The POG Modeling Technique Applied to Electrical Systems

UNIVERSITÀ DEGLI STUDI
DI MODENA E REGGIO EMILIA

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Outline

- Main characteristics of the Power-Oriented Graphs (POG) modelling technique

- POG modelling examples:
  1. DC motor connected to an hydraulic pump
  2. Three-phase brushless motor
  3. Three-phase asynchronous motor
  4. Electronic control of a multi-phase lighting system.

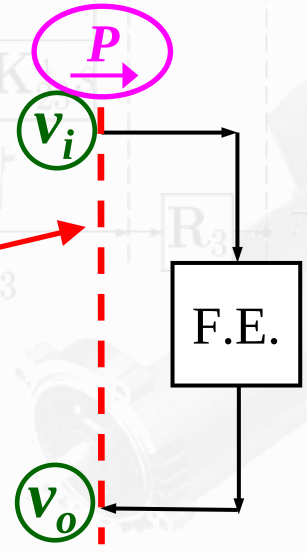
- Conclusions

POG Dynamic Modeling: Physical sections

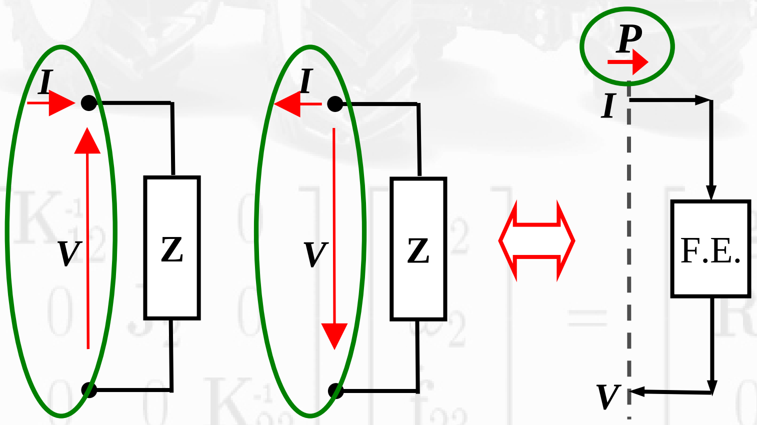
The physical elements (F.E.) interact with the external world through sections. Each section is characterized by two power variables v_i e v_o.

In POG a section is denoted by using a dashed line.

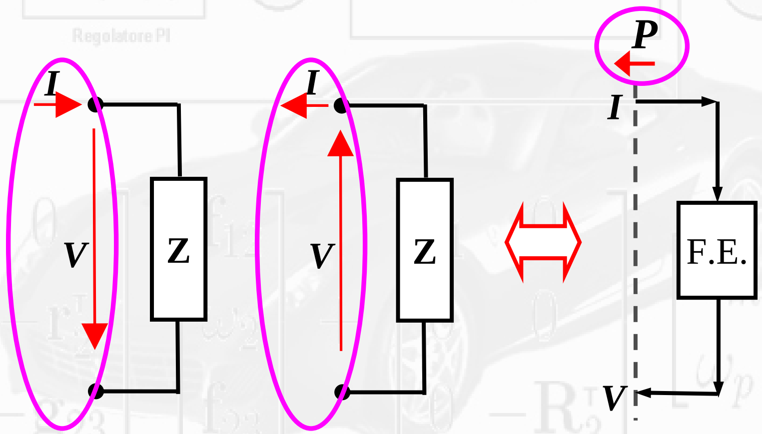
Each power variable has its own positive direction. The power flowing through a section can be positive or negative. An arrow over the dashed line is used for denoting the positive direction of the power P.



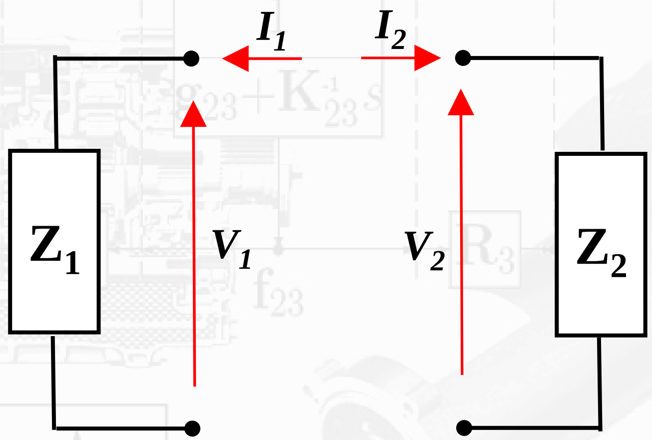
The power enters into the element:



The power exits from the element:



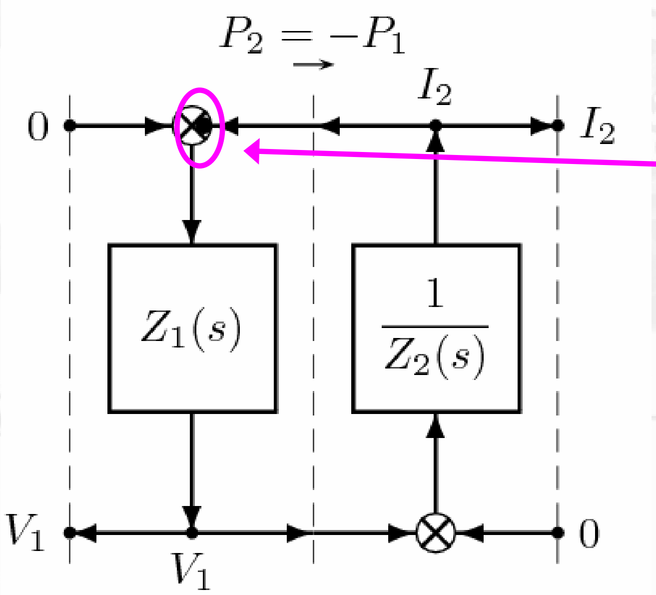
POG Dynamic Modeling: Connections



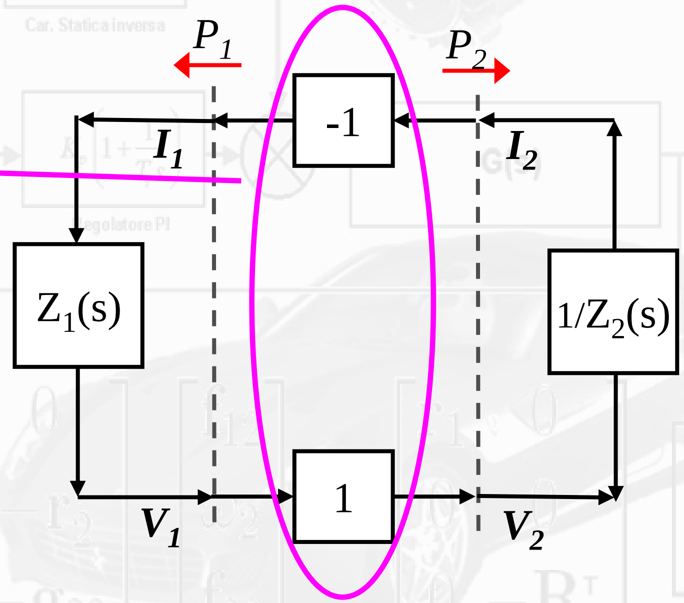
Example: connection of two electrical elements Z_1, Z_2.

If the powers P_1, P_2 enter into the two electrical elements, the variables I_1, V_1, I_2, V_2 cannot have all the same positive direction.

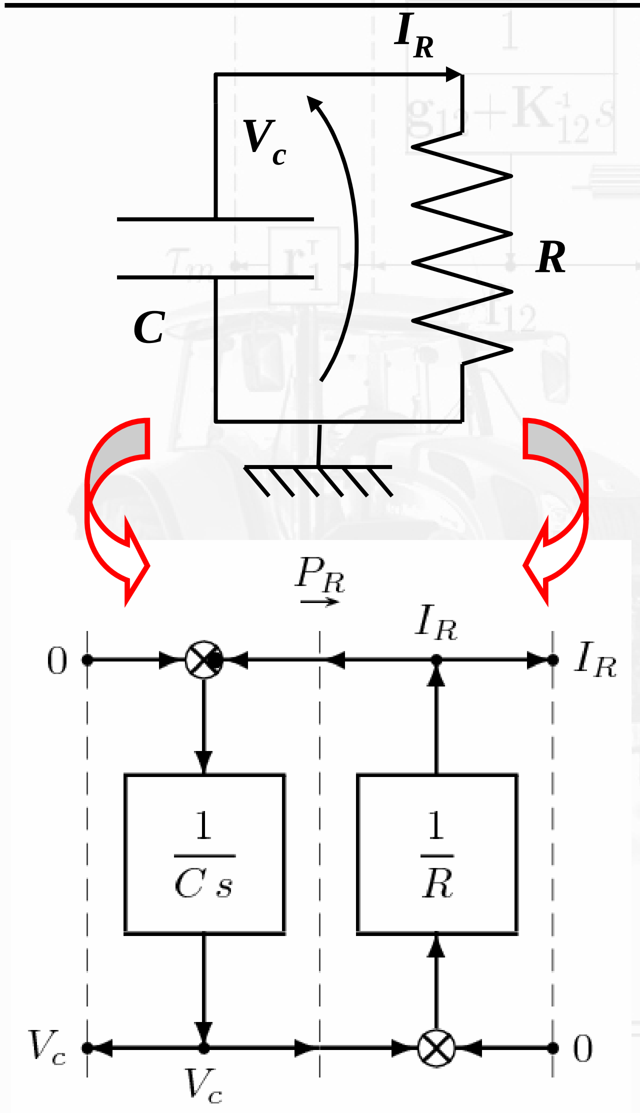
In this case a "connection block" is used for converting the power variables.

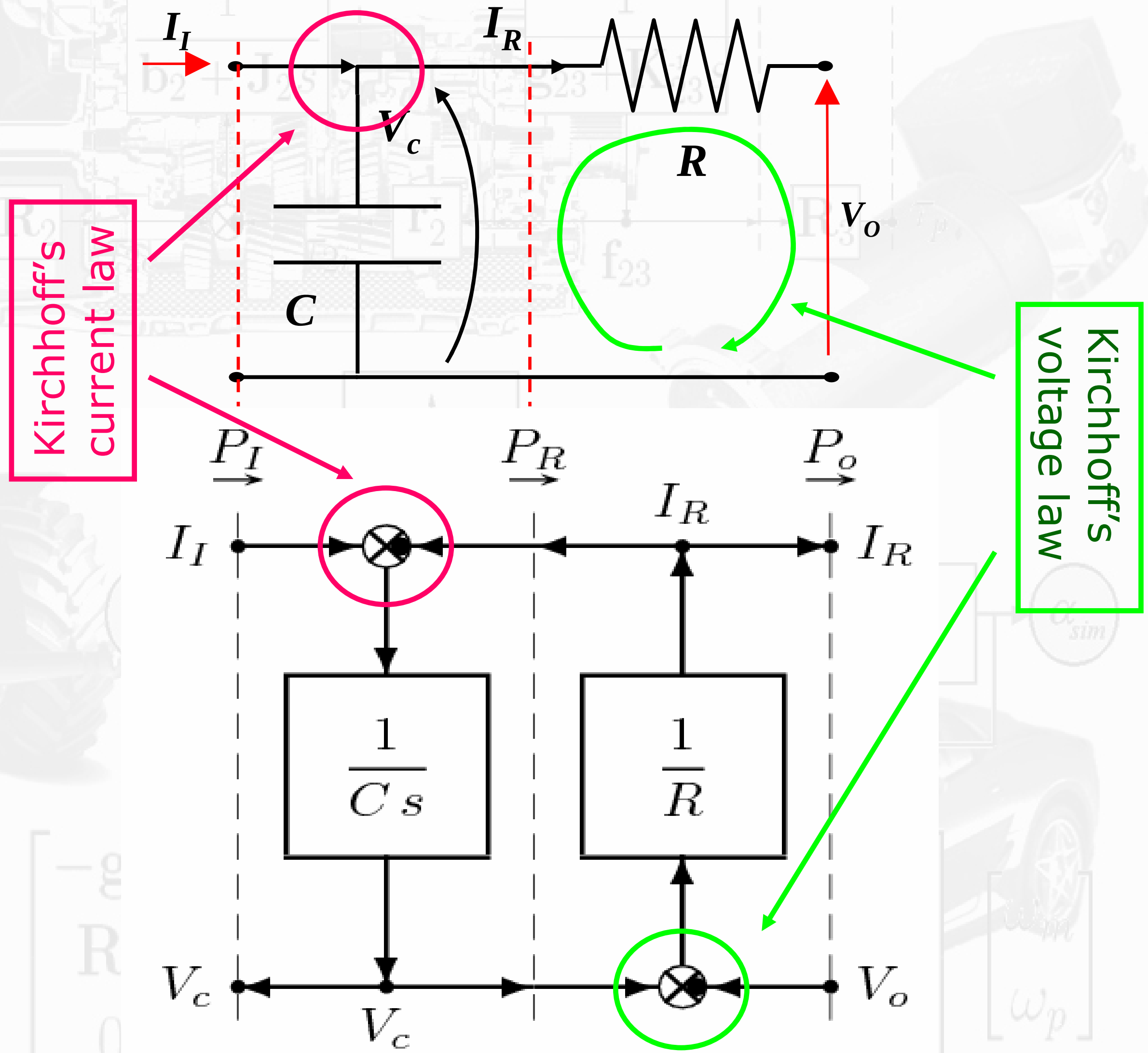


Equivalent description (scalar case)



Dynamic Modeling: Electrical examples





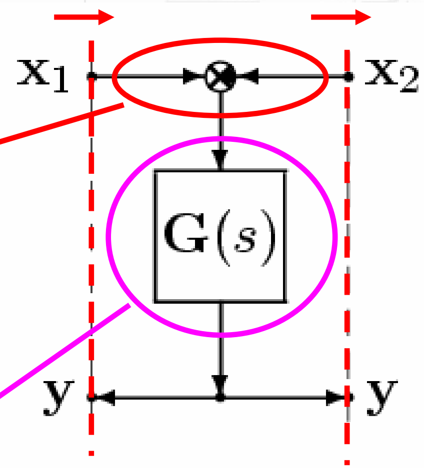
Introduction
Power-Oriented Graphs (POG)

The Power-Oriented Graphs are "block diagrams" obtained by using a "modular" structure essentially based on the following two blocks:

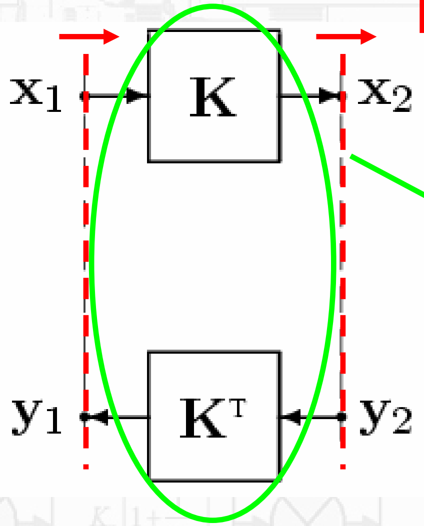
Positive power flows

3-Port Junctions:
- 0-Junction;
- 1-Junction;

1-Port Elements:
- Capacitor C;
- Inertia I;
- Resistor R;



Elaboration block



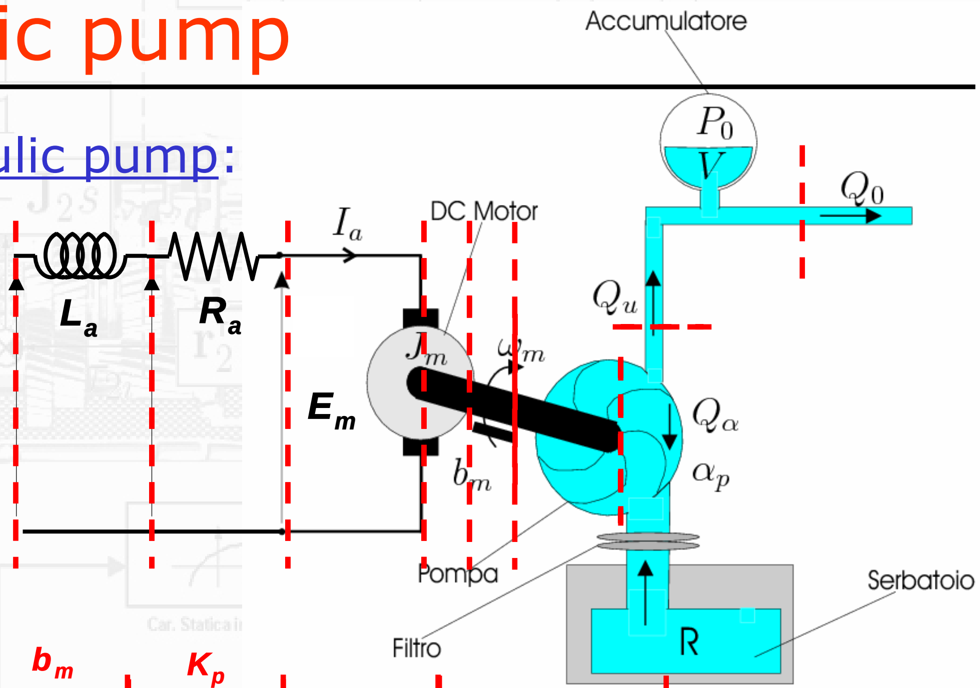
Connection block

2-Port Elements :
- Transformers TR;
- Girators GY;
- Modulated TR;
- Modulated GY;

- POG maintains a direct correspondence between pairs of system variables and real power flows: the product of the two variables involved in each dashed line of the graph has the physical meaning of ``power flowing through that section''.
- The Elaboration block can store and dissipate/generate energy.
- The Connection block can only "transform" the energy.

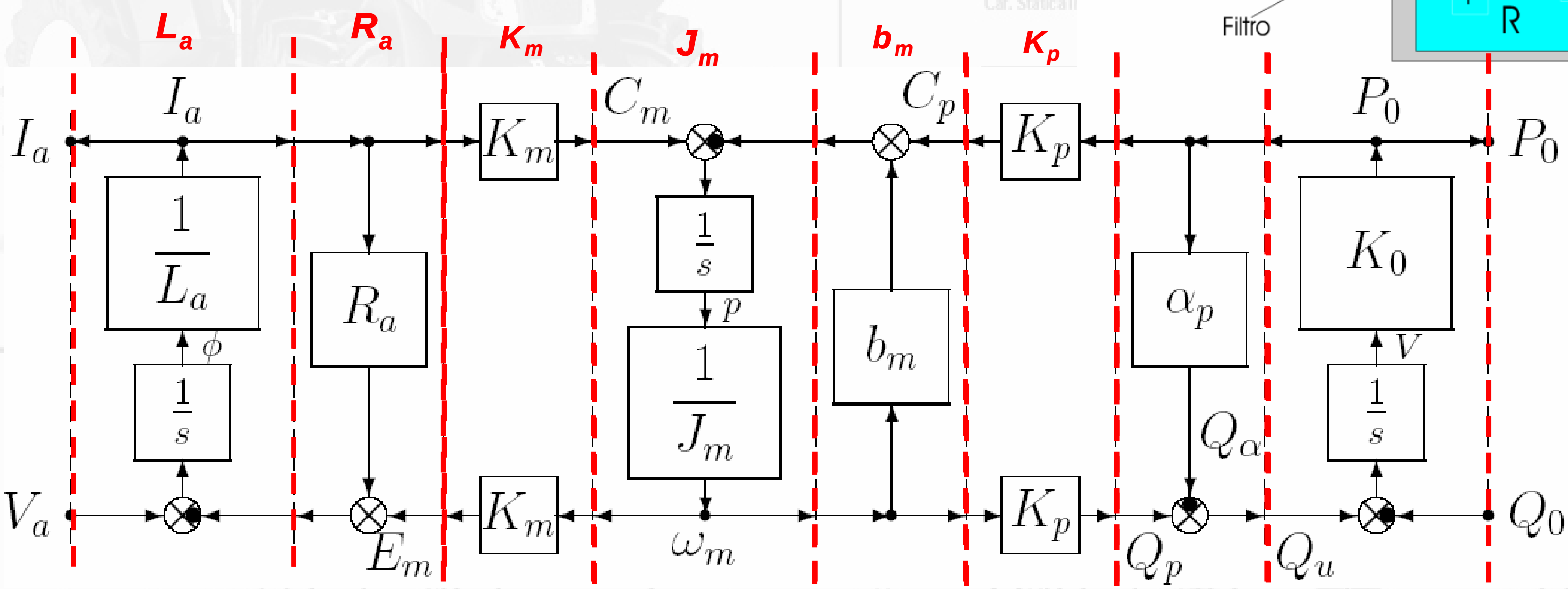
Example of POG modeling: DC electric motor with an hydraulic pump

A DC motor connected to an hydraulic pump:



There is a direct correspondence between the POG blocks and the physical elements …

The POG model:



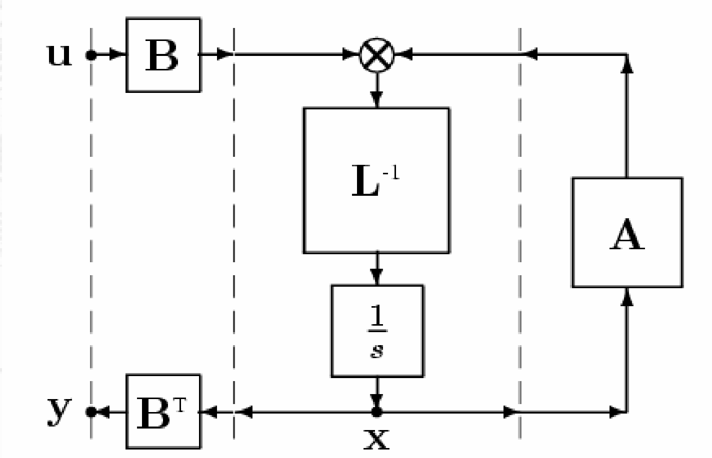
Introduction
Power-Oriented Graphs - LTI Systems

- Direct correspondence between POG and state space descriptions:

$$\begin{cases} \mathbf{L}\,\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \\ \mathbf{y} = \mathbf{B}^{\mathrm{T}}\mathbf{x} \end{cases}$$

Stored Energy: $E_s = \frac{1}{2}\mathbf{x}^{\mathrm{T}}\mathbf{L}\,\mathbf{x}$

Dissipating Power: $P_d = \mathbf{x}^{\mathrm{T}}\mathbf{A}\,\mathbf{x}$



A "power" state space description of the DC motor with hydraulic pump:

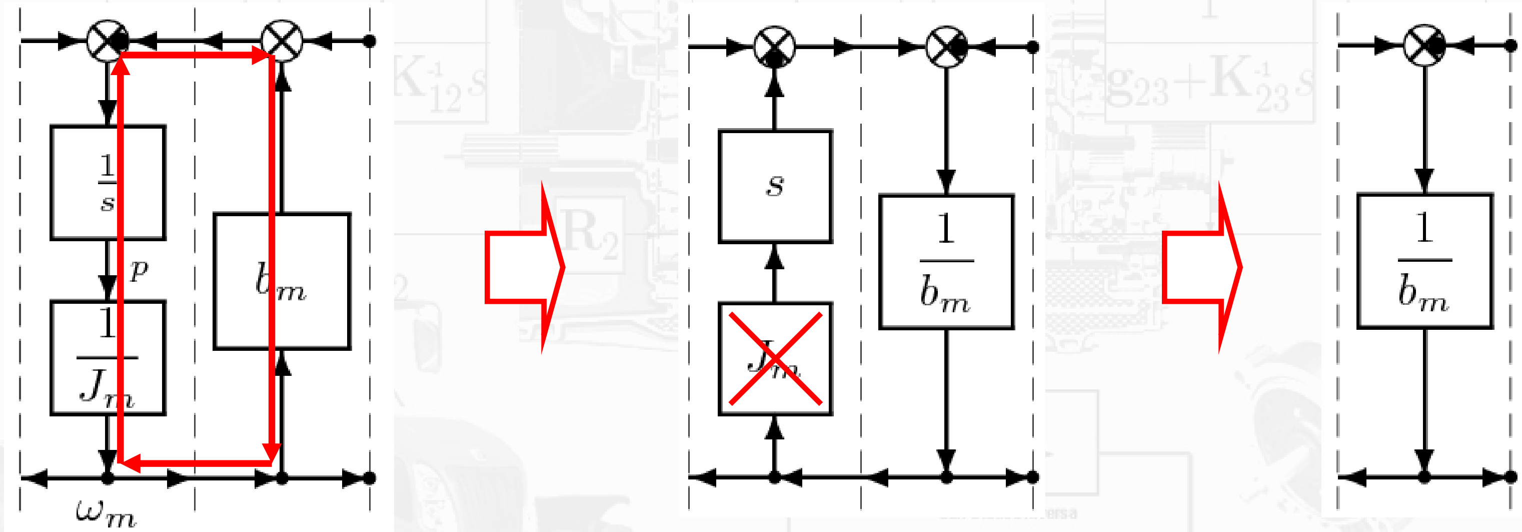
$$\underbrace{\begin{bmatrix} L_a & 0 & 0 \\ 0 & J_m & 0 \\ 0 & 0 & \frac{1}{K_0} \end{bmatrix}}_{\mathbf{L}} \underbrace{\begin{bmatrix} \dot{I}_a \\ \dot{\omega}_m \\ \dot{P}_0 \end{bmatrix}}_{\dot{\mathbf{x}}} = \underbrace{\begin{bmatrix} -R_a & -K_m & 0 \\ K_m & -b_m & -K_p \\ 0 & K_p & -\alpha_p \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} I_a \\ \omega_m \\ P_0 \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix}}_{\mathbf{B}} \underbrace{\begin{bmatrix} V_a \\ Q_0 \end{bmatrix}}_{\mathbf{u}}$$

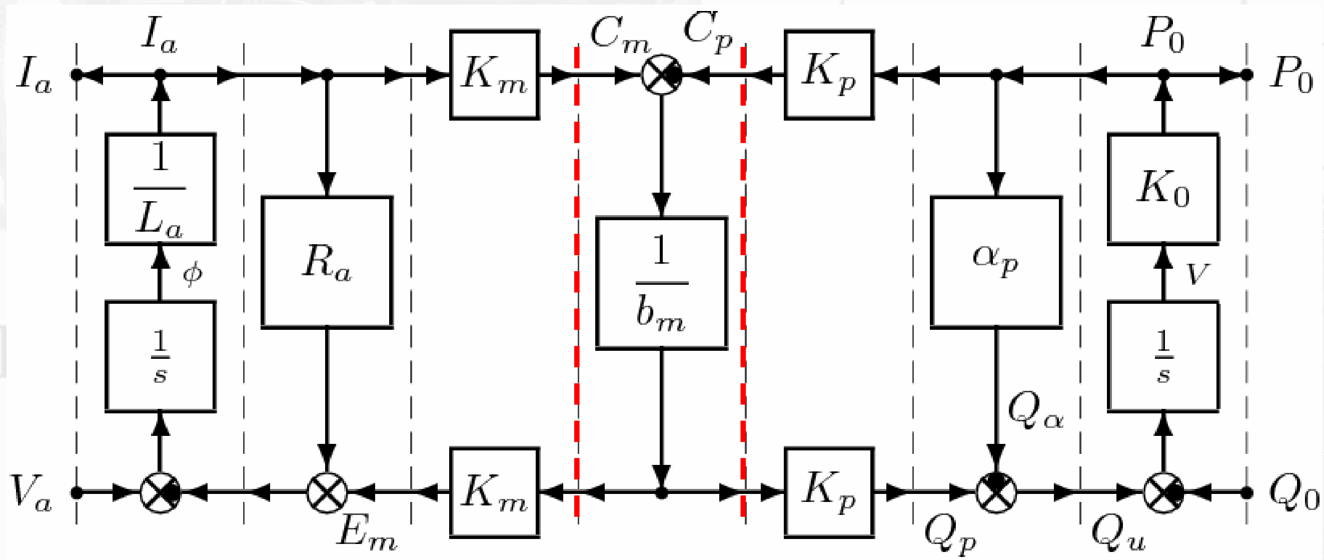
$$\mathbf{y} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{C}} \mathbf{x}$$

Which is the "reduced model" when J_m->0 ?

Two possible solutions:
1) graphically inverting a path …;
2) using a congruent transformation

POG modeling reduction: graphically inverting a path



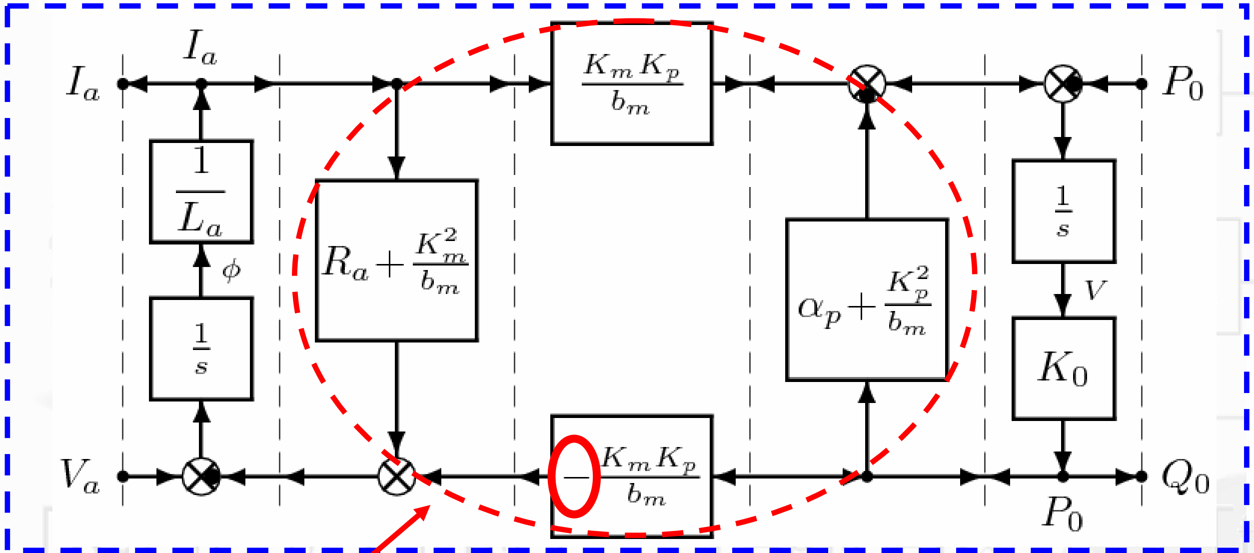


POG reduced model !

POG modeling reduction: using a "congruent" transformation

When an eigenvalue of matrix ***L*** goes to zero (or to infinity), the system degenerates towards a lower dynamic dimension system. The "reduced system " can be obtained by using a "congruent" transformation $x = \boldsymbol{T}z$ where ***T*** is a rectangular matrix:

$$\begin{cases} \mathbf{T}^T\mathbf{L}\mathbf{T}\dot{\mathbf{z}} = \mathbf{T}^T\mathbf{A}\mathbf{T}\mathbf{z} + \mathbf{T}^T\mathbf{B}\mathbf{u} \\ \mathbf{y} = \mathbf{B}^T\mathbf{T}\mathbf{z} \end{cases} \Leftrightarrow \begin{cases} \overline{\mathbf{L}}\dot{\mathbf{z}} = \overline{\mathbf{A}}\mathbf{z} + \overline{\mathbf{B}}\mathbf{u} \\ \mathbf{y} = \overline{\mathbf{B}}^T\mathbf{z} \end{cases}$$



$$J_m = 0$$

$$K_m I_a - b_m \omega_m - K_p P_0 = 0$$

$$\omega_m = \frac{K_m}{b_m} I_a - \frac{K_p}{b_m} P_0$$

$$\underbrace{\begin{bmatrix} I_a \\ \omega_m \\ P_0 \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} 1 & 0 \\ \frac{K_m}{b_m} & -\frac{K_p}{b_m} \\ 0 & 1 \end{bmatrix}}_{\mathbf{T}} \underbrace{\begin{bmatrix} I_a \\ P_0 \end{bmatrix}}_{\mathbf{z}}$$

Dissipation 2D element

$$\begin{bmatrix} L_a & 0 \\ 0 & \frac{1}{K_0} \end{bmatrix} \begin{bmatrix} \dot{I}_a \\ \dot{P}_0 \end{bmatrix} = \begin{bmatrix} -R_a - \frac{K_m^2}{b_m} & \frac{K_m K_p}{b_m} \\ \frac{K_m K_p}{b_m} & -\alpha_p - \frac{K_p^2}{b_m} \end{bmatrix} \begin{bmatrix} I_a \\ P_0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} V_a \\ Q_0 \end{bmatrix}$$

POG modeling of Electrical Motors

Let us consider Electric Motors "energetically" characterized only by:

1) the magnetic flux "***L** I*" generated by the stator and/or rotor currents I_s and I_r;

2) the magnetic flux "**j**($\mathbf{q}_r$)" of the permanent magnets (if present);

3) the momentum "J_r $\boldsymbol{w}_r$" generated by rotor velocity $\boldsymbol{w}_r$;

<u>The Energy *K* stored in the system</u> can be expressed as follows:

$$K = \frac{1}{2}\dot{\mathbf{q}}^{\mathrm{T}}\,\mathbf{L}(\theta_r)\,\dot{\mathbf{q}} + \dot{\mathbf{q}}^{\mathrm{T}}\,\varphi(\theta_r) \qquad \mathbf{L}(\theta_r) = \mathbf{L}(\theta_r)^{\mathrm{T}} > 0$$

where $\dot{\mathbf{q}} = [\ \mathbf{I}_s \quad \mathbf{I}_s \quad \omega_r\ ]$ and $\mathbf{q}_r$ is the rotor angular position.

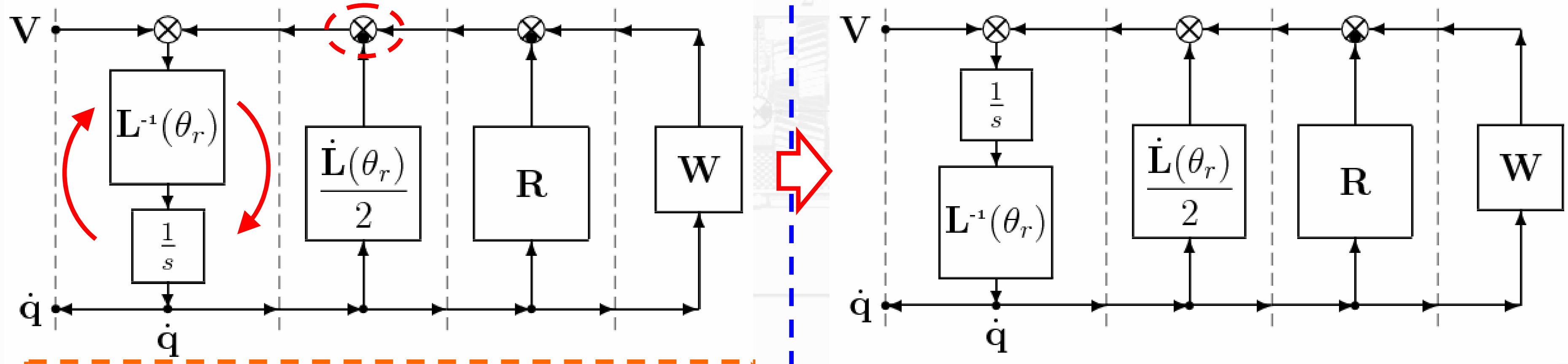
<u>The dynamic equations of the system are</u>:

$$\mathbf{L}\,\ddot{\mathbf{q}} + \frac{1}{2}\dot{\mathbf{L}}\,\dot{\mathbf{q}} = \mathbf{V} - \mathbf{R}\,\dot{\mathbf{q}} + \underbrace{\left[\frac{\partial(\frac{1}{2}\dot{\mathbf{q}}^{\mathrm{T}}\,\mathbf{L} + \varphi^{\mathrm{T}})}{\partial \mathbf{q}^{\mathrm{T}}} - \frac{\partial(\frac{1}{2}\mathbf{L}\,\dot{\mathbf{q}} + \varphi)}{\partial \mathbf{q}}\right]}_{\mathbf{W}}\dot{\mathbf{q}}$$

Where **R** is a symmetric matrix (energy "dissipation/generation") and **W** is a skew-symmetric matrix (energy "redistribution"): $\mathbf{R} = \mathbf{R}^{\mathrm{T}}$, $\mathbf{W} = -\mathbf{W}^{\mathrm{T}}$

POG modeling of Electrical Motors

Two different but equivalent POG graphical representations:



$$\mathbf{L}(\theta_r)\,\ddot{\mathbf{q}} + \frac{1}{2}\dot{\mathbf{L}}(\theta_r)\,\dot{\mathbf{q}} = \mathbf{V} - \mathbf{R}\,\dot{\mathbf{q}} + \mathbf{W}\,\dot{\mathbf{q}}$$

$$\frac{d}{dt}\left[\mathbf{L}(\theta_r)\,\dot{\mathbf{q}}\right] = \mathbf{V} + \frac{1}{2}\dot{\mathbf{L}}(\theta_r)\,\dot{\mathbf{q}} - \mathbf{R}\,\dot{\mathbf{q}} + \mathbf{W}\dot{\mathbf{q}}$$

The dynamic equations can be easily interpreted from a "power" point of view.

Multiplying $\dot{\mathbf{q}}^{\mathrm{T}}$ on the left of the first equation one obtains:

$$\underbrace{\dot{\mathbf{q}}^{\mathrm{T}}\,\mathbf{L}\,\ddot{\mathbf{q}} + \frac{1}{2}\dot{\mathbf{q}}^{\mathrm{T}}\,\dot{\mathbf{L}}\,\dot{\mathbf{q}}}_{\dot{K}} = \underbrace{\dot{\mathbf{q}}^{\mathrm{T}}\,\mathbf{V}}_{P_e} - \underbrace{\dot{\mathbf{q}}^{\mathrm{T}}\,\mathbf{R}\,\dot{\mathbf{q}}}_{P_d} + \underbrace{\dot{\mathbf{q}}^{\mathrm{T}}\,\mathbf{W}\,\dot{\mathbf{q}}}_{0}$$

Stored energy variation ($\dot{K}$)

Entering power (P_e)

Dissipated power (P_d)

Redistributed power (0)

Brushless motor: the three-phase stator circuit

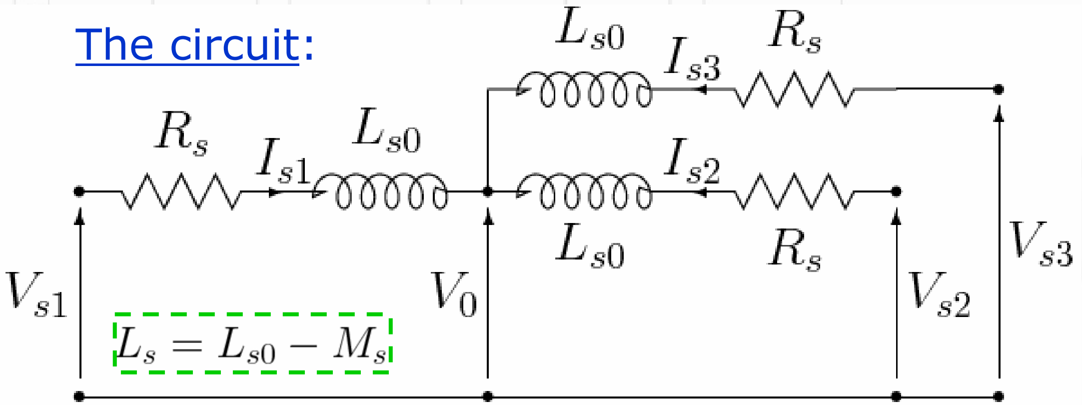
The constraint:

$$I_{s1} + I_{s2} + I_{s3} = 0$$

The dynamic equations:

$$ {}^t\mathbf{L}_s \, {}^t\dot{\mathbf{I}}_s = -\, {}^t\mathbf{R}_s \, {}^t\mathbf{I}_s + {}^t\mathbf{V}_s - \mathbf{V}_0 $$

The circuit:



that is, expanded:

$$\underbrace{\begin{bmatrix} L_{s0} & M_s & M_s \\ M_s & L_{s0} & M_s \\ M_s & M_s & L_{s0} \end{bmatrix}}_{{}^t\mathbf{L}_s} \underbrace{\begin{bmatrix} \dot{I}_{s1} \\ \dot{I}_{s2} \\ \dot{I}_{s3} \end{bmatrix}}_{{}^t\dot{\mathbf{I}}_s} = -\underbrace{\begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix}}_{{}^t\mathbf{R}_s} \underbrace{\begin{bmatrix} I_{s1} \\ I_{s2} \\ I_{s3} \end{bmatrix}}_{{}^t\mathbf{I}_s} + \underbrace{\begin{bmatrix} V_{s1} - V_0 \\ V_{s2} - V_0 \\ V_{s3} - V_0 \end{bmatrix}}_{{}^t\mathbf{V}_s - \mathbf{V}_0}$$

Static “dq”

By using a congruent transformation ${}^t\mathbf{I}_s = {}^t\mathbf{T}_b \, {}^b\mathbf{I}_s$ one obtains the “reduced system”:

$$ {}^t\mathbf{T}_b = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} $$

⇨

$$\underbrace{\begin{bmatrix} L_s & 0 \\ 0 & L_s \end{bmatrix}}_{{}^b\mathbf{L}_s} \underbrace{\begin{bmatrix} \dot{I}_{sd} \\ \dot{I}_{sq} \end{bmatrix}}_{{}^b\dot{\mathbf{I}}_s} = -\underbrace{\begin{bmatrix} R_s & 0 \\ 0 & R_s \end{bmatrix}}_{{}^b\mathbf{R}_s} \underbrace{\begin{bmatrix} I_{sd} \\ I_{sq} \end{bmatrix}}_{{}^b\mathbf{I}_s} + \underbrace{\begin{bmatrix} V_{sd} \\ V_{sq} \end{bmatrix}}_{{}^b\mathbf{V}_s}$$

Brushless motor: the rotating frame

By using a orthonormal transformation ${}^b\mathbf{I}_s = {}^b\mathbf{T}_\omega\, {}^\omega\mathbf{I}_s$ …

$$
{}^b\mathbf{T}_\omega = \begin{bmatrix} \cos\theta_r & -\sin\theta_r \\ \sin\theta_r & \cos\theta_r \end{bmatrix}
$$

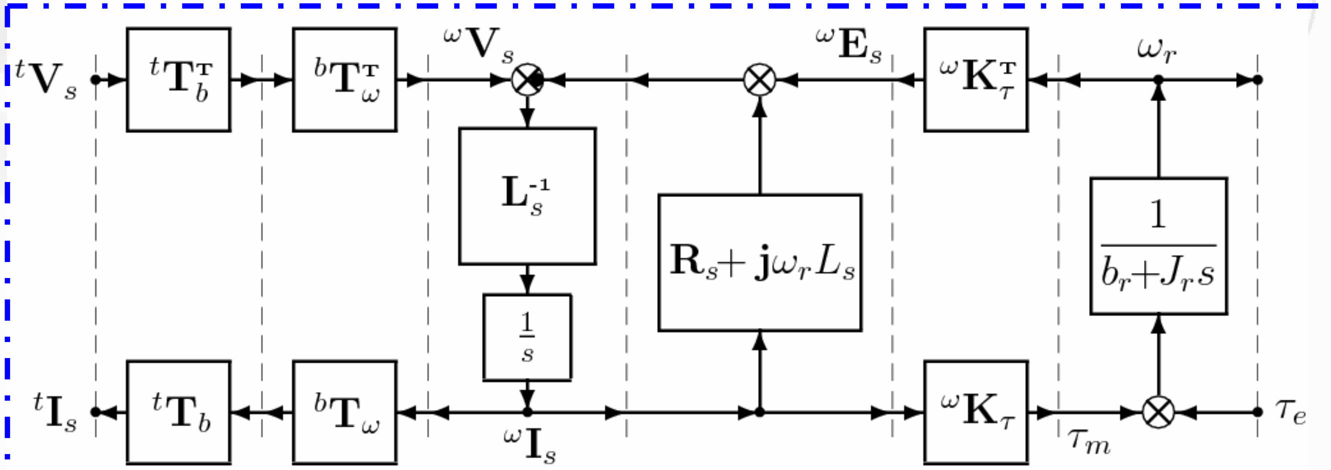
$$
\left[\begin{array}{c|c} \mathbf{L}_s & \mathbf{0} \\ \hline \mathbf{0} & J_r \end{array}\right]
\left[\begin{array}{c} {}^\omega\dot{\mathbf{I}}_s \\ \hline \dot{\omega}_r \end{array}\right]
= -\left[\begin{array}{c|c} {}^\omega\mathbf{R}_s + \mathbf{j}\omega_r L_s & {}^\omega\mathbf{K}_\tau^{\mathrm{T}} \\ \hline -\,{}^\omega\mathbf{K}_\tau & b_r \end{array}\right]
\left[\begin{array}{c} {}^\omega\mathbf{I}_s \\ \hline \omega_r \end{array}\right]
+ \left[\begin{array}{c} {}^\omega\mathbf{V}_s \\ \hline -\tau_e \end{array}\right]
$$

… one obtains the "two-phase rotating" dynamic model of the system.

$$
\left[\begin{array}{cc|c} L_s & 0 & 0 \\ 0 & L_s & 0 \\ \hline 0 & 0 & J_r \end{array}\right]
\left[\begin{array}{c} {}^\omega\dot{I}_{sd} \\ {}^\omega\dot{I}_{sq} \\ \hline \dot{\omega}_r \end{array}\right]
= -\left[\begin{array}{cc|c} R_s & -\omega_r L_s & \frac{\partial\, {}^\omega\varphi_d}{\partial\theta_r} \\ \omega_r L_s & R_s & \frac{\partial\, {}^\omega\varphi_q}{\partial\theta_r} \\ \hline -\frac{\partial\, {}^\omega\varphi_d}{\partial\theta_r} & -\frac{\partial\, {}^\omega\varphi_q}{\partial\theta_r} & b_r \end{array}\right]
\left[\begin{array}{c} {}^\omega I_{sd} \\ {}^\omega I_{sq} \\ \hline \omega_r \end{array}\right]
+ \left[\begin{array}{c} {}^\omega V_{sd} \\ {}^\omega V_{sq} \\ \hline -\tau_e \end{array}\right]
$$

Expanded form where $\vec{\varphi}(\theta_r)$ is the magnetic flux generated by the permanent magnets

POG dynamic model of the brushless motor



Brushless motor: sinusoidal magnetic flux

If the magnetic flux of the permanent magnets is sinusoidal …

$$ {}^{t}\vec{\varphi}(\theta_r) = \varphi_0 \begin{bmatrix} \cos\theta_r \\ \cos(\theta_r - \frac{2\pi}{3}) \\ \cos(\theta_r + \frac{2\pi}{3}) \end{bmatrix} $$

Three-phase

$$ {}^{b}\vec{\varphi}(\theta_r) = \sqrt{\frac{3}{2}}\varphi_0 \begin{bmatrix} \cos\theta_r \\ \sin\theta_r \end{bmatrix} $$

Two-phase static

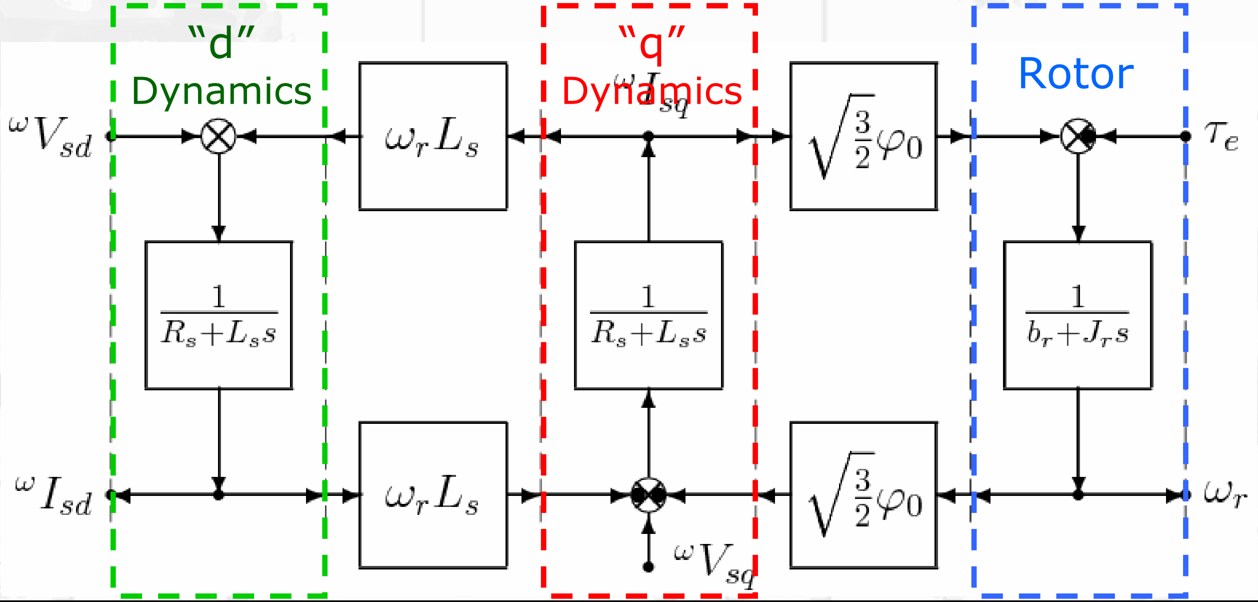
Two-phase rotating

$$ {}^{\omega}\vec{\varphi}(\theta_r) = \begin{bmatrix} \sqrt{\frac{3}{2}}\varphi_0 \\ 0 \end{bmatrix} $$

… the dynamic equations of brushless motor strongly simplify …

$$ \left[\begin{array}{cc|c} L_s & 0 & 0 \\ 0 & L_s & 0 \\ \hline 0 & 0 & J_r \end{array}\right] \left[\begin{array}{c} {}^{\omega}\dot{I}_{sd} \\ {}^{\omega}\dot{I}_{sq} \\ \hline \dot{\omega}_r \end{array}\right] = - \left[\begin{array}{cc|c} R_s & -\omega_r L_s & 0 \\ \omega_r L_s & R_s & \sqrt{\frac{3}{2}}\varphi_0 \\ \hline 0 & -\sqrt{\frac{3}{2}}\varphi_0 & b_r \end{array}\right] \left[\begin{array}{c} {}^{\omega}I_{sd} \\ {}^{\omega}I_{sq} \\ \hline \omega_r \end{array}\right] + \left[\begin{array}{c} {}^{\omega}V_{sd} \\ {}^{\omega}V_{sq} \\ \hline -\tau_e \end{array}\right] $$

… as well as the POG graphical representation.



Asyncronous three-phase motor: the stator and rotor circuits

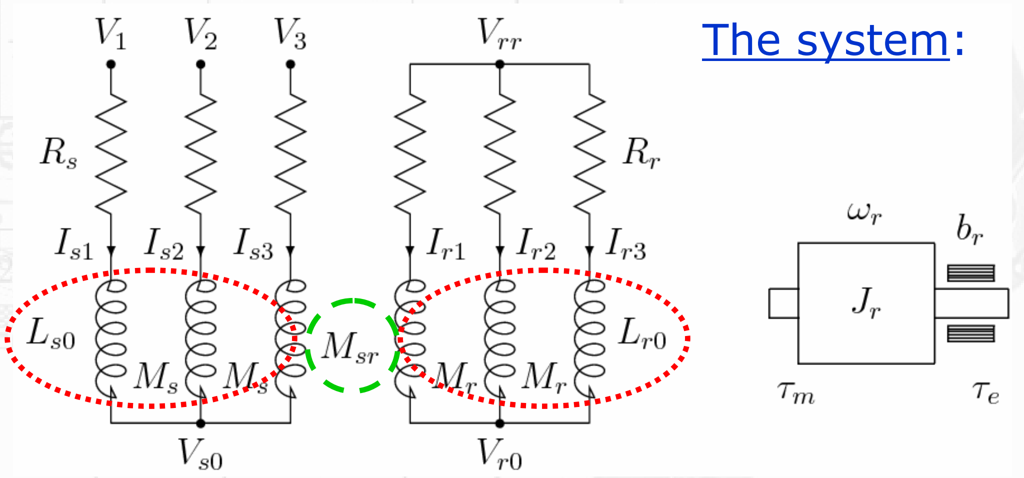
The variables. Stator and rotor currents and voltages:

$$ {}^t\mathbf{I}_s,\ {}^t\mathbf{I}_r,\ {}^t\mathbf{V}_s,\ {}^t\mathbf{V}_r $$

The constraints:

$$ I_{s1} + I_{s2} + I_{s3} = 0 $$
$$ I_{r1} + I_{r2} + I_{r3} = 0 $$

The system:



The dynamic equations:

$$ \frac{d}{dt}\left(\begin{bmatrix} {}^t\mathbf{L}_s & {}^t\mathbf{M}_{sr}^{\mathrm{T}} \\ {}^t\mathbf{M}_{sr} & {}^t\mathbf{L}_r \end{bmatrix} \begin{bmatrix} {}^t\mathbf{I}_s \\ {}^t\mathbf{I}_r \end{bmatrix} \right) = - \begin{bmatrix} {}^t\mathbf{R}_s & \mathbf{0} \\ \mathbf{0} & {}^t\mathbf{R}_r \end{bmatrix} \begin{bmatrix} {}^t\mathbf{I}_s \\ {}^t\mathbf{I}_r \end{bmatrix} + \begin{bmatrix} {}^t\mathbf{V}_s \\ {}^t\mathbf{V}_r \end{bmatrix} $$

where ${}^t\mathbf{R}_s = R_s \mathbf{I}_3$, ${}^t\mathbf{R}_r = R_r \mathbf{I}_3$ and

$$ {}^t\mathbf{L}_s = \begin{bmatrix} L_{s0} & M_s & M_s \\ M_s & L_{s0} & M_s \\ M_s & M_s & L_{s0} \end{bmatrix} \quad {}^t\mathbf{L}_r = \begin{bmatrix} L_{r0} & M_r & M_r \\ M_r & L_{r0} & M_r \\ M_r & M_r & L_{r0} \end{bmatrix} $$

Stator and Rotor Self-Inductances

$$ {}^t\mathbf{M}_{sr} = M_{sr} \begin{bmatrix} \cos(\theta_r) & \cos(\theta_r - \frac{2\pi}{3}) & \cos(\theta_r + \frac{2\pi}{3}) \\ \cos(\theta_r + \frac{2\pi}{3}) & \cos(\theta_r) & \cos(\theta_r - \frac{2\pi}{3}) \\ \cos(\theta_r - \frac{2\pi}{3}) & \cos(\theta_r + \frac{2\pi}{3}) & \cos(\theta_r) \end{bmatrix} $$

Stator / Rotor Mutual-Inductances

Asyncronous motor: dynamic model

Applying the two-phase "static" transformation (6->4):

$$\begin{bmatrix} {}^t\mathbf{I}_s \\ {}^t\mathbf{I}_r \end{bmatrix} = \begin{bmatrix} {}^t\tilde{\mathbf{T}}_b & \mathbf{0} \\ \mathbf{0} & {}^t\tilde{\mathbf{T}}_b \end{bmatrix} \begin{bmatrix} {}^b\mathbf{I}_s \\ {}^b\mathbf{I}_r \end{bmatrix}$$

where

$${}^t\tilde{\mathbf{T}}_b = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

… and then the two-phase "rotating" transformation ($\theta_d = \theta_s - \theta_r$):

$$\begin{bmatrix} {}^b\mathbf{I}_s \\ {}^b\mathbf{I}_r \end{bmatrix} = \begin{bmatrix} \mathbf{e}^{\mathbf{j}\theta_s} & \mathbf{0} \\ \mathbf{0} & \mathbf{e}^{\mathbf{j}\theta_d} \end{bmatrix} \begin{bmatrix} {}^\omega\mathbf{I}_s \\ {}^\omega\mathbf{I}_r \end{bmatrix}$$

where

$$\mathbf{j} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \qquad \mathbf{e}^{\mathbf{j}\theta} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

… one obtains the following "full" dynamic model:

$$\left[\begin{array}{cc|c} \mathbf{L}_s & \mathbf{L}_{sr} & \mathbf{0} \\ \mathbf{L}_{sr} & \mathbf{L}_r & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{0} & J_r \end{array}\right] \left[\begin{array}{c} {}^\omega\dot{\mathbf{I}}_s \\ {}^\omega\dot{\mathbf{I}}_r \\ \hline \dot{\omega}_r \end{array}\right] = - \left[\begin{array}{cc|c} \mathbf{R}_s + \mathbf{j}\omega_s L_s & \mathbf{j}(\omega_s - \frac{\omega_r}{2}) L_{sr} & \mathbf{j}\,{}^\omega\mathbf{I}_r \frac{1}{2} L_{sr} \\ \mathbf{j}(\omega_s - \frac{\omega_r}{2}) L_{sr} & \mathbf{R}_r + \mathbf{j}\omega_d L_r & -\mathbf{j}\,{}^\omega\mathbf{I}_s \frac{1}{2} L_{sr} \\ \hline {}^\omega\mathbf{I}_r^{\mathrm{T}}\, \mathbf{j} \frac{1}{2} L_{sr} & -{}^\omega\mathbf{I}_s^{\mathrm{T}}\, \mathbf{j} \frac{1}{2} L_{sr} & b_r \end{array}\right] \left[\begin{array}{c} {}^\omega\mathbf{I}_s \\ {}^\omega\mathbf{I}_r \\ \hline \omega_r \end{array}\right] + \left[\begin{array}{c} {}^\omega\mathbf{V}_s \\ \mathbf{0} \\ \hline -\tau_e \end{array}\right]$$

where $\mathbf{R}_s = R_s \mathbf{I}_2$, $\mathbf{R}_r = R_r \mathbf{I}_2$, $\mathbf{L}_s = L_s \mathbf{I}_2$, $\mathbf{L}_r = L_r \mathbf{I}_2$, $\mathbf{L}_{sr} = L_{sr} \mathbf{I}_2$,

$L_s = L_{s0} - M_s$, $L_r = L_{r0} - M_r$, $L_{sr} = \frac{3}{2} M_{sr}$.

Asyncronous motor: dynamic model

Dynamic model in a compact form:

$$\underbrace{\left[\begin{array}{c|c} {}^{\omega}\mathbf{L}_e & \mathbf{0} \\ \hline \mathbf{0} & J_r \end{array}\right]}_{{}^{\omega}\mathbf{L}} \underbrace{\left[\begin{array}{c} {}^{\omega}\dot{\mathbf{I}}_e \\ \hline \dot{\omega}_r \end{array}\right]}_{{}^{\omega}\dot{\mathbf{I}}} = - \underbrace{\left[\begin{array}{c|c} {}^{\omega}\mathbf{R}_e - \Omega_{sd} & {}^{\omega}\mathbf{K}_\tau^{\mathrm{T}} \\ \hline -{}^{\omega}\mathbf{K}_\tau & b_r \end{array}\right]}_{({}^{\omega}\mathbf{R} + {}^{\omega}\mathbf{W})} \underbrace{\left[\begin{array}{c} {}^{\omega}\mathbf{I}_e \\ \hline \omega_r \end{array}\right]}_{{}^{\omega}\mathbf{I}} + \underbrace{\left[\begin{array}{c} {}^{\omega}\mathbf{V}_e \\ \hline -\tau_e \end{array}\right]}_{{}^{\omega}\mathbf{V}}$$

The energy matrix represents the stored energy:

$$E_s = \frac{1}{2}\, {}^{\omega}\mathbf{I}^{\mathrm{T}}\, {}^{\omega}\mathbf{L}\, {}^{\omega}\mathbf{I}$$

The symmetric part of the system matrix represents the energy dissipations:

$${}^{\omega}\mathbf{R} = \left[\begin{array}{c|c} {}^{\omega}\mathbf{R}_e & \mathbf{0} \\ \hline \mathbf{0} & b_r \end{array}\right]$$

The skew symmetric part of the system matrix represents the energy redistribution:

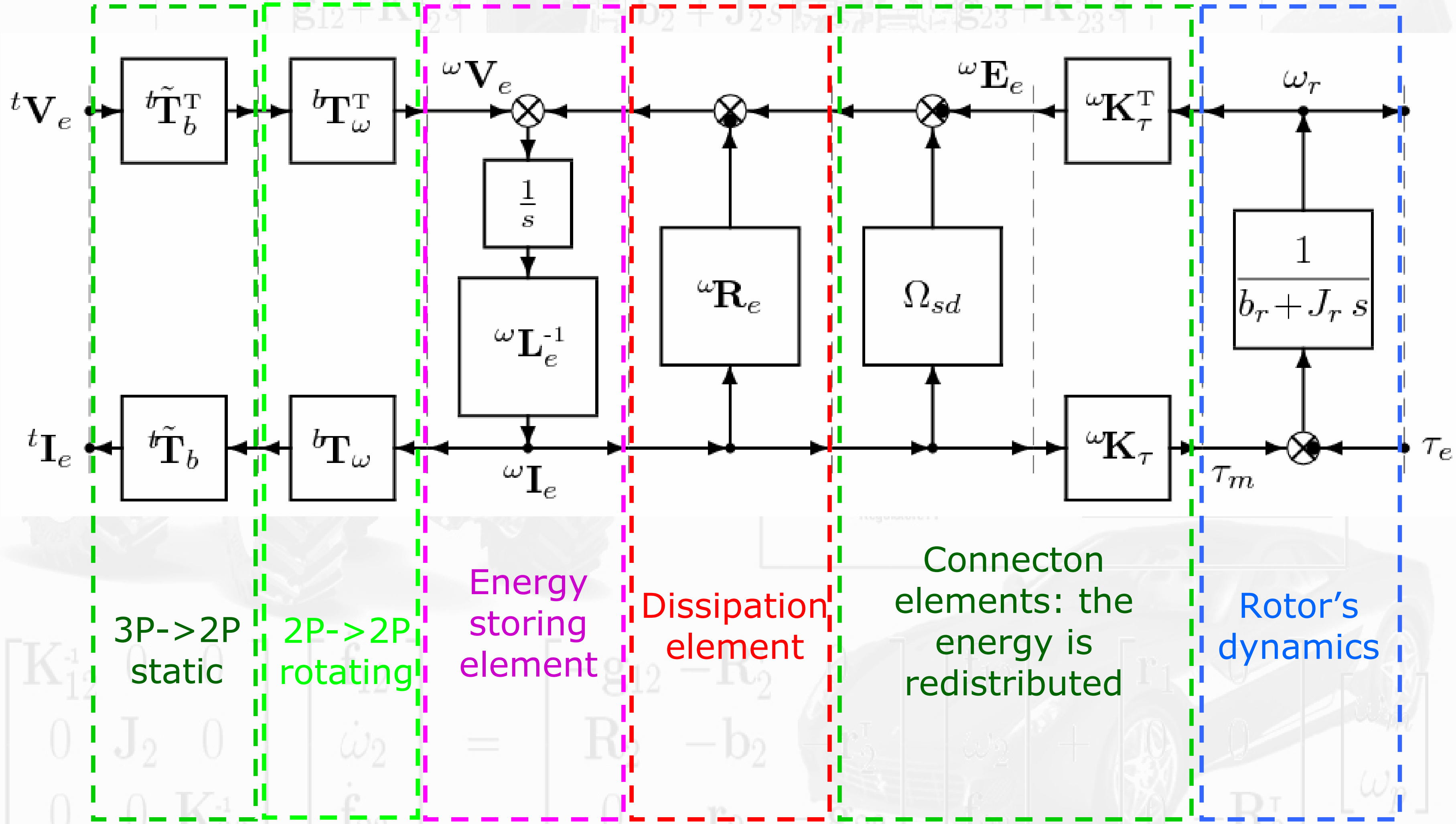
$${}^{\omega}\mathbf{W} = \left[\begin{array}{c|c} -\Omega_{sd} & {}^{\omega}\mathbf{K}_\tau^{\mathrm{T}} \\ \hline -{}^{\omega}\mathbf{K}_\tau & 0 \end{array}\right]$$

The active torque applied to the rotor:

$$\tau_m = \frac{\partial E_a}{\partial \theta_r} = \left[\; -{}^{\omega}\mathbf{I}_r^{\mathrm{T}}\, \mathbf{j}\tfrac{1}{2}L_{sr} \quad {}^{\omega}\mathbf{I}_s^{\mathrm{T}}\, \mathbf{j}\tfrac{1}{2}L_{sr} \;\right] {}^{\omega}\mathbf{I}_e = {}^{\omega}\mathbf{K}_\tau\, {}^{\omega}\mathbf{I}_e$$

Asyncronous motor: POG model

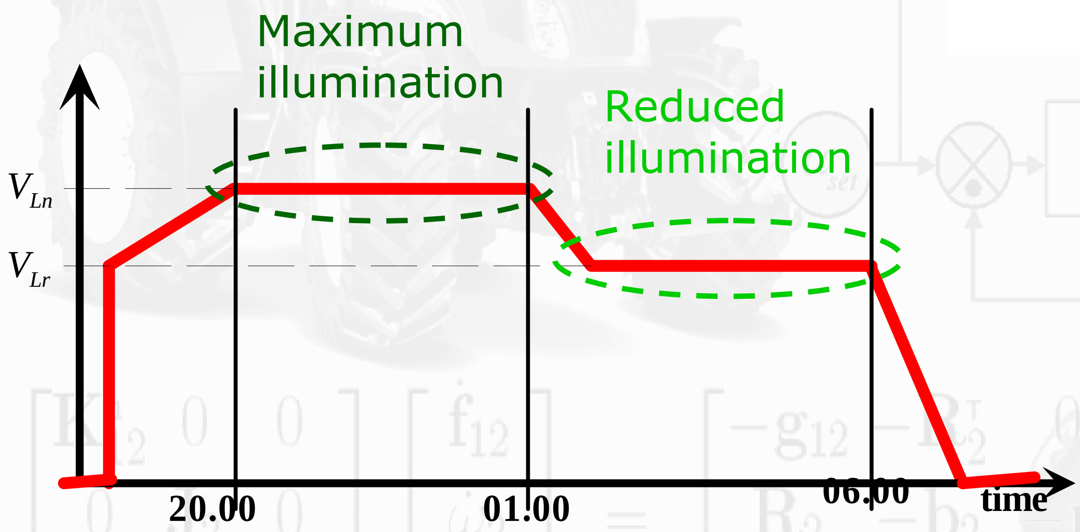
The POG graphical representation:



POG modeling and control of a Lighting System

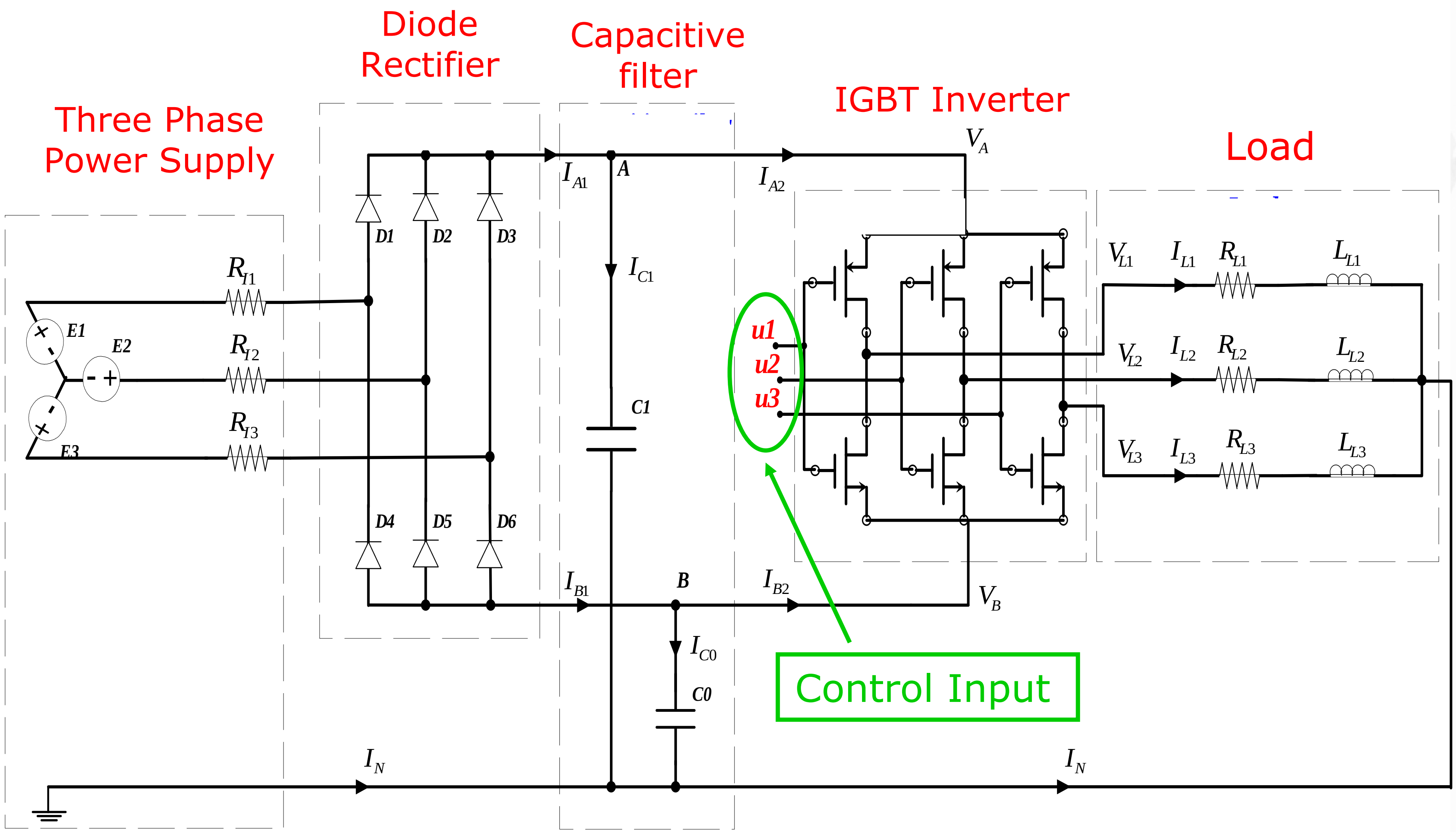
Typical lighting systems:
Electro-mechanical control.

Problem:
Control the Load Voltages using a Power Electronic circuit.

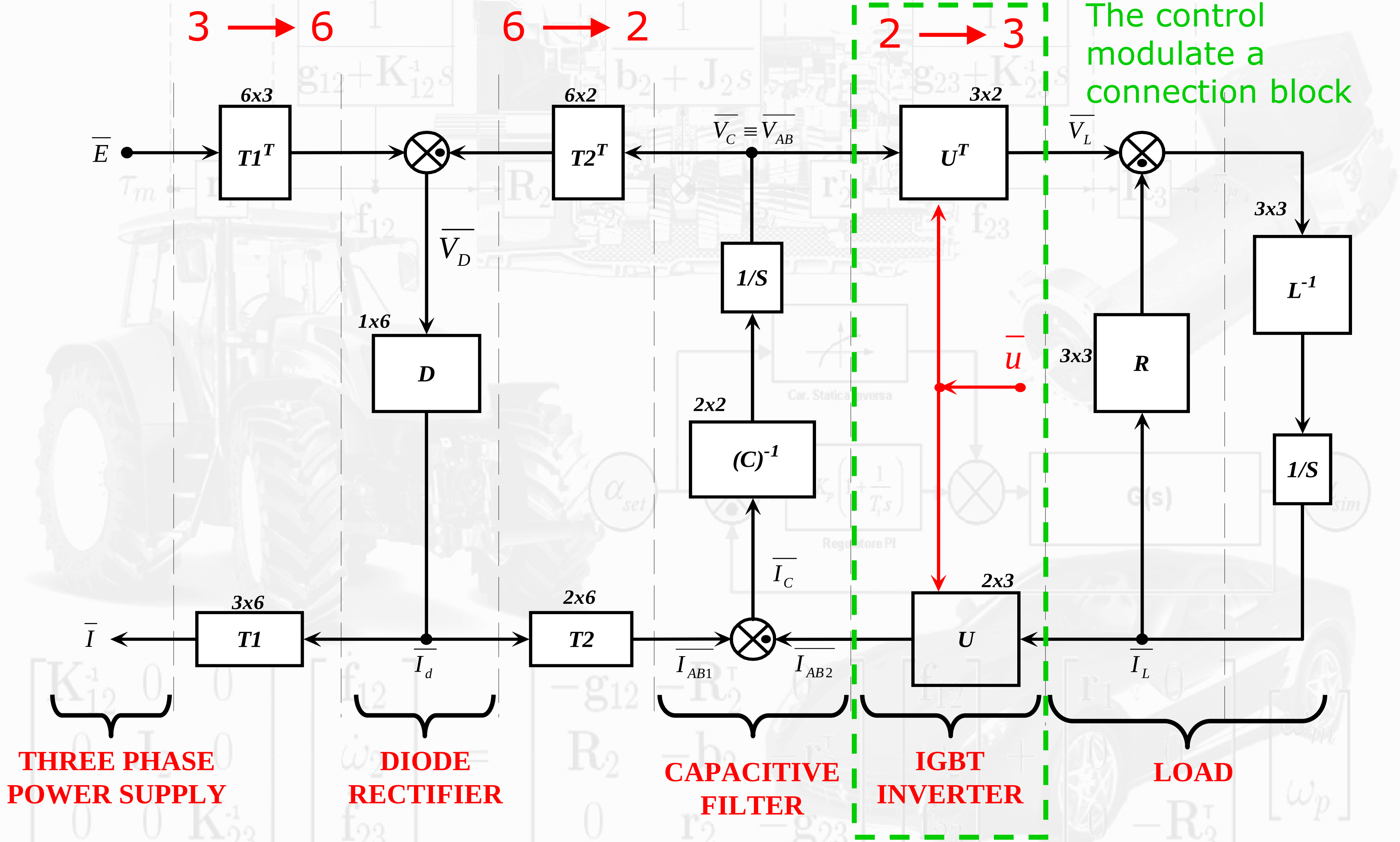


Time behaviour of the Load Voltages

Electric scheme of the lighting system



The POG Model of the lighting system



The POG Model: the connection blocks

The matrices ***T*₁** (3x6) and ***T*₂** (2x6) :

$$\mathbf{T}_1 = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix} \qquad \mathbf{T}_2 = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & -1 \end{bmatrix}$$

The IGBT modulated matrix ***U*** *(2xm)*:

$$\mathbf{U} = \begin{bmatrix} 1-u_1 & 1-u_2 & \dots & 1-u_m \\ u_1 & u_2 & \dots & u_m \end{bmatrix}$$

Matrix U is a function of the control vector $\mathbf{u} = [u_1, u_2, \dots u_m]$. If the IGBT are PWM controlled with an high switching frequency:

$$v_{Li}(t) = u_i\, v_B(t) + (1 - u_i)\, v_A(t)$$

where $\boldsymbol{v}_A$ and $\boldsymbol{v}_B$ are the capacitor's voltages

Simulation results

Requested load

Sinusoidal load voltages

- *Interval 1:* u=0
- *Interval 2:* $V_L=0$

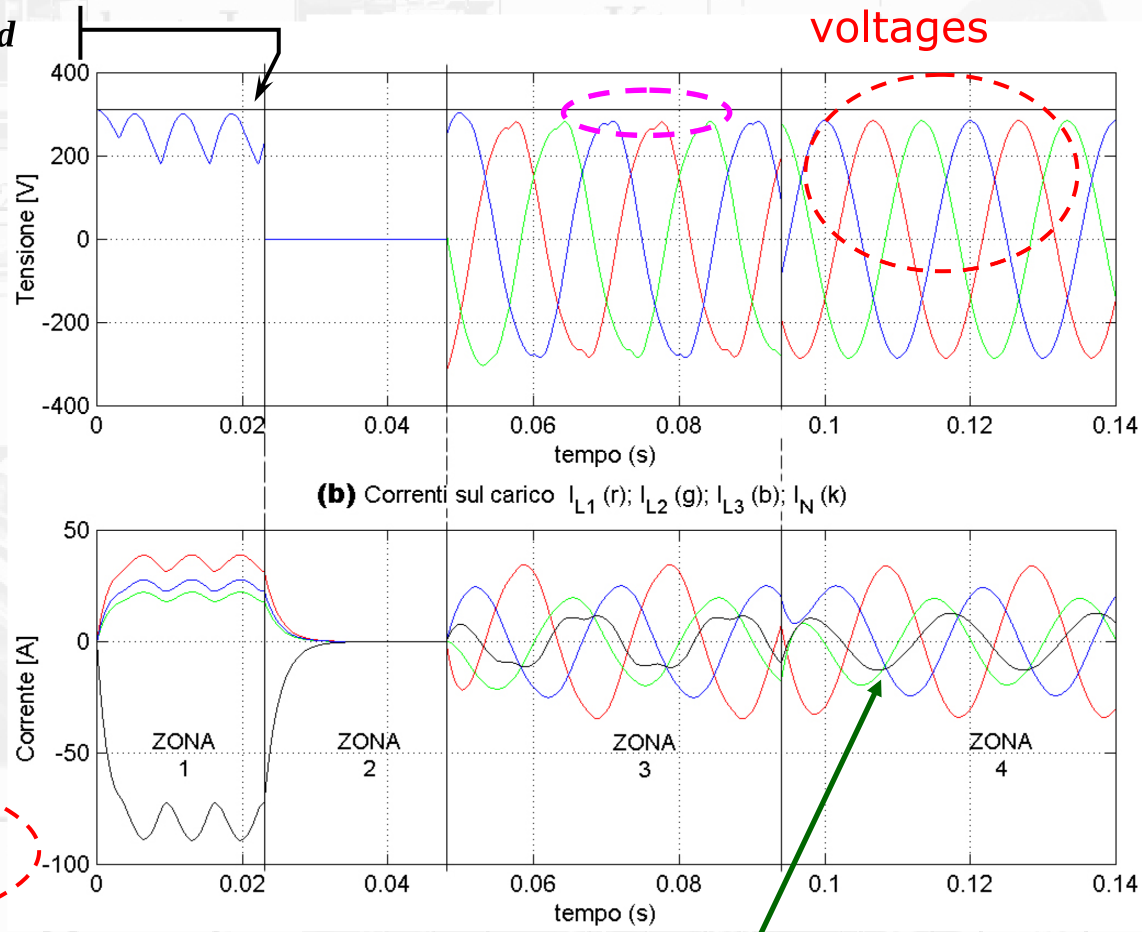
$$u_i(t) = \frac{V_A(t)}{V_A(t) - V_B(t)}$$

- *Interval 3:*

$$u_i(t) = A_u \sin(2\pi 50t + \varphi_{ui}) + \tfrac{1}{2}$$

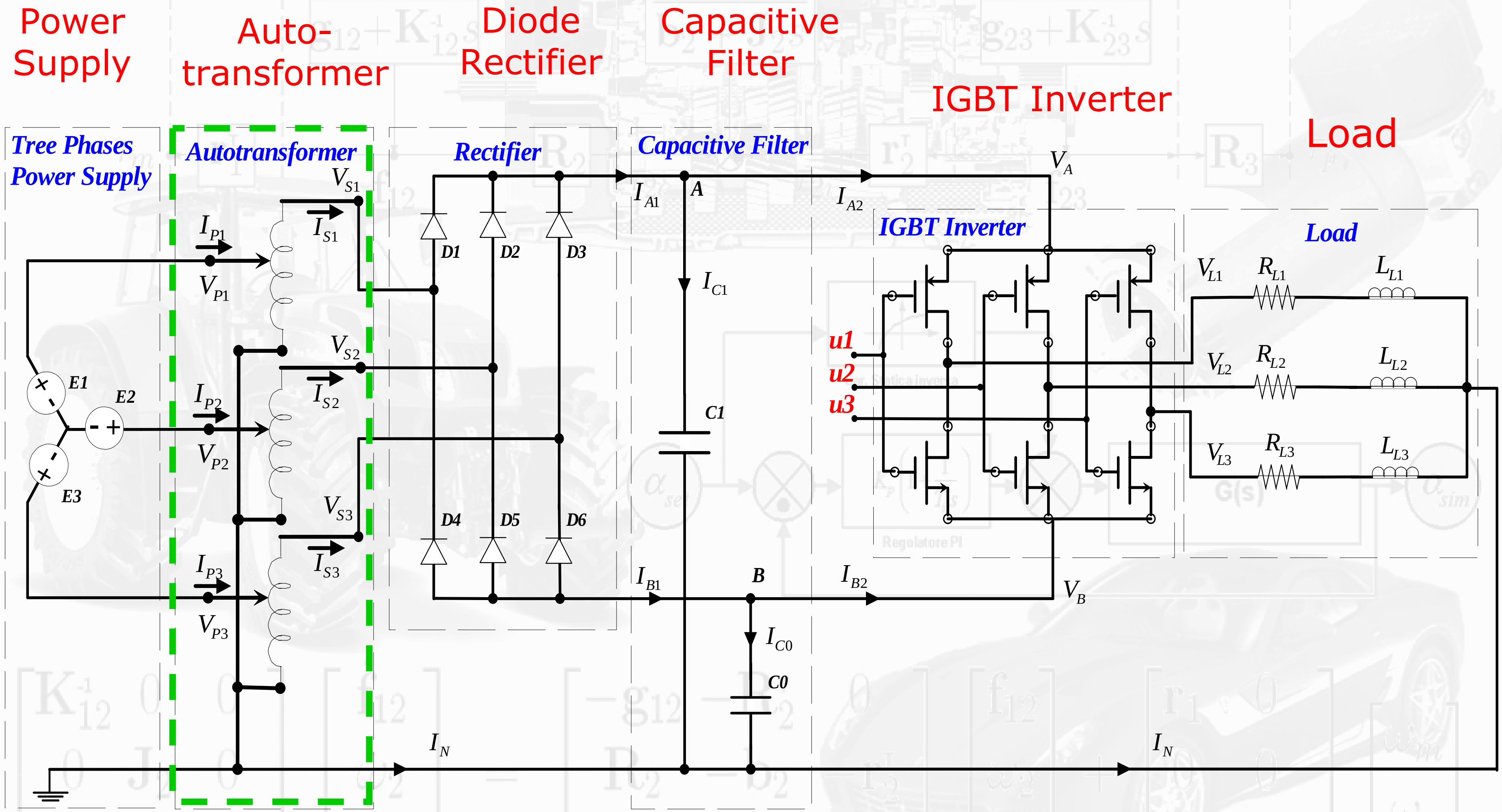
- *Interval 4:*

$$u_i(t) = \frac{A_L \sin\left(2\pi 50t + (i-1)\tfrac{2}{3}\pi\right) - V_A(t)}{V_B(t) - V_A(t)}$$

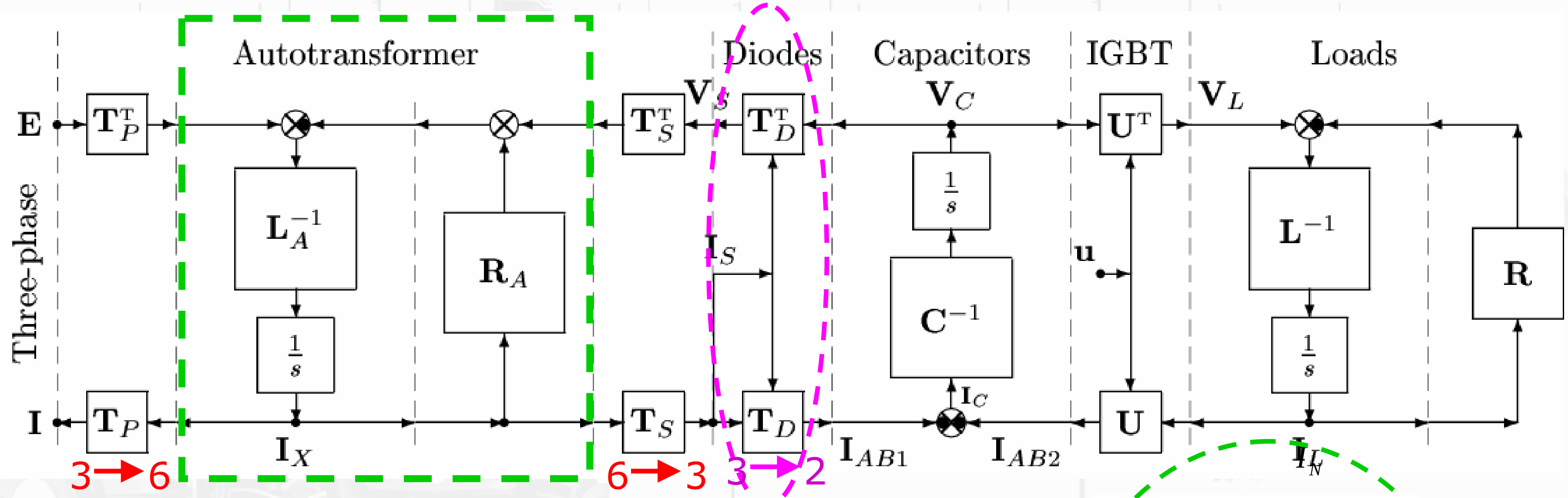


Drawback: this nonlinear control law doesn't minimize the neutral current and doesn't provide the maximum required voltage (230 Vrms)

Electical Circuit Of The System With Autotransformer



POG Model of the system with Autotransfomer



$$\mathbf{T}_P = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}, \quad \mathbf{T}_S = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{L}_A = \begin{bmatrix} L_{p1} & M_1 & 0 & 0 & 0 & 0 \\ M_1 & L_{s1} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{p2} & M_2 & 0 & 0 \\ 0 & 0 & M_2 & L_{s2} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{p3} & M_3 \\ 0 & 0 & 0 & 0 & M_3 & L_{s3} \end{bmatrix}$$

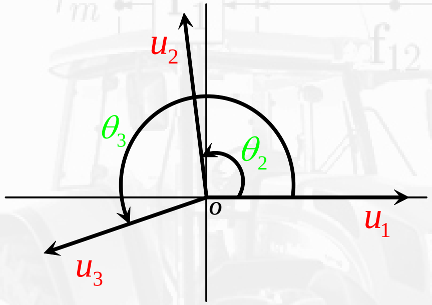
$$\mathbf{T}_D = \begin{bmatrix} \frac{1-\mathrm{sgn}(i_{s1})}{2} & \frac{1-\mathrm{sgn}(i_{s2})}{2} & \frac{1-\mathrm{sgn}(i_{s3})}{2} \\ \frac{1+\mathrm{sgn}(i_{s1})}{2} & \frac{1+\mathrm{sgn}(i_{s2})}{2} & \frac{1+\mathrm{sgn}(i_{s3})}{2} \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} C_1 & 0 \\ 0 & C_0 \end{bmatrix}$$

$$\mathbf{R}_A = \begin{bmatrix} R_{p1} & 0 & 0 & 0 & 0 & 0 \\ 0 & R_{s1} & 0 & 0 & 0 & 0 \\ 0 & 0 & R_{p2} & 0 & 0 & 0 \\ 0 & 0 & 0 & R_{s2} & 0 & 0 \\ 0 & 0 & 0 & 0 & R_{p3} & 0 \\ 0 & 0 & 0 & 0 & 0 & R_{s3} \end{bmatrix}$$

"Ideal" diodes without dissipations

Neutral Current Minimization: simulation results

The neutral current can be minimized acting on the relative angles θ_2 and θ_3 of the control signals



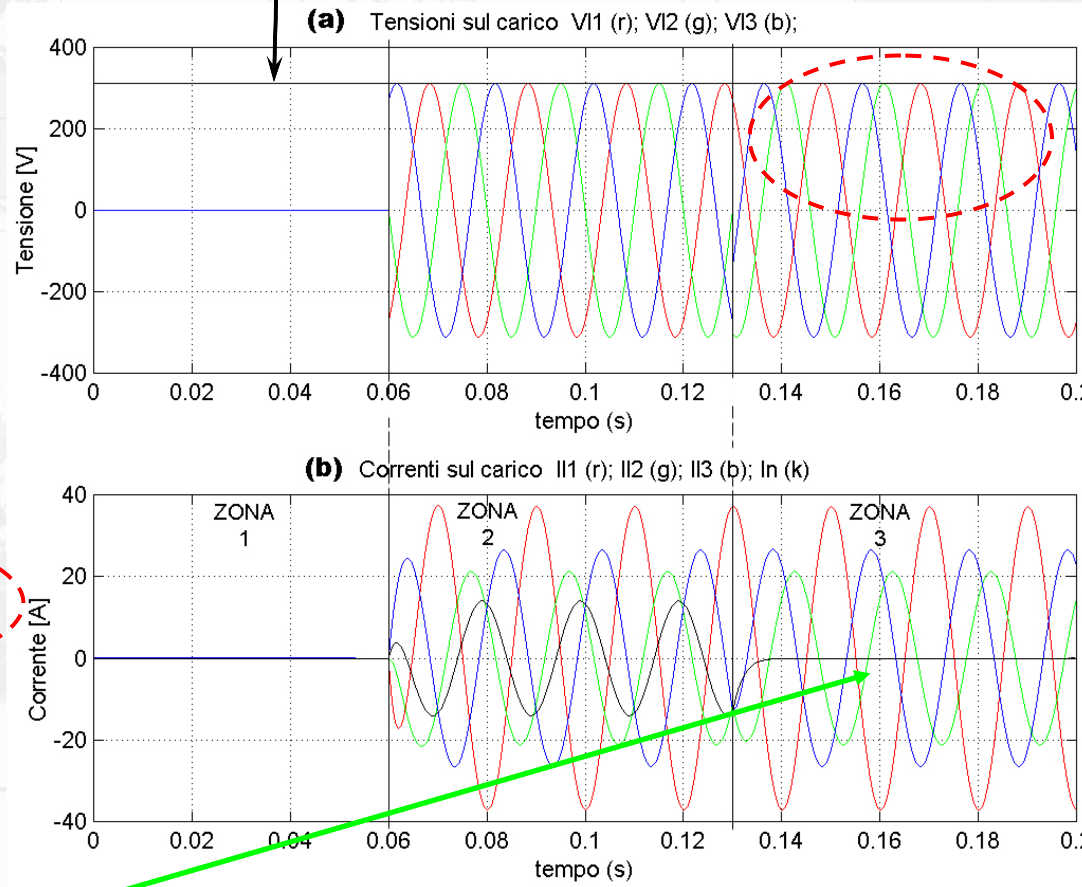
Control low:

$$u_i(t) = \frac{A_L \sin\left(2\pi 50 \cdot t + \theta i\right) - V_A(t)}{V_B(t) - V_A(t)}$$

An adaptation control low can be used to find the optimal relative angles $\theta 2$ and $\theta 3$ that minimize the neutral current

Requested load

Desired sinusoidal voltages



Conclusions

- Power-Oriented Graphs (POG) are a simple and powerful graphical technique that can be used for modeling all types of physical systems involving power flows.

- POG are easily understandable, simple to use and suitable both for teaching and for research.

Thank you for your attention!