Dynamic Modeling and Control of a Car Transmission System

R. Zanasi*, A. Visconti†, G. Sandoni*, R. Morselli*

*DSI, Dipartimento di Scienze dell'Ingegneria,
University of Modena and Reggio Emilia, Via Vignolese 905, 41100 Modena, Italy,
Phone: +39 59 2056161, Fax +39 59 2056126; E-mail: roberto.zanasi@unimo.it

†Ferrari SpA, Innovation Department, Via Abetone Inf.re 4, 41053 Maranello, Modena, Italy,
Phone: +39 536 949402, Fax +39 536 949594; E-mail: AVisconti@ferrari.it

Abstract— **In the paper, the dynamic model of a car transmission system is taken into account and a simple control strategy for controlling the transmitted torque is presented. All the main components of the transmission system (the engine, the clutch, the gear-box, the differential, etc.) have been modeled in details by using the graphical modeling technique named Power-Oriented Graph. A particular attention has been paid to the model of the clutch system because of its importance in the modulation of the transmitted torque. A simple control strategy for controlling the torque transmitted during the "start" of the car is also presented. Simulation results show the usefulness of the model and the effectiveness of the presented control strategy.**

I. Introduction

One of the key elements for controlling the global dynamic behavior of a car is the transmission system. In fact, a good control of a car can be obtained by properly controlling the torque transmitted to the vehicle. The component of the transmission system that directly modulates and controls the transmitted torque is the clutch. The availability of a good dynamic model for the clutch and for the whole transmission system is an important element for designing good strategies for controlling the dynamic behavior of the car during all the critical working situations.

In the paper, all the main components of the transmission system (the engine, the clutch, the gear-box, etc.) are modeled, at different levels of details, by using a graphical modeling technique named Power-Oriented Graph (POG) [1], [2] and [3]. The main idea of this graphical technique is to use the "power interaction" between sub-systems as basic element for modeling. The "bond graph" technique [4], [5] is based on the same idea, but uses a different graphical representation. By keeping "coupled" the variables which are "conjugate" with respect to power, these graphical techniques provide graphical dynamic models which, usually, are intuitive and easy to use.

The component of the transmission system that has been modeled with more details because of its importance in controlling the transmitted torque is the clutch with in series the torsional dumper-spring. Unfortunately, the simulation of this block is critical due to the presence of two coulomb fractions in series in the same dynamic element: the dynamic order of the block changes according to the particular working condition. To simplify the simulation of this element, a particularly efficient state space transformation has been used.

A fast "start" of the car is one of the more critical situation for the control of the transmitted torque. A simple control strategy for controlling the torque transmitted during the start of the car through the use of the clutch is also presented. The effectiveness of the control strategy is tested by simulation.

The paper is organized as follows: the basic concepts of the POG graphical technique are reported in Section I-A. The dynamic models of all the components of the car transmission system are described in Section II. A simplified model of the system is reported in Section III. The control strategy for controlling the clutch and the transmitted torque together with some simulation results are presented in Section IV.

A. Power-Oriented Graphs: basic concepts

The "Power-Oriented Graphs" are "signal flow graphs" combined with a particular "modular" structure essentially based on the two blocks shown in Fig. 1. The basic characteristic of this modular structure is the direct correspondence between pairs of system variables and real power flows: the product of the two variables involved in each dashed line of the graph has the physical meaning of "power flowing through the section". The two basic blocks

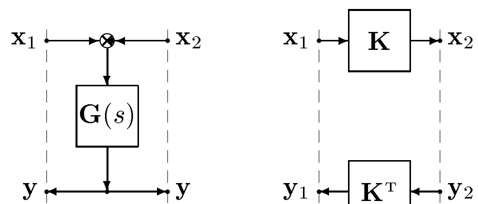

Fig. 1. Basic blocks: elaboration block and connection block.

shown in Fig. 1 are named "elaboration block" and "connection block". There is no restriction on variables $\mathbf{x}$ and $\mathbf{y}$ other than the fact that the inner product $\langle \mathbf{x}, \mathbf{y} \rangle = \mathbf{x}^T \mathbf{y}$ must have the physical meaning of a "power". The elaboration and connection blocks are suitable for representing both scalar and vectorial systems. In the vectorial case, $\mathbf{G}(s)$ and $\mathbf{K}$ are matrices: $\mathbf{G}(s)$ is always square, $\mathbf{K}$ can also be rectangular. While the elaboration block can store and dissipate energy (i.e. springs, masses and

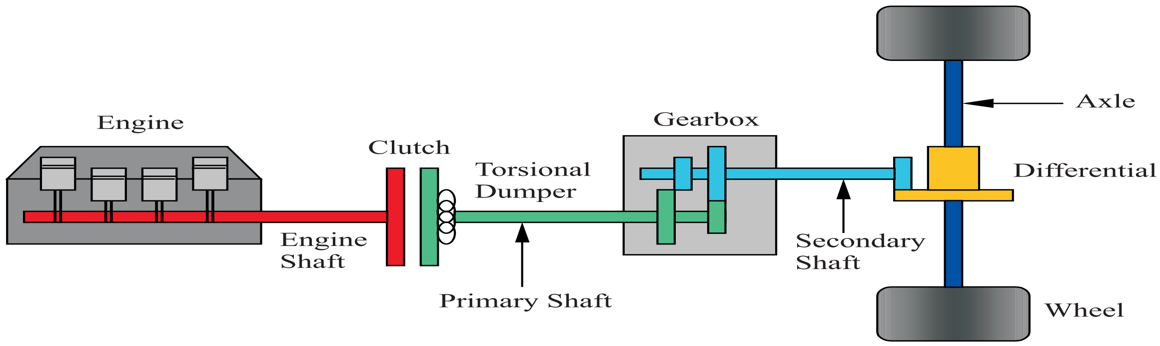

Fig. 2. Simplified representation of a car transmission system.

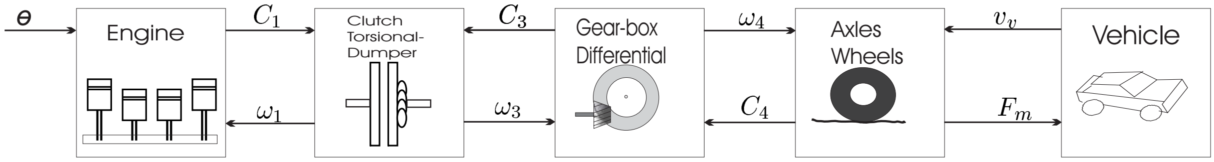

Fig. 3. Power-Oriented graphical representation of the transmission system.

dampers), the connection block can only "transform" the energy, that is, transform the system variables from one type of energy-field to another (i.e. any type of gear reduction). Main characteristics of the POG technique are: a direct correspondence between the POG blocks and the real parts of the system; the POG schemes can be easily transformed, both graphically and mathematically; the state space mathematical model of a system can be "directly" obtained from the corresponding POG representation. For a more detailed description of the POG graphical technique, please refer to [1], [2] and [3].

II. Dynamic Model of the Transmission System

A simplified representation of a car transmission system is shown in Fig. 2. It is composed by: the engine, the clutch, the torsional dumper-spring, the gearbox, the differential, the axles and the wheels. A Power-Oriented graphical representation of the transmission system is shown in Fig. 3. The variables which are present in this figure have the following meaning: θ is the position of the inlet throttle, C_1 is the output engine torque, ω_1 is the engine shaft angular velocity, C_3 is the low gear torque, ω_3 is the low gear shaft angular velocity, C_4 is the axles torque, ω_4 is the axles shaft angular velocity, v_v is the vehicle velocity and F_m is the force acting on the ground. The block scheme of Fig. 3 clearly shows the power connections between the main components of the transmission system. In the following sections, the five components of the transmission system are described in more details.

A. The engine

For the engine, the following simplified first order model has been considered:

$$C_1 = C_G - b_1(\omega_1), \qquad C_G(s) = \frac{K}{1+\tau_p\, s}\,\theta(s)$$

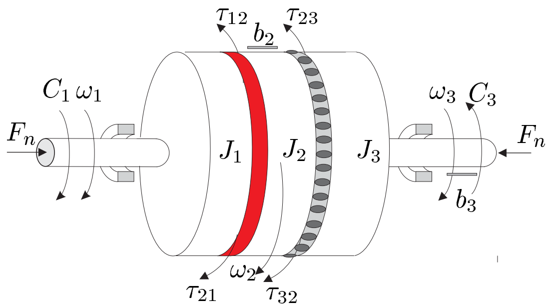

Fig. 4. Schematic representation of the clutch with the torsional dumper-spring.

where θ the throttle position, C_G is the generated torque, C_1 is the output available torque, ω_1 is the engine angular velocity, $b_1 = b_1(\omega_1)$ is a non-linear function that represents the friction acting on the engine shaft, K and τ_p are parameters describing the first order engine dynamics.

B. Clutch with torsional dumper-spring

Among all the elements of a transmission system, the clutch is the most difficult to be modeled due to the presence of a torsional dumper-spring between the clutch disk and the primary shaft of the gear. The goal of this element is to filter the torque spikes generated by the engine. A schematic representation of the clutch with the torsional dumper-spring is shown in Fig. 4. The differential equations describing the clutch with torsional dumper-spring, during the slipping, are:

$$\left\{\begin{array}{rcl} J_1\,\dot{\omega}_1 &=& C_1 - k_{12}\,\mathrm{sgn}(\omega_1-\omega_2) \\ J_2\,\dot{\omega}_2 &=& -b_2\,\omega_2 + k_{12}\,\mathrm{sgn}(\omega_1-\omega_2) \\ && -k_{23}\,\mathrm{sgn}(\omega_2-\omega_3) - \phi_e(\theta_d) \\ J_3\,\dot{\omega}_3 &=& -C_3 - b_3\,\omega_3 + k_{23}\,\mathrm{sgn}(\omega_2-\omega_3) \\ && +\phi_e(\theta_d) \\ \dot{\theta}_d &=& \omega_2-\omega_3 \end{array}\right. \qquad (1)$$

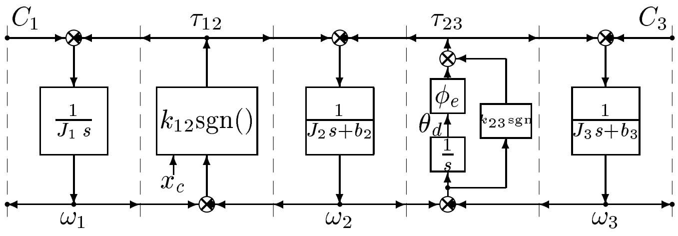

Fig. 5. POG graphical representation of the torsional dumper-spring.

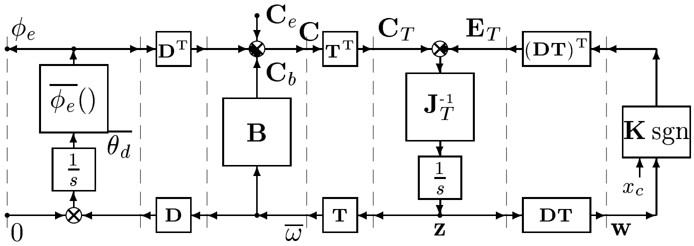

Fig. 6. Transformed POG graphical representation of the clutch with torsional dumper-spring.

where: J_1, J_2, J_3 are the inertia momenta of the engine shaft and flywheel, torsional dumper disk and transmission shaft, respectively; C_1, C_3 are the engine torque and the resistant external torque; ω_1, ω_2, ω_3 are the angular velocities of the engine, torsional dumper disk and primary shaft; b_1, b_2, b_3 are the viscous friction coefficients of the engine shaft, torsional dumper disk and transmission shaft; k_{12}, k_{23} are the coulomb friction: k_{23} is assumed to be constant, while k_{12} is a function of the clutch pedal position x_c (see Fig. 12); $\phi_e = \phi_e(\theta_d)$ is the elastic torque characteristics of the torsional dumper-spring as function of the relative position $\theta_d = \theta_2 - \theta_3$ of the disk. A POG graphical representation of the equations (1) is shown in Fig. 5. The simulation of system (1) is quite critical due to the presence of the two coulomb frictions $k_{12}\,\mathrm{sgn}()$ and $k_{23}\,\mathrm{sgn}()$. The system (1) is a "dynamic changing structure system". In fact while functioning, it changes the order and the type of its differential equations. The simulation of this type of systems can be simplified by using a state space transformation proposed in [6] and here briefly summarized. If we introduce the following vectors and matrices:

$$\mathbf{C}_e = \begin{bmatrix} C_1 \\ 0 \\ -C_3 \end{bmatrix}, \quad \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}, \quad \overline{\omega} = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix},$$

$$\overline{\theta_d} = \begin{bmatrix} \theta_1 - \theta_2 \\ \theta_2 - \theta_3 \end{bmatrix}, \; \mathbf{J} = \begin{bmatrix} J_1 & 0 & 0 \\ 0 & J_2 & 0 \\ 0 & 0 & J_3 \end{bmatrix}, \; \mathbf{B} = \begin{bmatrix} b_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & b_3 \end{bmatrix},$$

$$\mathbf{D} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}, \; \mathbf{K} = \begin{bmatrix} k_{12} & 0 \\ 0 & k_{23} \end{bmatrix}, \; \overline{\phi_e} = \begin{bmatrix} 0 & 0 \\ 0 & \phi_e \end{bmatrix}$$

system (1) can be rewritten in matrix form $\mathbf{J}\,\dot{\overline{\omega}} = \mathbf{C} - \mathbf{E}$ as follows:

$$\mathbf{J}\,\dot{\overline{\omega}} = \underbrace{\mathbf{C}_e - \mathbf{B}\,\overline{\omega} - \mathbf{D}^{\mathrm{T}}\overline{\phi_e}(\overbrace{\mathbf{D}\,\theta}^{\overline{\theta_d}})}_{\mathbf{C}} - \underbrace{\mathbf{D}^{\mathrm{T}}\,\mathbf{K}\;\mathrm{sgn}(\overbrace{\mathbf{D}\,\overline{\omega}}^{\mathbf{w}})}_{\mathbf{E}} \qquad (2)$$

System (2) can be simplified by using the following congruent space state transformation:

$$\overline{\omega} = \mathbf{T}\,\mathbf{z}, \qquad \mathbf{T} = \left[\begin{array}{c|cc} 1 & \frac{J_2+J_3}{\Delta} & \frac{J_3}{\Delta} \\ 1 & -\frac{J_1}{\Delta} & \frac{J_3}{\Delta} \\ 1 & -\frac{J_1}{\Delta} & -\frac{J_1+J_2}{\Delta} \end{array}\right]$$

$$\mathbf{z} = \mathbf{T}^{-1}\overline{\omega} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} \frac{J_1\omega_1 + J_2\omega_2 + J_3\omega_3}{\Delta} \\ \omega_1 - \omega_2 \\ \omega_2 - \omega_3 \end{bmatrix}$$

where $\Delta = J_1 + J_2 + J_3$. The components of the new state vector $\mathbf{z}$ have the following physical meaning: z_1 is the weighted mean velocity (the weights in this case are the inertias), while z_2 and z_3 are the relative velocities of one inertia with respect to the following. By using transformation $\overline{\omega} = \mathbf{Tz}$, system (2) can be rewritten as $\mathbf{J}_T\,\dot{\mathbf{z}} = \mathbf{C}_T(\mathbf{z}) - \mathbf{E}_T$, that is:

$$\underbrace{\mathbf{T}^{\mathrm{T}}\mathbf{J}\mathbf{T}}_{\mathbf{J}_T}\,\dot{\mathbf{z}} = \underbrace{\mathbf{T}^{\mathrm{T}}\mathbf{C}}_{\mathbf{C}_T} - \underbrace{(\mathbf{DT})^{\mathrm{T}}\mathbf{K}\,\mathrm{sgn}(\overbrace{\mathbf{DTz}}^{\mathbf{w}})}_{\mathbf{E}_T} \qquad (3)$$

A POG graphical representation of the transformed systems (3) is shown in Fig. 6. The first part of the block scheme describes how the torque $\mathbf{C}$ is obtained as a function of the external torque $\mathbf{C}_e$, the viscous coefficients $\mathbf{B}$ and the elastic contributions $\overline{\phi}_e(\overline{\theta_d})$; the second part of the block scheme directly describes equation (3). The block scheme of Fig. 6 is quite general and it can also be applied to "dynamic changing structure systems" of higher order. In the three dimensional case, equation (3), left multiplied by matrix $\mathbf{J}_T^{-1}$, simplifies as:

$$\begin{cases} \dot{z}_1 = \frac{C_{T1}}{\Delta} \\ \begin{bmatrix} \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} \frac{J_1+J_2}{J_1 J_2} & -\frac{1}{J_2} \\ -\frac{1}{J_2} & \frac{J_2+J_3}{J_2 J_3} \end{bmatrix} \begin{bmatrix} C_{T2} - k_{12}\,\mathrm{sgn} z_2 \\ C_{T3} - k_{23}\,\mathrm{sgn} z_3 \end{bmatrix} \end{cases} \qquad (4)$$

The state space transformation $\overline{\omega} = \mathbf{T}\,\mathbf{z}$ decomposes the original system in two independent parallel systems: variable z_1 is not influenced by variables z_2 and z_3, and viceversa. The second part of system (4), describing the relative dynamics, can be interpreted as a two dimensional *Variable Structure System* that, due to the presence of the switching inputs $k_{i-1,i}\,\mathrm{sgn}(z_i)$, converges towards the sliding surfaces $z_2 = 0, z_3 = 0$. When one of the sliding surfaces $z_i = 0$ is reached, a *sliding mode* can arise in the system. When the system is in sliding mode the term $k_{j-1,j}\ \mathrm{sgn}(z_j)$ switches at infinite frequency and the variable z_j is kept at zero. In this condition the dynamic dimension of the model decreases of one unit. The simulation of the transformed model (4) is easier than the original one. For a detailed description of the transformation method and the simulation algorithm, please refer to [6].

C. Gear-box and differential

The POG block scheme of the gear-box and the differential is shown in Fig. 7: C_3 is the primary-gear torque; C_4

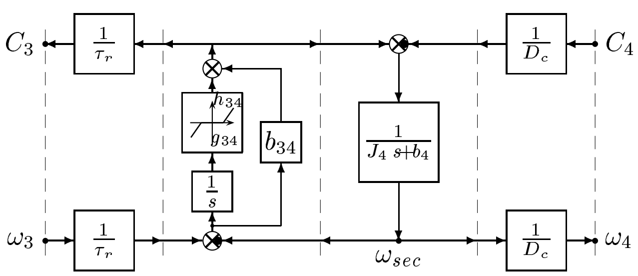

Fig. 7. POG block scheme of the gear-box and the differential.

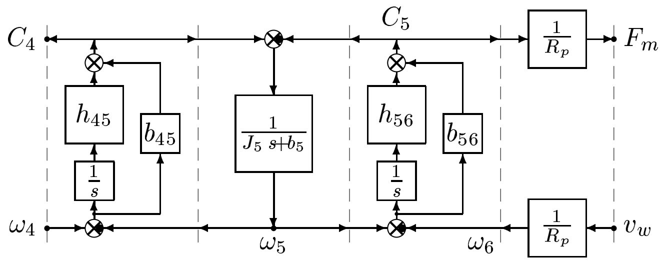

Fig. 8. POG block scheme of the axles and the wheels.

is the torque at the output of the differential; the gear-box, in first approximation, has been modeled as a transformation ratio τ_r which is a function of the engaged gear; g_{34} is the semi-amplitude of the teeth backlash; h_{34} is the teeth stiffness. The transformation ratio τ_r is defined as the ratio between the primary shaft angular velocity ω_3 and the secondary shaft angular velocity ω_{sec}: $\tau_r = \frac{\omega_3}{\omega_{sec}}$. The differential is assumed to be an ideal block that transforms the angular velocity ω_{sec} into the input semi-axle angular velocity ω_4: $\omega_4 = \frac{\omega_{sec}}{D_c}$. Parameter g_{34} describes the global backlash of the gear-box and the differential teeth; J_4 is total inertia of the secondary shaft and the differential, b_{34} is the relative friction coefficient of the gear-box, and b_4 is the viscous friction coefficient of the differential.

D. Axles and Wheels

The POG block scheme of the axles and the wheels is shown in Fig. 8. The presence of a high torque at low velocity leads to think that axles and wheels have to be considered as elastic elements. The deformation of the axles and the wheels is taken into account with the stiffness coefficients h_{45} and h_{56}, while their dumps are described by the friction coefficients b_{45} and b_{56}. Moreover, in Fig. 8, C_4 is the axles torque, C_5 is the torque transmitted to the wheels, ω_4 is the axles angular velocity, ω_5 is the wheels angular velocity, F_m is the force acting between wheels and ground, v_w is the wheels linear velocity, R_p is the wheel radius, J_5 and b_5 are the global inertia momentum and viscous friction coefficient of the axles and the wheels.

E. Vehicle

The POG block scheme of the vehicle is shown in Fig. 9. The vehicle dynamics is described by the following variables: v_v is the vehicle velocity, $v_r = v_w - v_v$ is the relative velocity, k_w is the wheel-ground friction coefficient, M_v is

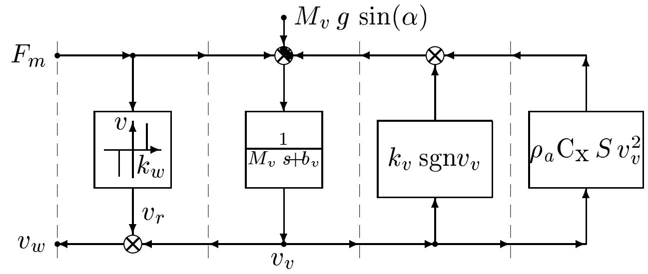

Fig. 9. POG block scheme of the vehicle.

the overall vehicle mass, b_v is the vehicle friction coefficient, k_v is the volvent friction force, g is the gravity acceleration, ρ_a is the air density, C_X is the air penetration coefficient, S is the trasverse section of the vehicle, α is the slope of the ground. In Fig. 9, the first block represents the sliding of the wheels over the ground when the transmitted force F_m exceeds the coulomb friction k_w.

III. Model Reduction

The dynamic model presented in Section II could be too much complex, so a simplified model could be useful to simulate the behavior of the transmission system. A four order simplified POG model of the transmission system is shown in Fig. 10. This reduced model has been obtained with the following assumptions: the torsional dumper-spring is assumed to have an infinite stiffness, so $\omega_2 = \omega_3$ and $k_{13}(x_c) = k_{12}(x_c)$; inertia J_{25} is obtained assuming $h_{34}, h_{45}, h_{56} \to \infty$, and the overall stiffness h_t is computed assuming $J_4, J_5 \to 0$:

$$\begin{aligned} J_{25} &= J_2 + J_3 + \frac{J_4}{\tau_r^2} + \frac{J_5}{\tau_r^2\,D_c^2}, & g_t &\simeq g_{34}, \\ h_t^{-1} &= \frac{1}{h_{34}} + \frac{\tau_r^2\,D_c^2}{\frac{h_{45}\,h_{56}}{h_{45}+h_{56}}}, & \tau_t &= \tau_r\,D_c \end{aligned}$$

This simplified model is used in the following section for testing the proposed control strategy.

IV. Clutch control strategy

Both the complete transmission model and the simplified one shown in Fig. 10 can be used in simulation to test the clutch control system. Since the most challenging problem is the behavior of the vehicle during the start, this chapter proposes a simple control strategy that automatically acts on the clutch to obtain the desired performance, then it is tested by simulation experiments. When the car is moving and the clutch is completely engaged, the torque τ_{13} transmitted by the clutch is equal to the output torque $C_1 = C_G - b_1(\omega_1)$ supplied by the engine: $\tau_{13} = C_1$. The driver, using the accelerator, modifies the supplied torque C_1 and consequently the car dynamics. When the clutch is slipping (see Fig. 11) the torque $\tau_{13} = k_{13}(x_c)\,\mathrm{sgn}(\omega_1 - \omega_3)$ completely determines the dynamics of the car, but the dynamics of the engine shaft is:

$$J_1 \dot{\omega}_1 = C_1 - k_{13}(x_c)\,\mathrm{sgn}(\omega_1 - \omega_3) = C_1 - \tau_{13} \tag{5}$$

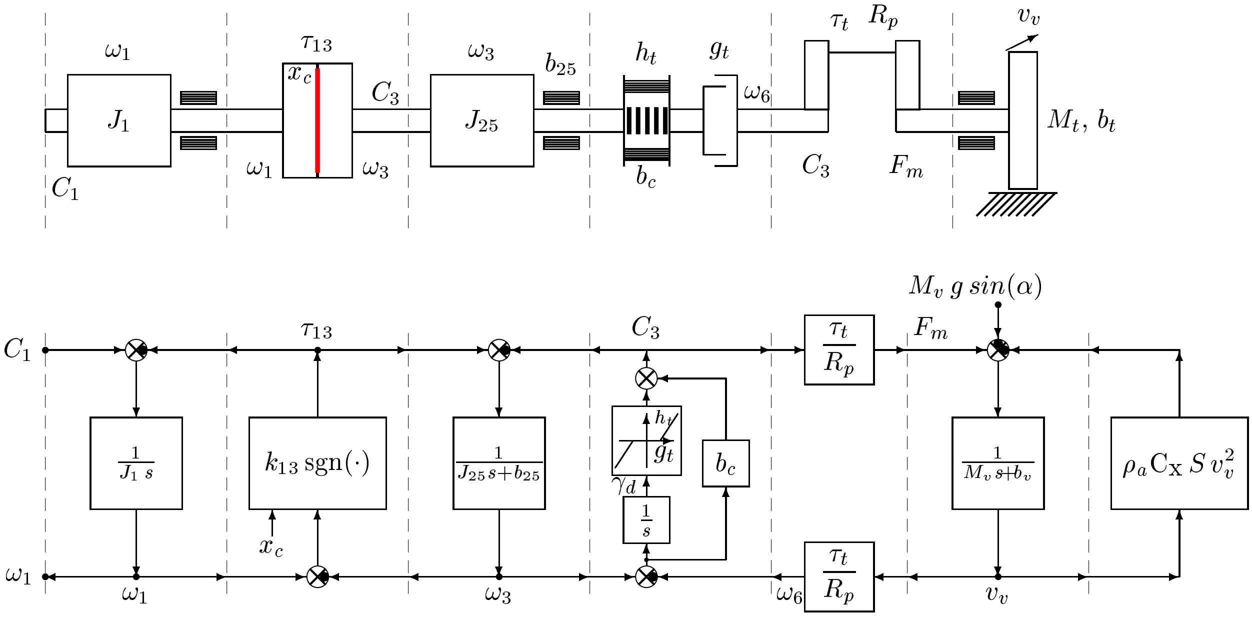

Fig. 10. Four order simplified POG model of the transmission system.

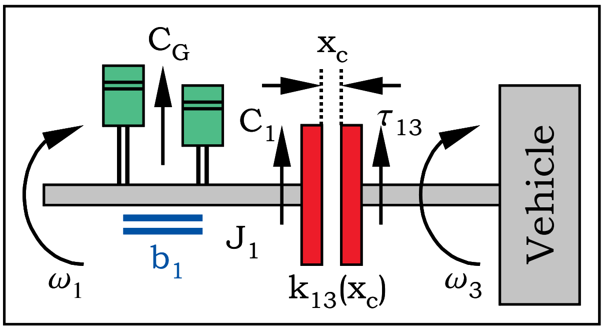

Fig. 11. Simplified scheme of a transmission

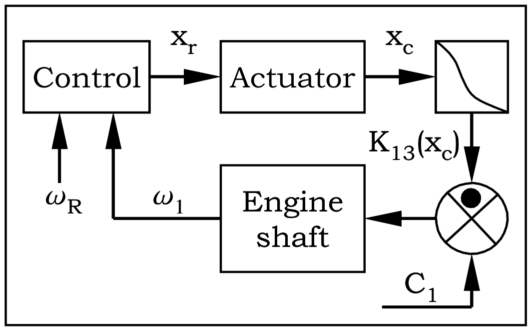

Fig. 13. Clutch control scheme

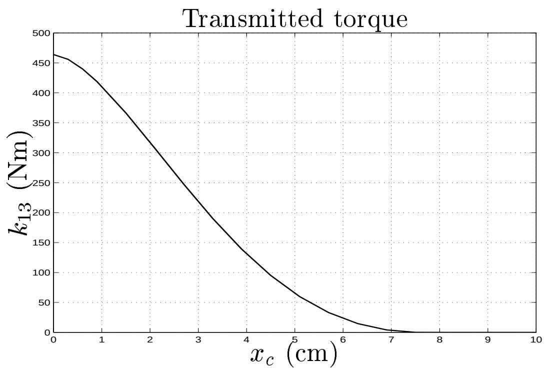

Fig. 12. Clutch pedal position x_c and corresponding transmitted torque k_{13}.

where $k_{13}(x_c)$ is the torque transmitted by the coulomb friction and it is a function of the position x_c of the clutch pedal. Figure 12 shows an example of function $k_{13}(x_c)$. If the clutch control system is able to act on k_{13} through x_c in such a way that the velocity ω_1 of the engine shaft follows a slowly variable reference ω_r, it will be $\omega_1 \simeq \omega_r$ and $\dot{\omega}_1 \simeq \dot{\omega}_r \simeq 0$. Then, from eq. (5) it follows that $C_1 \simeq \tau_{13}$.

Namely the control system gives to the car the same dynamics as when the clutch is engaged. The driver does not feel the difference because it is always the supplied engine torque C_1 that determines the dynamics of the car. Moreover, the proposed control, based on a velocity reference ω_r, prevents engine shut down or shaft speed spikes. These considerations lead to the control scheme shown in Fig. 13, where the torque C_1 can be seen as a noise that must be faced. The key element of the proposed control strategy is represented by the electro-hydraulic actuator that acts on the position x_c of the clutch pedal. The performance of the system increases as the time response of the actuator decreases. There exist many kinds of such actuators with different features, see [7] and [8]. The simplest way to model them is to consider a second order system whose input is the position reference signal x_r and the output is the position x_c:

$$X_C(s) = \frac{\omega_a^2}{s^2 + 2\delta_a \omega_a s + \omega_a^2} X_R(s)$$

The control system has to define the reference signal x_r comparing the reference velocity ω_r with the actual shaft velocity ω_1. The control law depends mainly on the transfer function of the actuator; the simplest one is the following:

$$x_r = x_{r0} + k_r(\omega_r - \omega_1)\ \mathrm{sgn}(\omega_1 - \omega_3) \qquad (6)$$

where k_r is a positive gain, x_{r0} is the default position of the actuator when the angular velocity error is null. During a normal start $\omega_1 > \omega_3$, the gear primary shaft can act as a brake on the engine shaft and the magnitude of the braking effect is given by the clutch coulomb friction k_{13}. When $\omega_1 > \omega_r$ the engine shaft is too fast and the control law (6) decreases x_r such that, see Fig.12, the friction k_{13} increases and brakes the engine shaft and viceversa. Instead, if $\omega_3 > \omega_1$ the control law (6) must compute x_r according to the fact that the gear primary shaft can only increase the velocity of the engine shaft if the clutch is closed. Finally, when $\omega_3 = \omega_1$, the start procedure ends and the clutch can be entirely engaged. The simulation results obtained with $x_{r0} = 0.07$ m, $k_r = 3 \cdot 10^{-4} \frac{\mathrm{m}}{\mathrm{srad}}$, $\omega_a = 200 \frac{\mathrm{rad}}{\mathrm{s}}$ and $\delta_a = 0.8$ are shown in Fig. 14. The start begins at $t = 0.5$ s when the reference ω_r rises to 1600 rpm and then it remains constant. As the engine shaft velocity ω_1 becomes higher than the reference ω_r, the position reference x_r decreases to brake the engine and avoid a speed spike. According to eq. (5), almost all the torque C_1 is transmitted through the clutch friction to the vehicle and the speed increases. When $t \simeq 1.5$ s the driver, acting on the accelerator pedal, reduces the torque C_G. To avoid the engine shut down, the control law (6) increases the position of the clutch pedal. Despite the torque reduction, the velocity ω_1 still follows the reference ω_r and the torque τ_{13} transmitted to the vehicle is almost equal to C_1. Finally, when $t \simeq 2.5$ s, $\omega_1 = \omega_3$ and the clutch is entirely engaged.

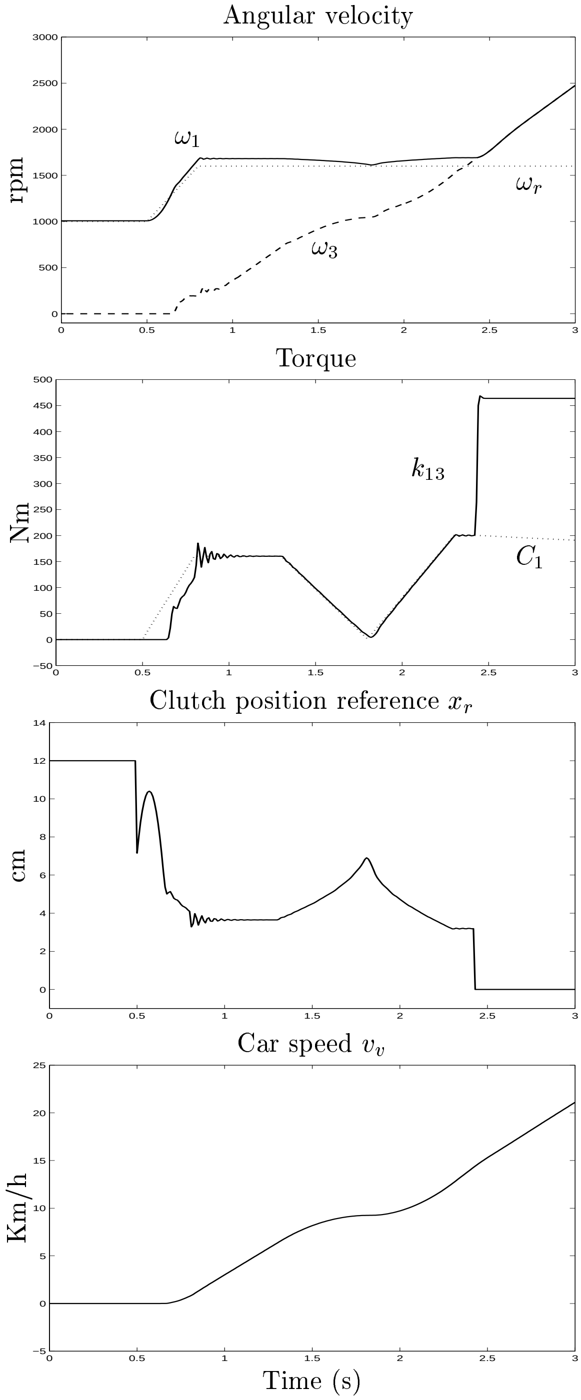

Fig. 14. Simulation results

V. Conclusions

The paper addressed the problem of modeling a car transmission system. All the components of the kinematics chain, from the engine to the wheels, have been considered. The Power-Oriented Graphs technique has been used for describing the power flows through the transmission system. A simplified low order model has been also proposed. The reduced model has been used to test a clutch control strategy, whose purpose is to control the torque transmitted during the start. The performance of the controlled model has been tested through simulation experiments.

References

[1] Zanasi, R.,"Power-Oriented Modeling of Dynamical System for Simulation", Symp. on Modeling and Control of Tech. Sys., Lille, France, 1991.

[2] Zanasi, R., "Power-Oriented Graphs for Modelling Robotic Systems", Second Int. Symposium MIM-S^2 '93, Special Session on "Intelligent Models in Robotics", Brussels, Belgium, 1993.

[3] R. Zanasi, "Dynamics of a n-links Manipulator by Using Power-Oriented Graphs", submitted for SYROCO '94, Capri, Italy, 19-21 September 1994.

[4] Paynter, H.M., "Analysis and Design of Engineering Systems", MIT-press, Camb., MA, 1961.

[5] Karnopp, D.C., Rosemberg R. C., "System dynamics: a unified approach", Wiley, N.Y, 1975.

[6] R. Zanasi, G. Sandoni, R Morselli *"Simulation of dynamic changing structure systems: the clutch example"* submitted to European Control Conference *ECC-2001*.

[7] R. Morselli, "Modellistica e controllo di una frizione per auto ad alte prestazioni di guida", Degree Thesis A.A. 1999-2000 Università degli studi di Modena e Reggio Emilia.

[8] G. Sandoni, "Modellistica dinamica e controllo di una frizione per F1 ", Degree Thesis A.A. 1998-1999 Univerità degli studi di Modena e Reggio Emilia.

[9] J.H. Taylor, *Rigorous Handling of State Events in MATLAB*, 1995 IEEE International Conference on Control Application, pp 156-161, 1995.

[10] S. B. Lee, E. A. Misawa and D. A. Lucca *Sliding Mode Compensation of Dry Friction*, Proceedings of the 1996 IEEE International Conference on Control Applications, Dearborn,Mi September 15-18, 1996, Page(s) 809-813.