:



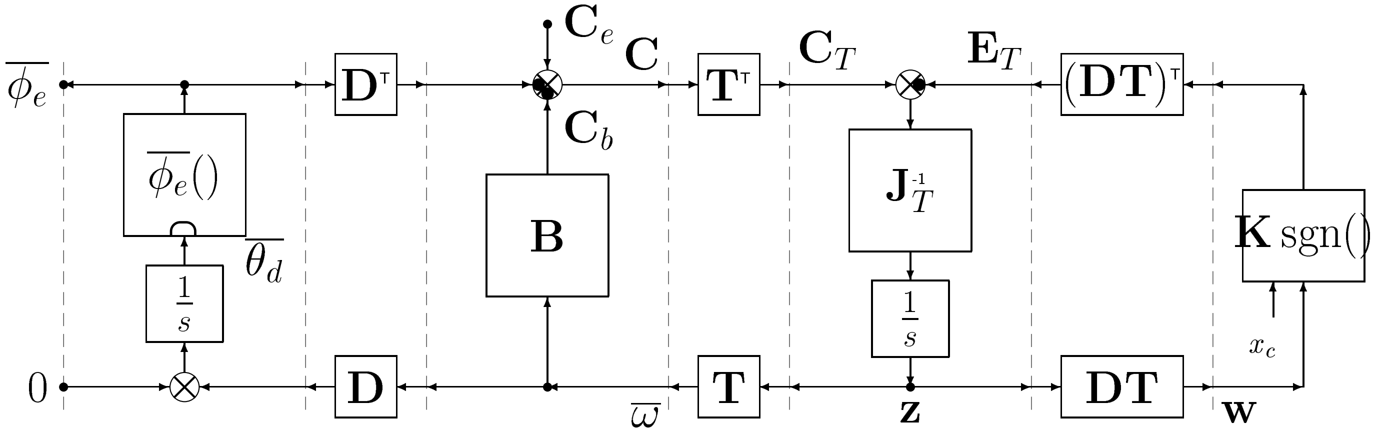
- Angular velocities $\omega_1(t)$, $\omega_2(t)$ and $\omega_3(t)$:



- Note that $\omega_3(t)$ is smoother that $\omega_2(t)$ due to the presence of the torsional damper-spring.

Clutch with torsional dumper-spring as a Variable structure system

- Transformed POG graphical representation of the clutch with torsional dumper-spring



- Algorithm:

loop $(z_j = 0)_{j=2,3,4}$

$\tau_{j-1,j} = M_j^{-1}\left[M_i\left(F_{i-1,i} - k_{i-1,i}\ \mathrm{sgn}(z_i)\right)\right] + F_{j-1,j}$

if $|\tau_{j-1,j}| < k_{j-1,j}$

$(\dot{z}_j)_{k+1} = 0$

else

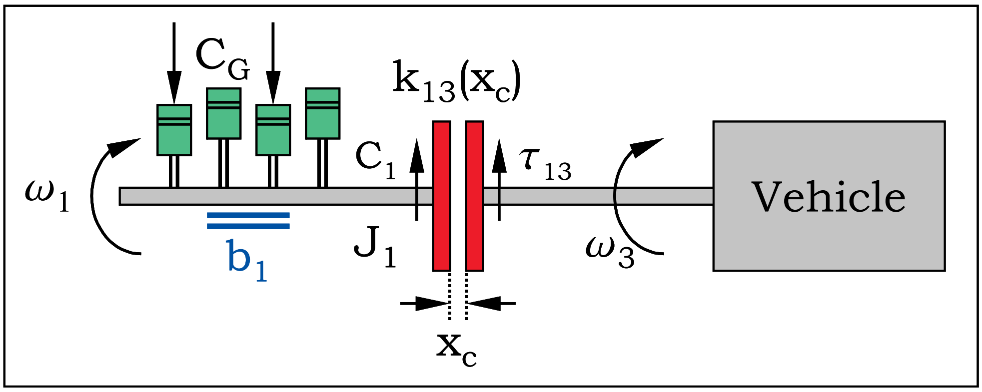
$(\dot{z}_j)_{k+1} = M_{ii}^{-1}\left(F_{i-1,i} - k_{i-1,i}\ \mathrm{sgn}(z_i)\right)$

end if

end loop

Engine and vehicle dynamic during a start

- Simplified scheme of the transmission system



- $C_1 = C_G - b_1(\omega_1)$ = engine supplied torque = torque generated - torque lost.
- $k_{13}(x_c)$ = torque transmitted through the clutch; k_{13} is a function of the clutch pedal position x_c.
- τ_{13} = torque transmitted to the vehicle

$$\tau_{13} = \begin{cases} C_1 & \text{if the clutch is engaged} \\ k_{13}(x_c)\,\mathrm{sgn}(\omega_1 - \omega_3) & \text{if the clutch is slipping} \end{cases}$$

- Control Problem: find a clutch control strategy such that:

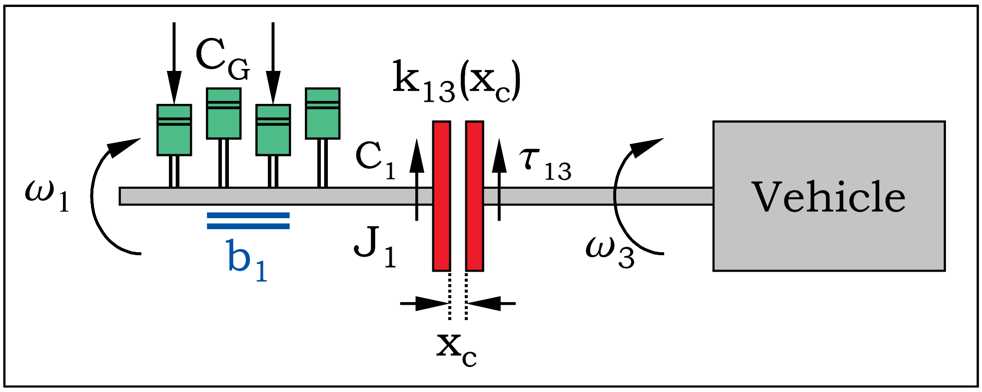
$$\tau_{13} \cong C_1$$

in every operative conditions, also when the clutch is slipping.

- The control system gives to the car the same dynamics both when the clutch is engaged and when the clutch is slipping.

Proposed clutch control strategy

- Simplified scheme of the transmission system



- When the clutch is slipping the dynamic of the engine shaft is:

$$J_1\dot{\omega}_1 = C_1 - k_{13}(x_c)\ \mathrm{sgn}(\omega_1 - \omega_3) = C_1 - \tau_{13}$$

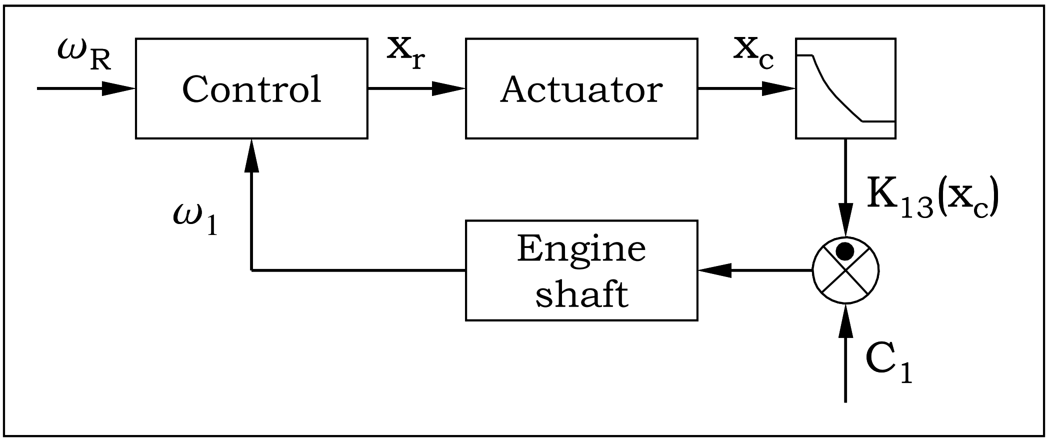
- The proposed clutch control strategy acts on k_{13} through x_c in order to force ω_1 to follow a slowly variable reference ω_r ($\dot{\omega}_r \cong 0$).
- If $\omega_1 \cong \omega_r \Rightarrow \dot{\omega}_1 \cong \dot{\omega}_r \cong 0$
- When $\dot{\omega}_1 \cong 0$ the dynamics of the engine shaft becomes:

$$J_1\dot{\omega}_1 = C_1 - \tau_{13} \cong 0 \Rightarrow C_1 \cong \tau_{13}$$

- The proposed control strategy solves the control problem. Moreover it prevents engine shut down or shaft speed spikes.

Clutch control scheme

- Clutch control scheme



- Engine shaft: dynamics when the clutch is slipping:

$$J_1\dot{\omega}_1 = C_1 - k_{13}(x_c)\ \operatorname{sgn}(\omega_1 - \omega_3) = C_1 - \tau_{13}$$

- Actuator: electro-hydraulic system that acts on the position x_c of the clutch pedal. Basic second order linear model:

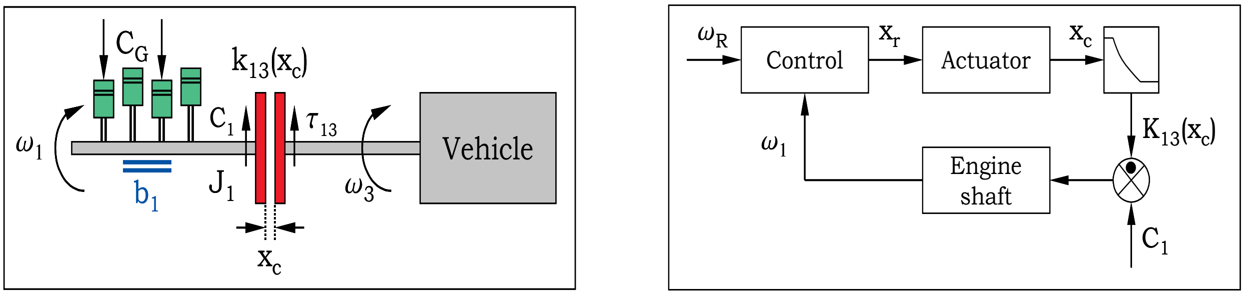
$$X_C(s) = \frac{\omega_a^2}{s^2 + 2\delta_a\omega_a s + \omega_a^2} X_R(s)$$

- Control law: the simplest one is

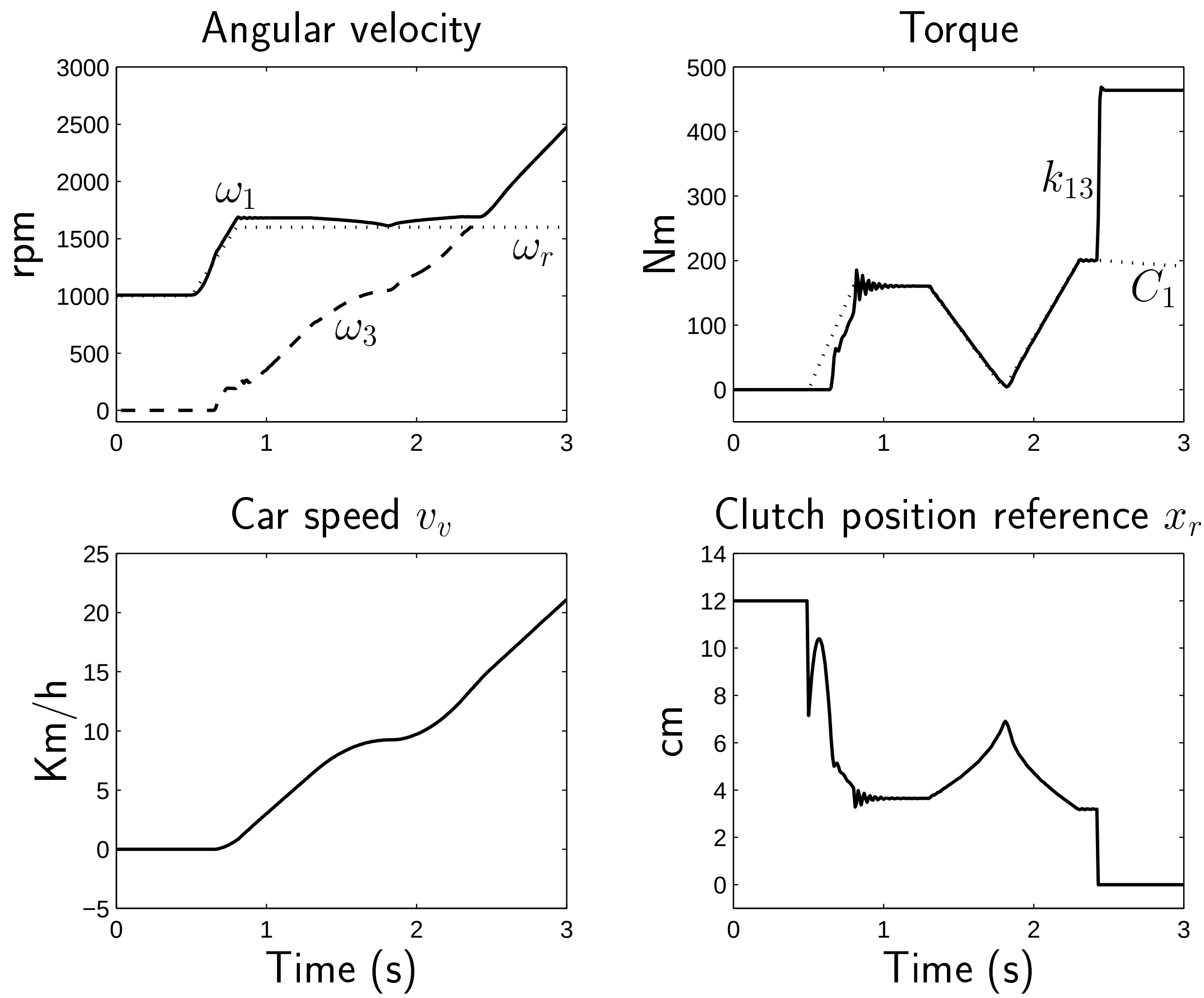
$$x_r = x_{r0} + k_r(\omega_r - \omega_1)\ \operatorname{sgn}(\omega_1 - \omega_3)$$

where x_r is the clutch pedal position reference that should be followed by the actuator output x_c.

- Scheme of a transmission and clutch control scheme:



- Simulation results



- The engine speed ω_1 follows the reference ω_r also when the supplied torque C_1 varies.

- The torque k_{13} transmitted to the vehicle through the clutch is almost equal to the engine supplied torque C_1 until the clutch in engaged.

- The proposed car transmission model allows to see the vehicle speed during the start.

CONCLUSIONS

- A Car Transmission System has been modeled by using the POG Modeling Technique;
- The clutch with torsional dumper-spring has been modeled and simulated as variable dynamic dimension system;
- A Clutch control strategy has been proposed;