Modeling of Automotive Control Systems Using Power Oriented Graphs

UNIVERSITÀ DEGLI STUDI
DI MODENA E REGGIO EMILIA

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Outline

- Introduction on Power-Oriented Graphs (POG)
- POG and Port-Controlled Hamiltonian (PCH) systems
- Modified PCH: introduction of “direct dissipations”
- Example: Clutch Control System
- Conclusions

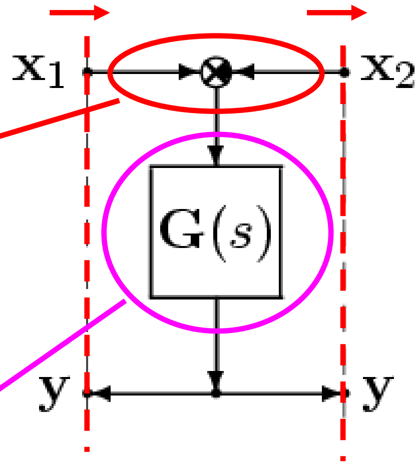
Introduction
Power-Oriented Graphs (POG)

The Power-Oriented Graphs are "block diagrams" obtained by using a "modular" structure essentially based on the following two blocks:

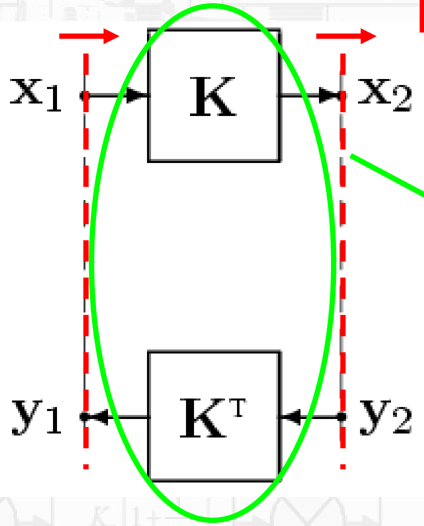
Positive power flows

3-Port Junctions:
- 0-Junction;
- 1-Junction;

1-Port Elements:
- Capacitor C;
- Inertia I;
- Resistor R;



Elaboration block



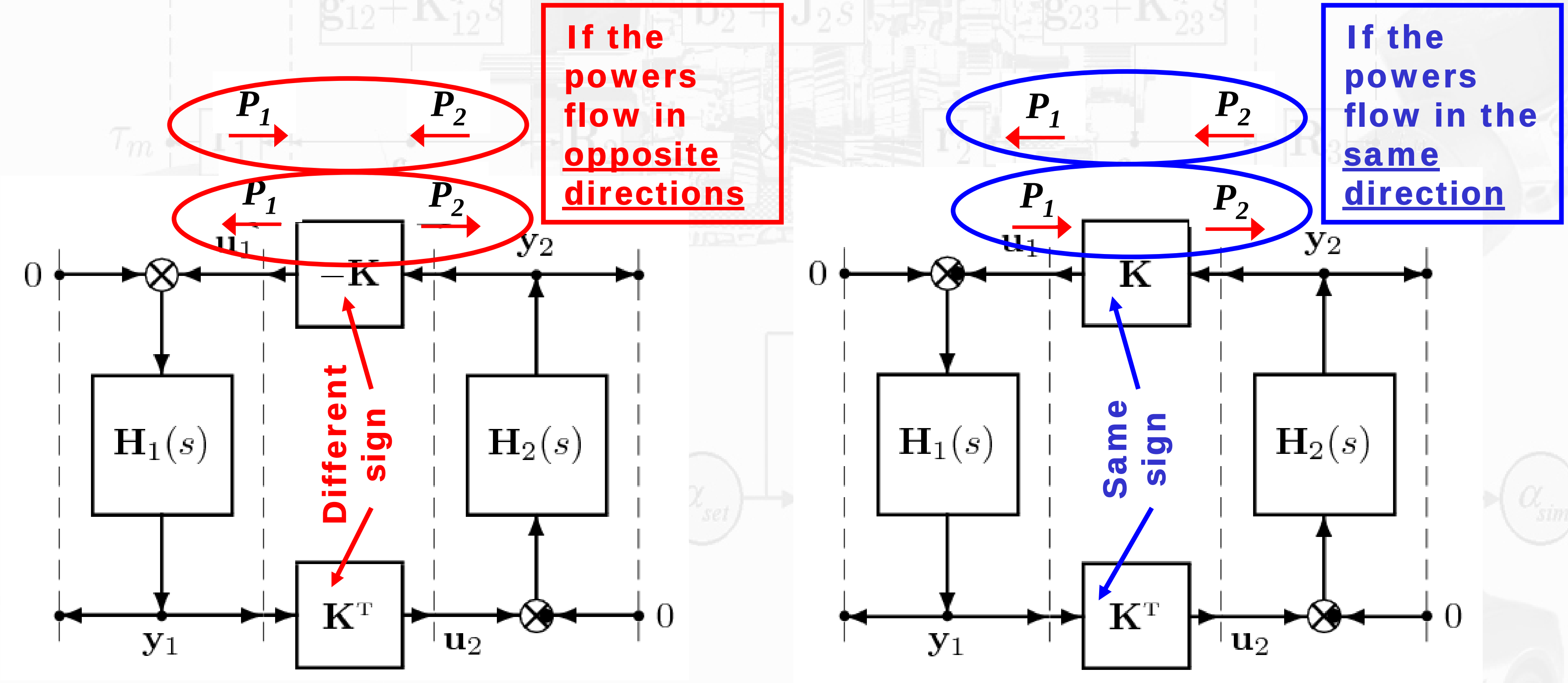
Connection block

2-Port Elements :
- Transformers TR;
- Girators GY;
- Modulated TR;
- Modulated GY;

- POG maintains a direct correspondence between pairs of system variables and real power flows: the product of the two variables involved in each dashed line of the graph has the physical meaning of ``power flowing through that section''.
- The Elaboration block can store and dissipate/generate energy.
- The Connection block can only "transform" the energy.

POG Dynamic Modeling: Connection blocks

In the general vector case the connection block is characterized by a matrix ***K*.**

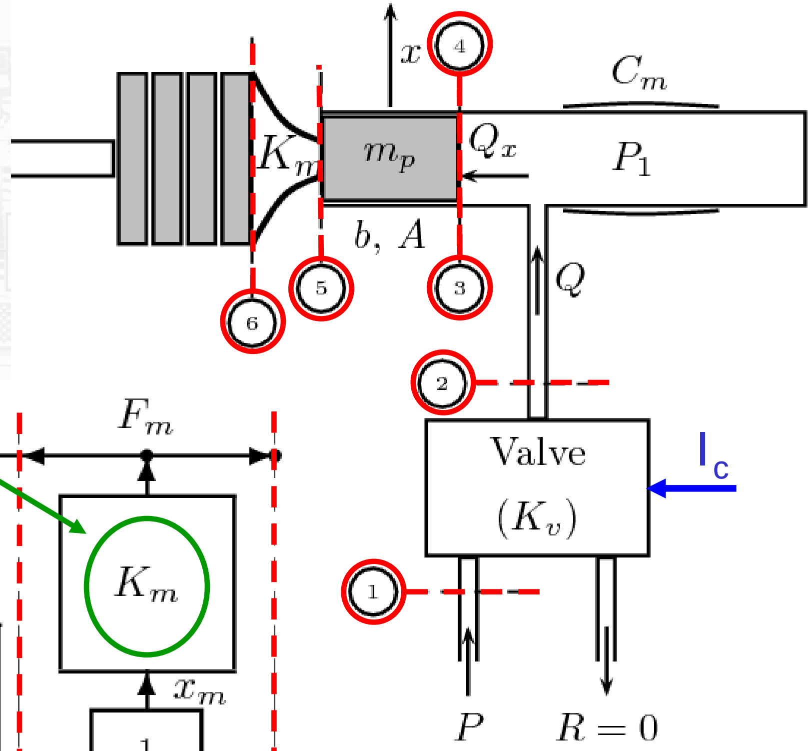


Matrix ***K*** is rectangular if systems $H_1(s)$ and $H_2(s)$ have different dimensions.

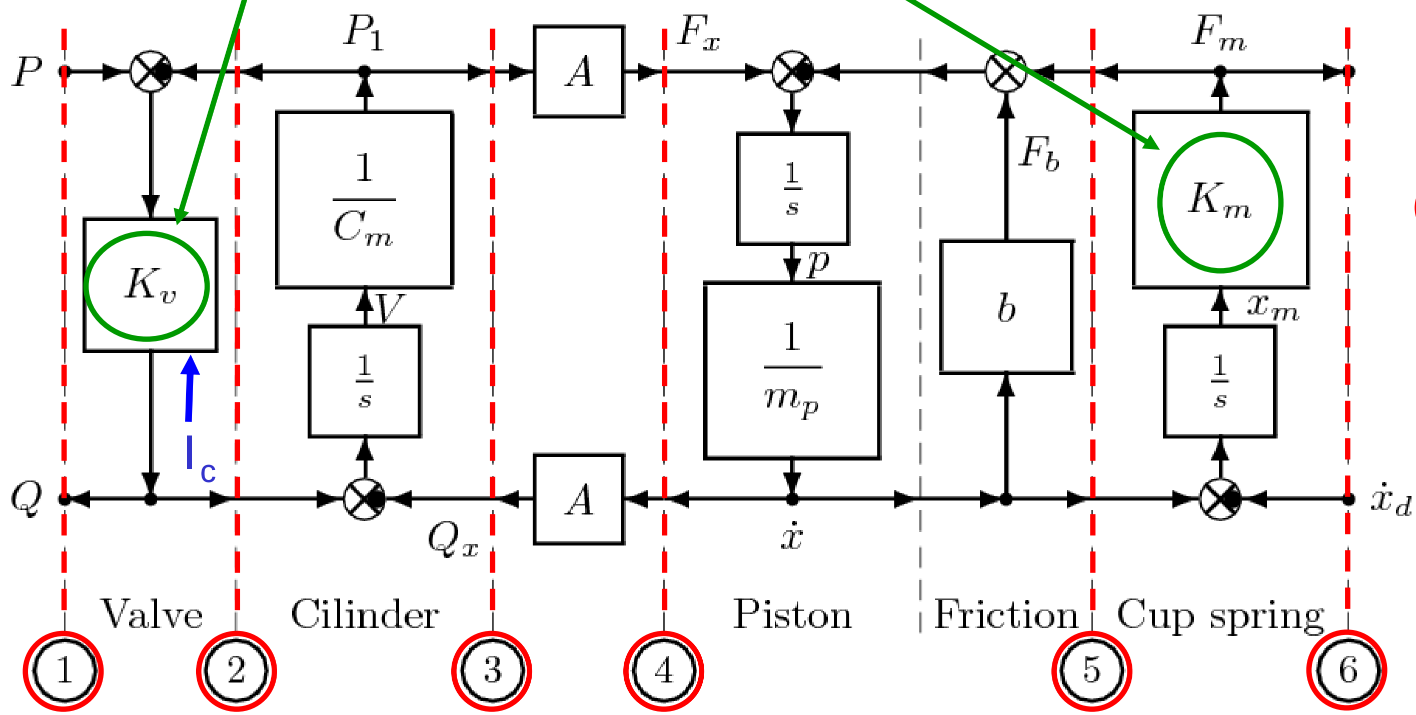
Matrix ***K=K(x,v)*** that can also be a function of state variables ***x*** or external variables ***v*.**

Example of POG modeling: an electro-hydraulic clutch

The electro-hydraulic clutch:



Strongly nonlinear elements



Control input

Dynamic Modeling of Physical Systems

Different Energy domains:

1) Electrical; 2) Mechanical (tras./rot.); 3) Hydraulic; etc.

The same dynamic structure:

- 2 "dynamic" elements D_1, D_2 that store energy;
- 1 "static" element R that dissipates (or generates) energy;
- 2 "energy variables" $q_1(t)$, $q_2(t)$ used for describing the stored energy;
- 2 "power variables" $v_1(t)$, $v_2(t)$ used for moving the energy;

		Electrical		**Mechanical**		**Hydraulic**	
	$\mathcal{D}_1$	C	Capacitor	M	Mass	C_I	Hyd. Capacitor
	q_1	Q	Charge	p	Momentum	V	Volume
Across-variables	v_1	V	Voltage	v	Velocity	P	Pressure
	$\mathcal{D}_2$	L	Inductor	E	Spring	L_I	Hyd. Inductor
	q_2	ϕ	Flux	x	Displacement	ϕ_I	Hyd. Flux
Through-variables	v_2	I	Current	F	Force	Q	Flow
	$\mathcal{R}$	R	Resistor	b	Friction	R_I	Hyd. Resistor

In POG the new symbols and the new definitions are "minimal"

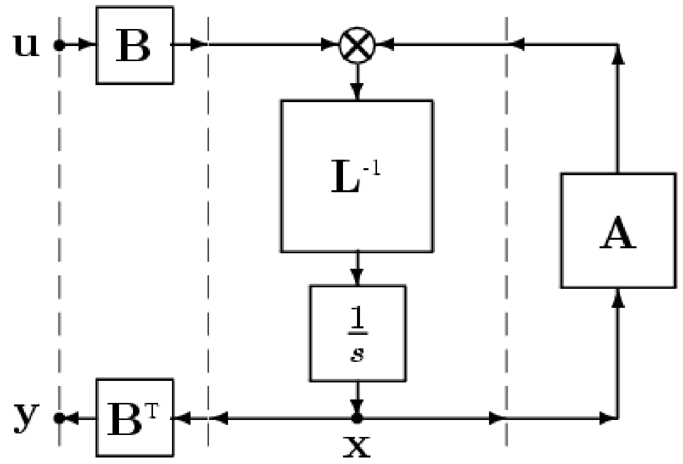
Introduction
Power-Oriented Graphs - LTI Systems

- The POG are suitable for representing both scalar and vectorial systems.
- A direct correspondence between POG and state space descriptions:

$$\begin{cases} \mathbf{L}\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \\ \mathbf{y} = \mathbf{B}^T\mathbf{x} \end{cases}$$

Stored Energy: $E_s = \frac{1}{2}\mathbf{x}^T\mathbf{L}\mathbf{x}$

Dissipating Power: $P_d = \mathbf{x}^T\mathbf{A}\mathbf{x}$



- When an eigenvalue of matrix ***L*** goes to zero (or to infinity), the system degenerates towards a lower dynamic dimension system. The “reduced” can be obtained by using “congruent” transformation $x = \mathbf{T}z$ (matrix ***T*** is constant and it can be rectangular):

$$\begin{cases} \mathbf{T}^T\mathbf{L}\mathbf{T}\dot{\mathbf{z}} = \mathbf{T}^T\mathbf{A}\mathbf{T}\mathbf{z} + \mathbf{T}^T\mathbf{B}\mathbf{u} \\ \mathbf{y} = \mathbf{B}^T\mathbf{T}\mathbf{z} \end{cases} \Leftrightarrow \begin{cases} \overline{\mathbf{L}}\dot{\mathbf{z}} = \overline{\mathbf{A}}\mathbf{z} + \overline{\mathbf{B}}\mathbf{u} \\ \mathbf{y} = \overline{\mathbf{B}}^T\mathbf{z} \end{cases}$$

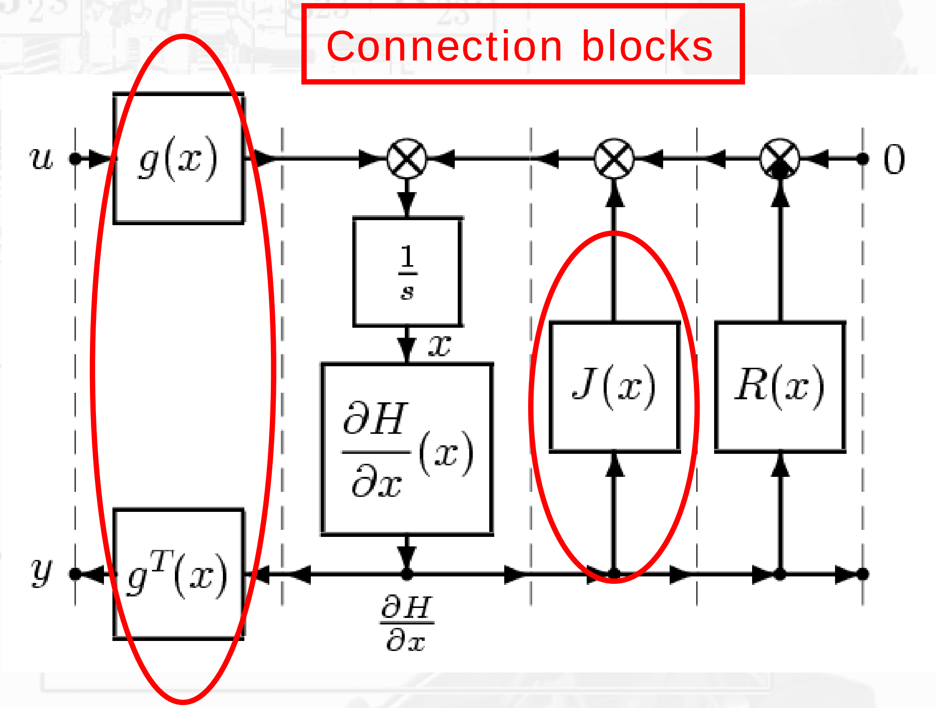
Drawback: the POG graphical schemes are not so compact as in Bond Graphs

Introduction
Port-Controlled Hamiltonian Systems

The Port-Controlled Hamiltonian Systems (PCH) can be used to model a wide variety of physical systems.

$$\begin{cases} \dot{x} = [J(x) - R(x)] \dfrac{\partial H}{\partial x}(x) + g(x)u \\ y = g^T(x) \dfrac{\partial H}{\partial x}(x) \end{cases}$$

$$J(x) = -J^T(x),\ R(x) = R^T(x) \geq 0.$$



Power balance:

$$y^T u = \frac{dH}{dt} + \frac{\partial H}{\partial x}^T R \frac{\partial H}{\partial x} \geq \frac{dH}{dt}$$

Basically, PCH are defined by:

- a geometric structure J(x), g(x), R(x) that captures the interconnection and dissipation laws;
- a Hamiltonian function H(x) given by the total energy stored of the system;
- a power port (u,y) that defines the exchanges of power between the system and the external world.

Port-Controlled Hamiltonian Systems
Interconnection of subsystems

Many PCH can be obtained connecting different subsystems. The general form of the power preserving interconnection element is:

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 & A \\ -A^T & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

Matrix **A** can also be time varying and/or state dependent.

Standard PCH cannot describe systems with a direct dissipation between the input **u** and the output **y**. A resistor is the simplest example.

Extended definition for PCH with "direct dissipation" and "direct interconnection":

$$\begin{cases} \dot{x} = [J_1(x,v) - R_1(x,v)] \dfrac{\partial H}{\partial x}(x) + g(x,v)\, u \\ y = g^T(x,v) \dfrac{\partial H}{\partial x}(x) + [R_2(x,v) - J_2(x,v)]\, u \end{cases}$$

direct interconnections

direct dissipations

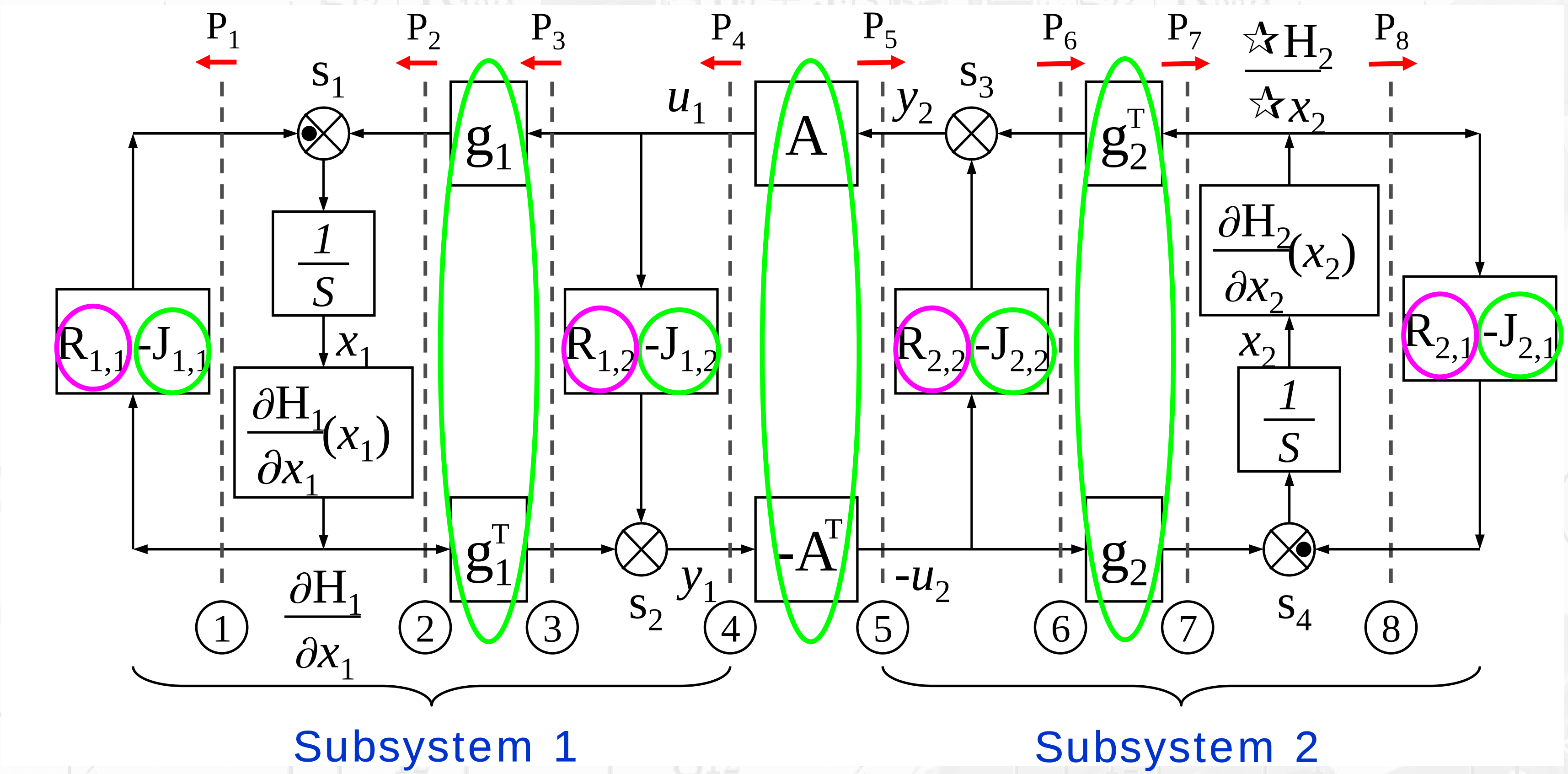
$$J_i(x,v) = -J_i^T(x,v), \quad R_i(x,v) = R_i^T(x,v) \geq 0$$

Power balance:

$$\frac{dH}{dt} = y^T u - \frac{\partial H}{\partial x}^T R_1(x,v) \frac{\partial H}{\partial x} - u^T R_2(x,v) u$$

Interconnected Systems
Relation between POG and PCH

The POG representation of two interconnected PCH:

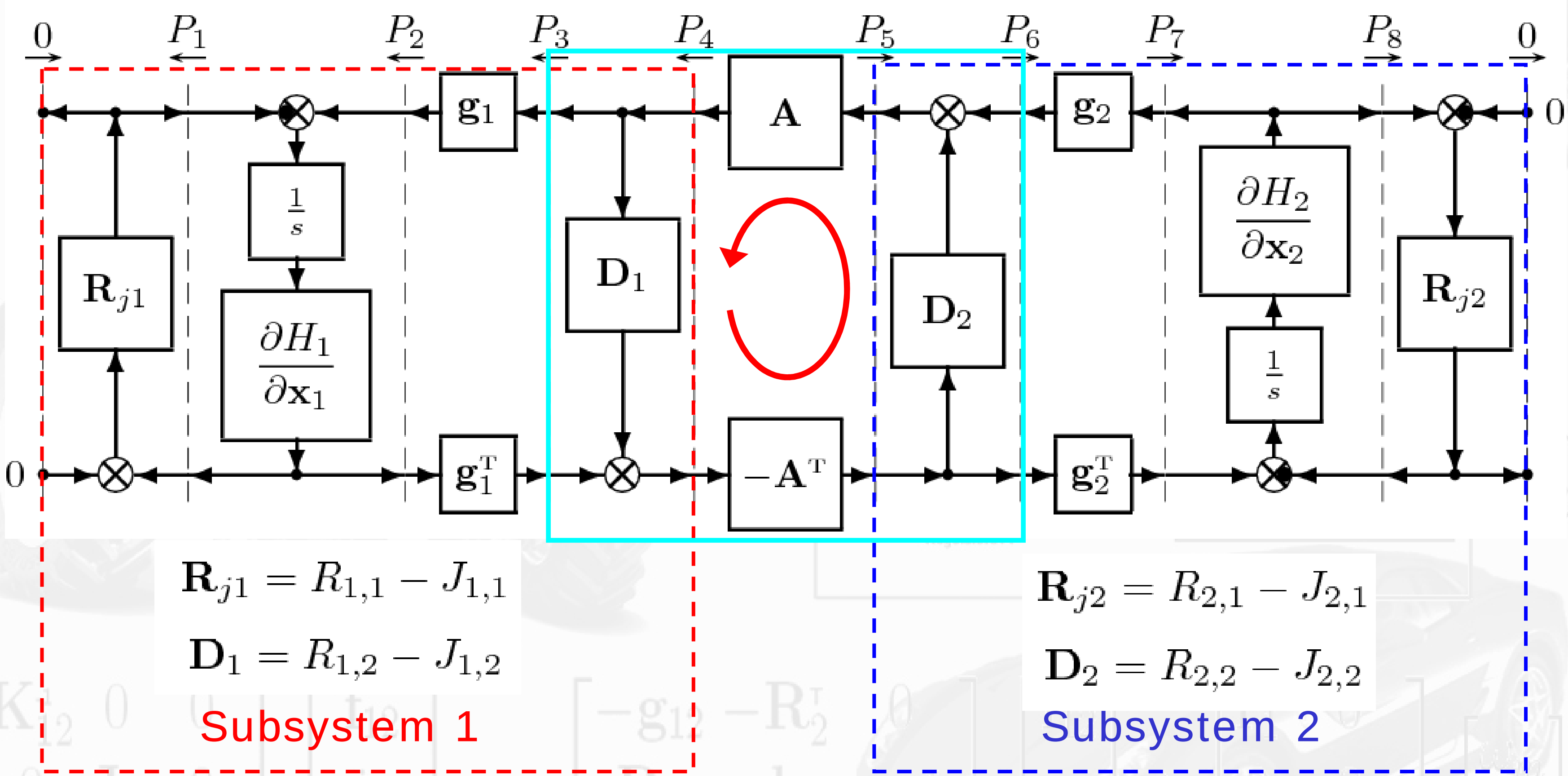


Dissipations

Connections

Interconnected Systems
Relation between POG and PCH

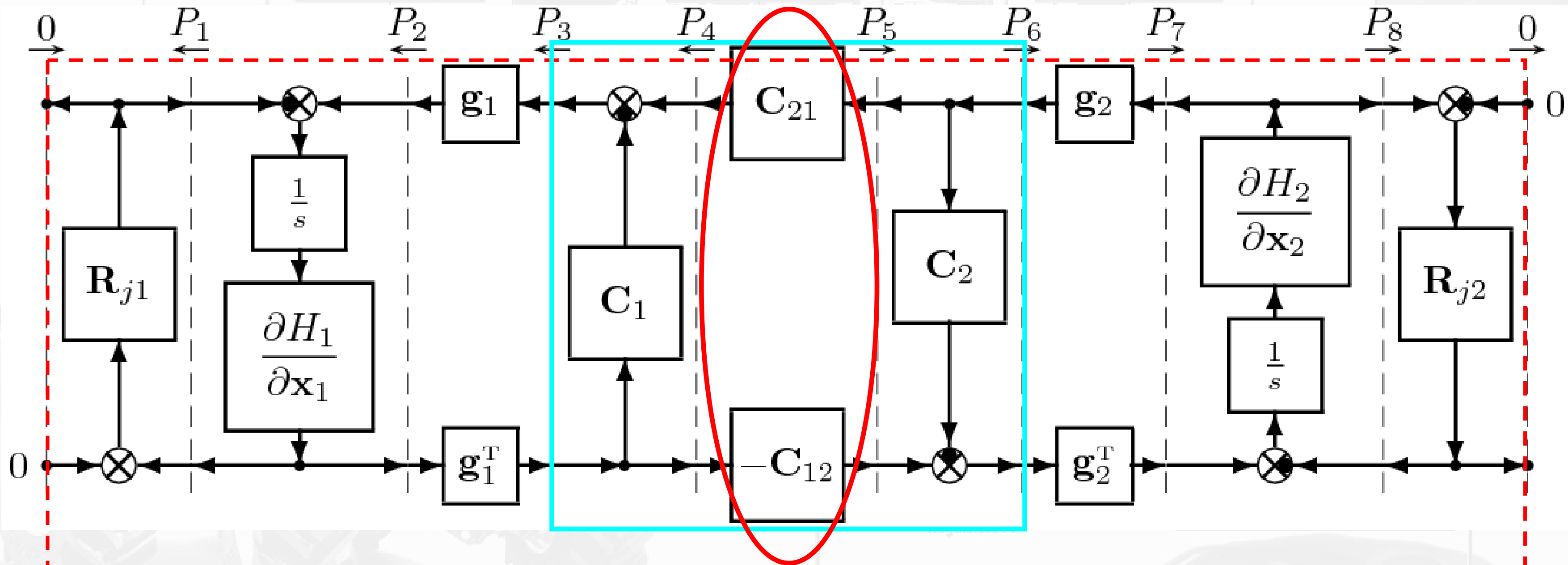
The POG representation of two interconnected PCH:



Note: in the center of the block scheme is present an algebric loop

Interconnected Systems
Relation between POG and PCH

The POG representation of two interconnected PCH without algebric loop:



$$\mathbf{C}_1 = (\mathbf{I} + \mathbf{A}\mathbf{D}_2\mathbf{A}^T\mathbf{D}_1)^{-1}\mathbf{A}\mathbf{D}_2\mathbf{A}^T$$
$$\mathbf{C}_2 = (\mathbf{I} + \mathbf{A}^T\mathbf{D}_1\mathbf{A}\mathbf{D}_2)^{-1}\mathbf{A}^T\mathbf{D}_1\mathbf{A}$$

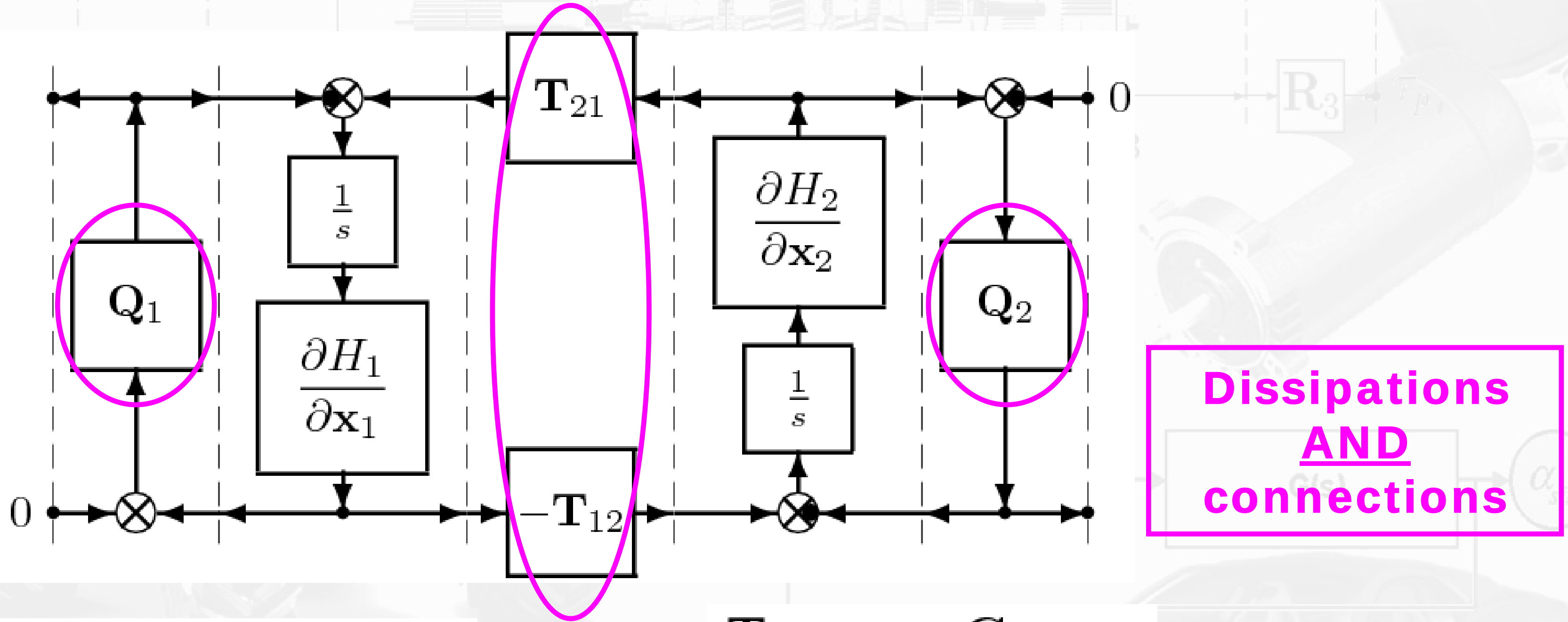
$$\mathbf{C}_{12} = (\mathbf{I} + \mathbf{A}^T\mathbf{D}_1\mathbf{A}\mathbf{D}_2)^{-1}\mathbf{A}^T$$
$$\mathbf{C}_{21} = (\mathbf{I} + \mathbf{A}\mathbf{D}_2\mathbf{A}^T\mathbf{D}_1)^{-1}\mathbf{A}$$

In general, it is NOT a connection block

$$\mathbf{C}_{21} \neq \mathbf{C}_{21}^T$$

Interconnected Systems
Relation between POG and PCH

Two interconnected PCH: the POG reduced system



$$\mathbf{Q}_1 = \mathbf{R}_{j1} + \mathbf{g}_1 \mathbf{C}_1 \mathbf{g}_1^{\mathrm{T}}$$
$$\mathbf{Q}_2 = \mathbf{R}_{j2} + \mathbf{g}_2 \mathbf{C}_2 \mathbf{g}_2^{\mathrm{T}}$$

$$\mathbf{T}_{21} = \mathbf{g}_1 \mathbf{C}_{21} \mathbf{g}_2$$
$$\mathbf{T}_{12} = \mathbf{g}_2^{\mathrm{T}} \mathbf{C}_{12} \mathbf{g}_1^{\mathrm{T}}$$

Subsystem 1 + Subsystem 2

POG and PCH Interconnected Systems
Example of three connected electric circuits

The RLR circuit:

$$\dot{\varphi} = [-r_{1,1}] \frac{\partial H_1}{\partial \varphi} + [1 \quad -1] \begin{bmatrix} V_{i1} \\ V_{i2} \end{bmatrix}$$

$$\begin{bmatrix} I_{o1} \\ I_{o2} \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \frac{\partial H_1}{\partial \varphi} + R_{1,2} \begin{bmatrix} V_{i1} \\ V_{i2} \end{bmatrix}$$

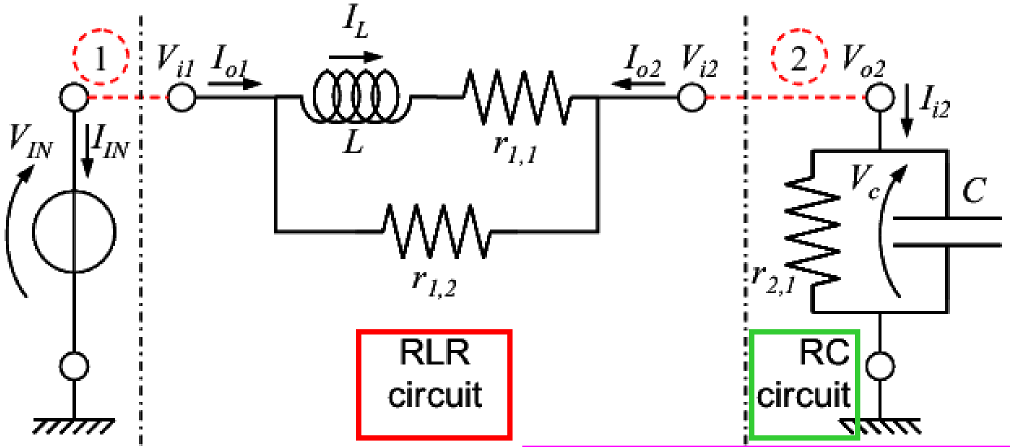
Matrix $R_{1,2}$:

$$R_{1,2} = \begin{bmatrix} 1/r_{1,2} & -1/r_{1,2} \\ -1/r_{1,2} & 1/r_{1,2} \end{bmatrix}$$

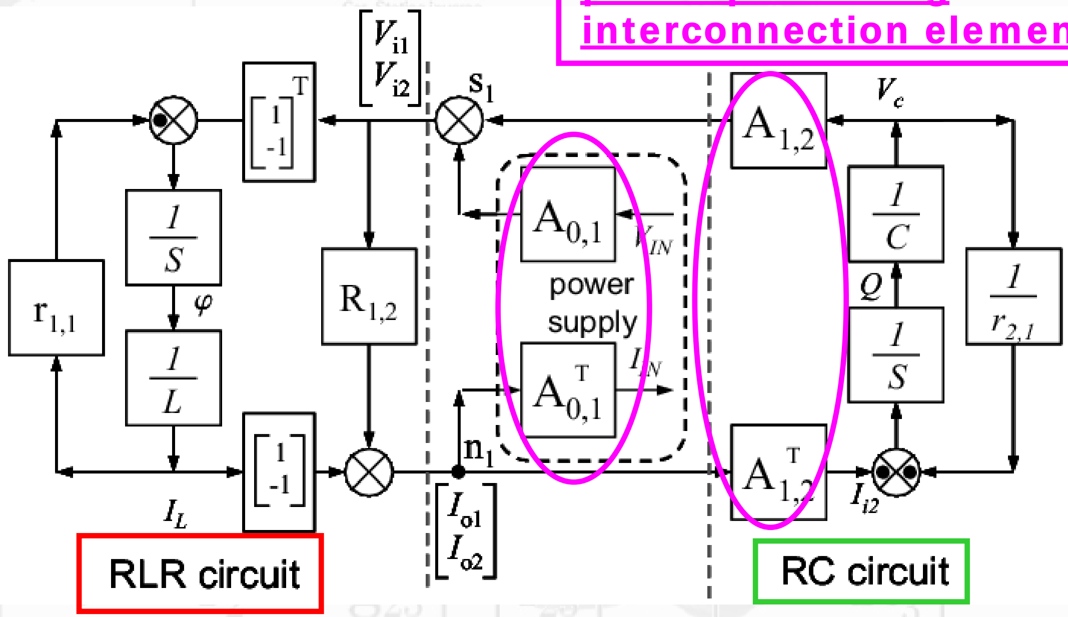
The RC circuit:

$$\dot{Q} = \left[-\frac{1}{r_{2,1}}\right] \frac{\partial H_2}{\partial Q} + I_{i,2}$$

$$V_{o2} = \frac{\partial H_2}{\partial Q} = V_c$$

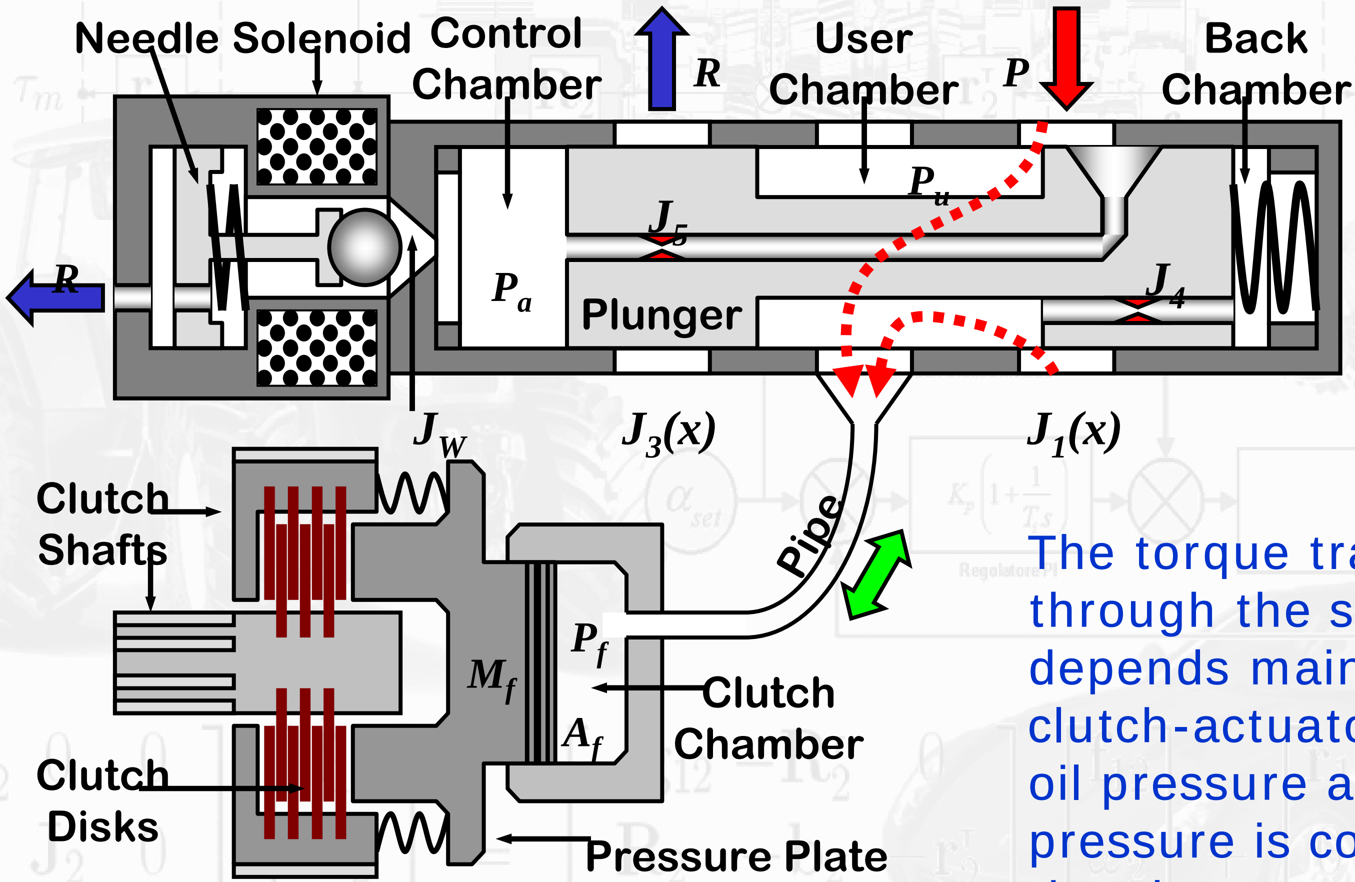


power preserving interconnection element



Application example: Electro-hydraulic clutch actuator

A simplified scheme of an Electro-hydraulic clutch actuator:

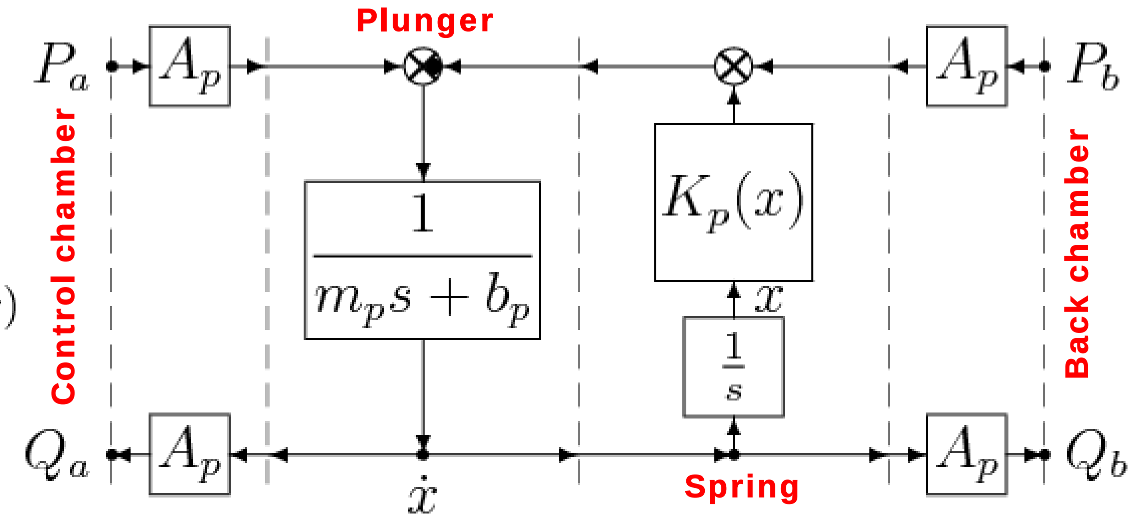


The torque transmitted through the shafts depends mainly on the clutch-actuator internal oil pressure and this pressure is controlled by the electro-valve.

Electro-hydraulic clutch actuator: the POG model

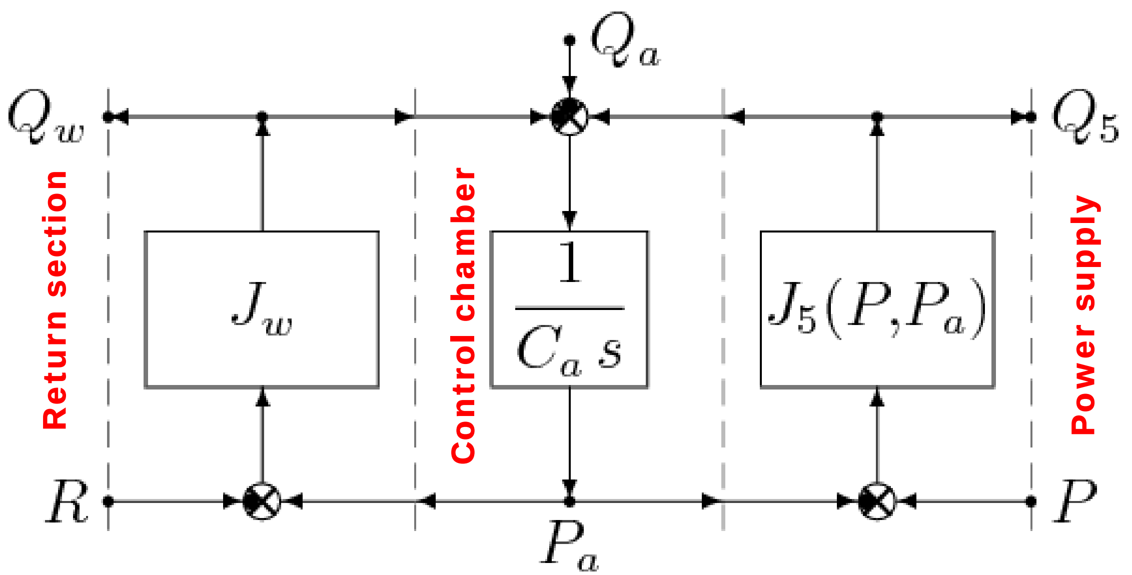
POG model of the valve plunger:

$$m_p \ddot{x} = (P_a - P_b)A_p - b_p \dot{x} - K_p(x)$$
$$Q_a = Q_b = A_p \dot{x}$$



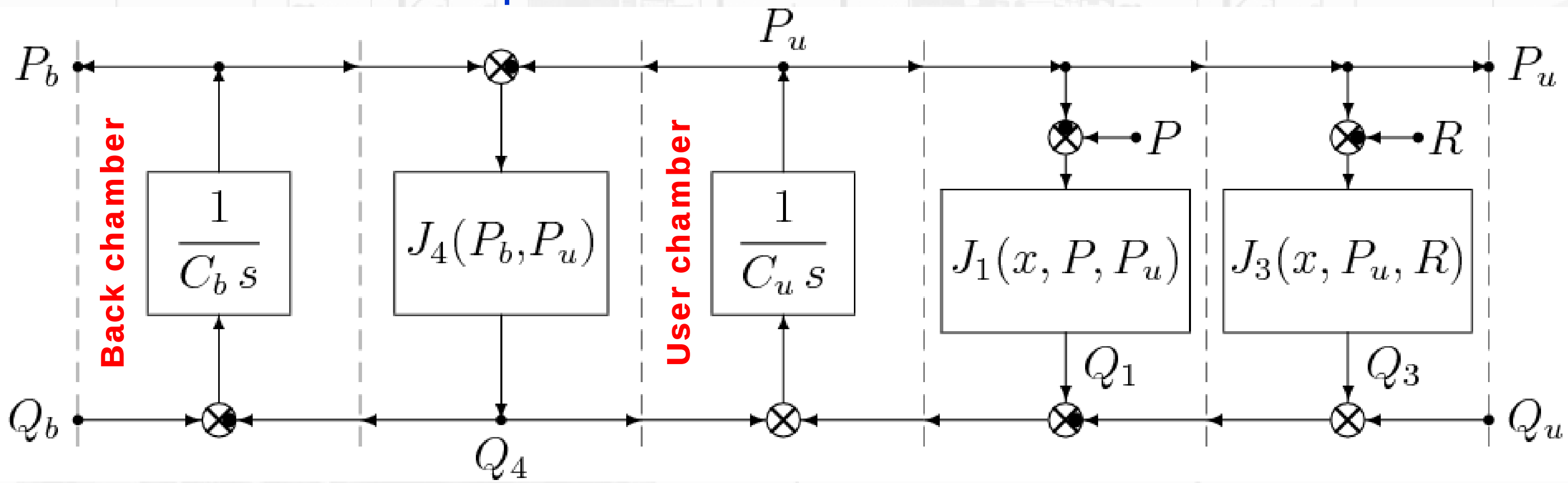
POG model of the control chamber:

$$C_a \dot{P}_a = Q_5 - Q_a - Q_w$$

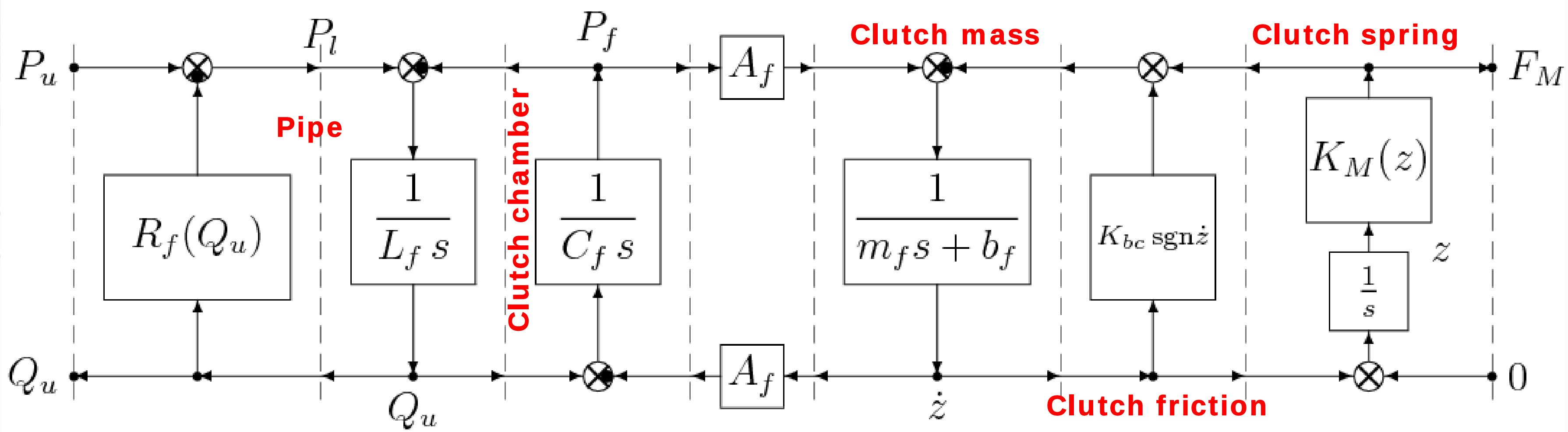


Electro-hydraulic clutch actuator: The POG model

Back chamber and output user chamber POG models:



Clutch actuator POG model:



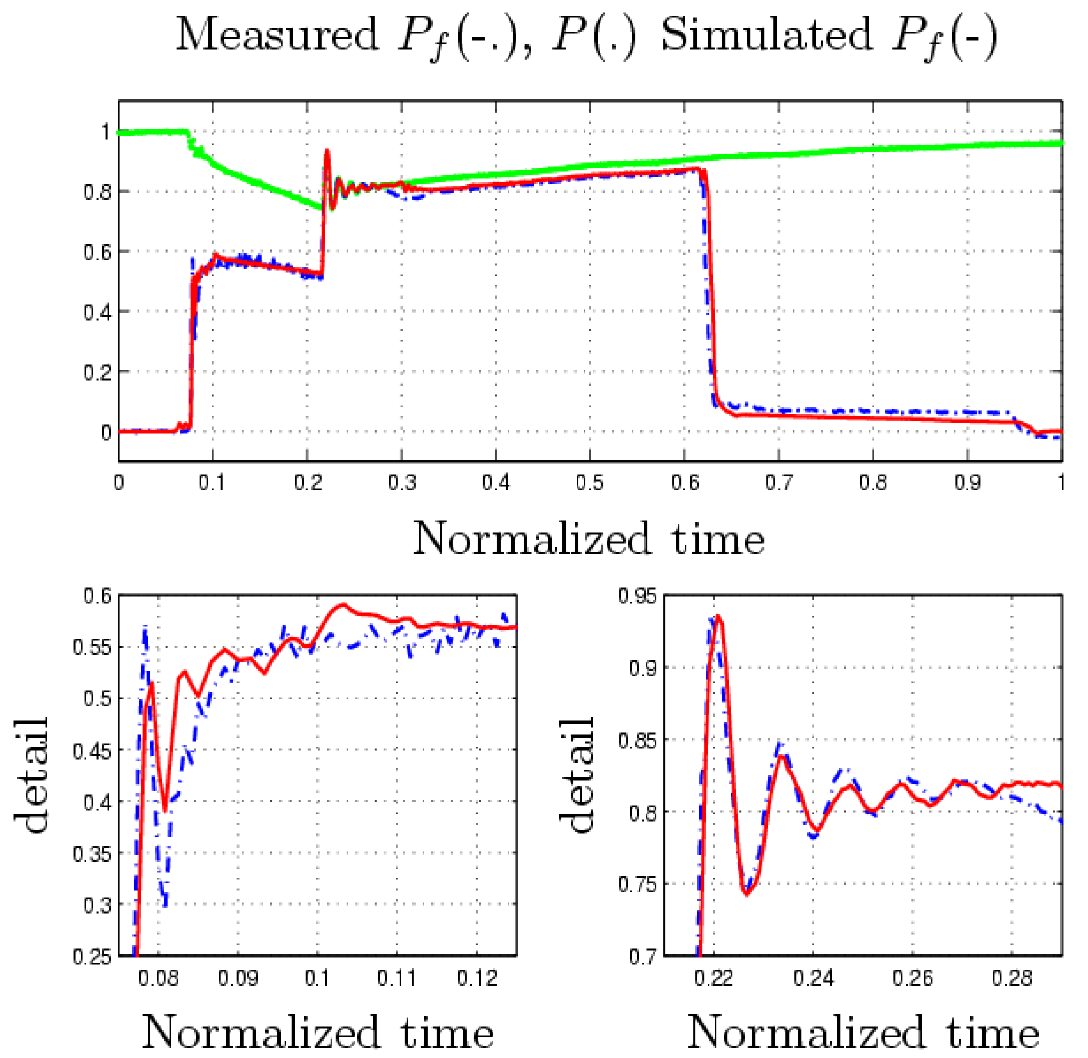
Electro-hydraulic clutch actuator: Simulation and experimental results (1/2)

Step input measurements.

Supply pressure P (dotted)

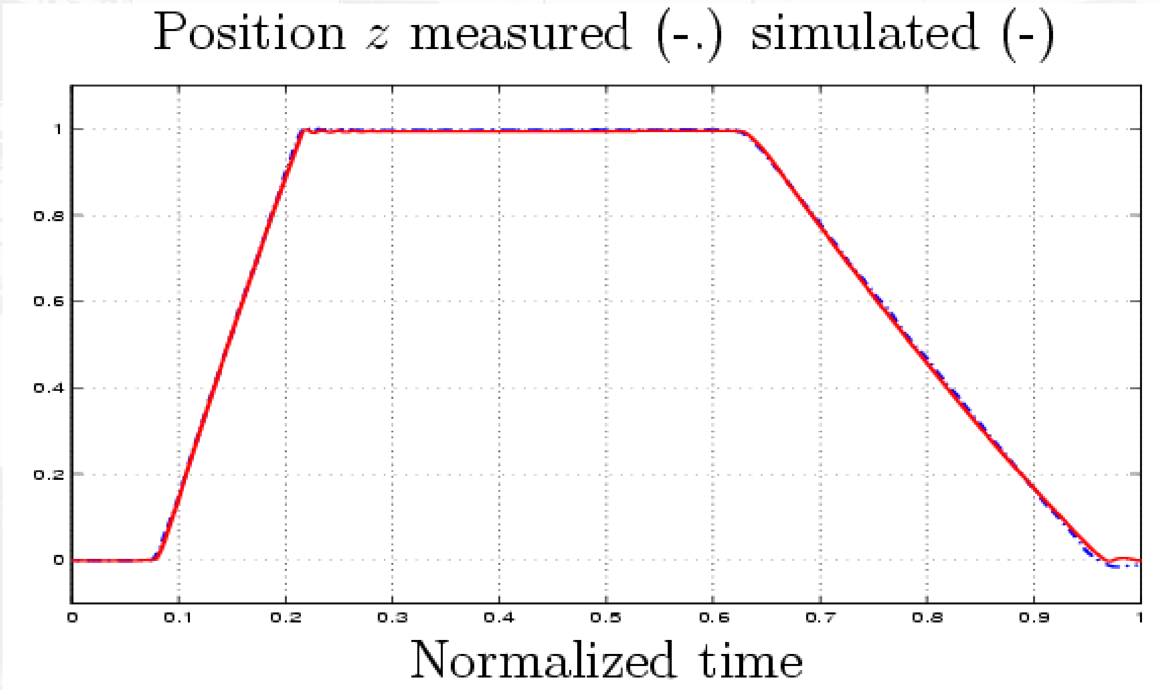
Clutch chamber pressure P_f :

- measured (dash-dotted)
- simulated (solid)



Electrovalves:

Electro-hydraulic clutch actuator: Simulation and experimental results (2/2)



Clutch attuator posizion *z*:

- measured (dash-dotted)
- simulated (solid)

The figure axes are normalized.

The simulations are very similar to the experimental data proving that the modeling approach is suitable to this kind of multi-domain systems.

Conclusions

- The paper has addressed the problem of modeling physical systems by using the Power-Oriented Graphs (POG) modeling technique.
- The Port-Controlled Hamiltonian (PCH) systems has been graphically described by using the POG modeling technique.
- A slight extension of the definition of PCH has been given introducing the “direct dissipations” and “direct connections”.
- The POG technique has been used for modeling a clutch control system. The obtained simulation results are very similar to the experimental data.

Thank you for your attention!