Traction system

The traction system of an automatic subway is taken as example in Fig. 1.

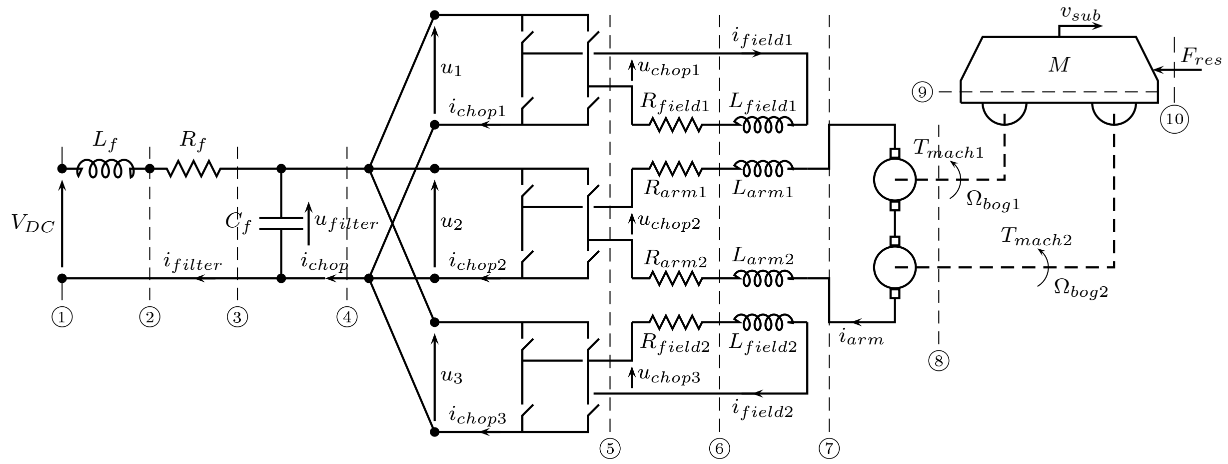

Figura 1: Power structure of the subway traction system.

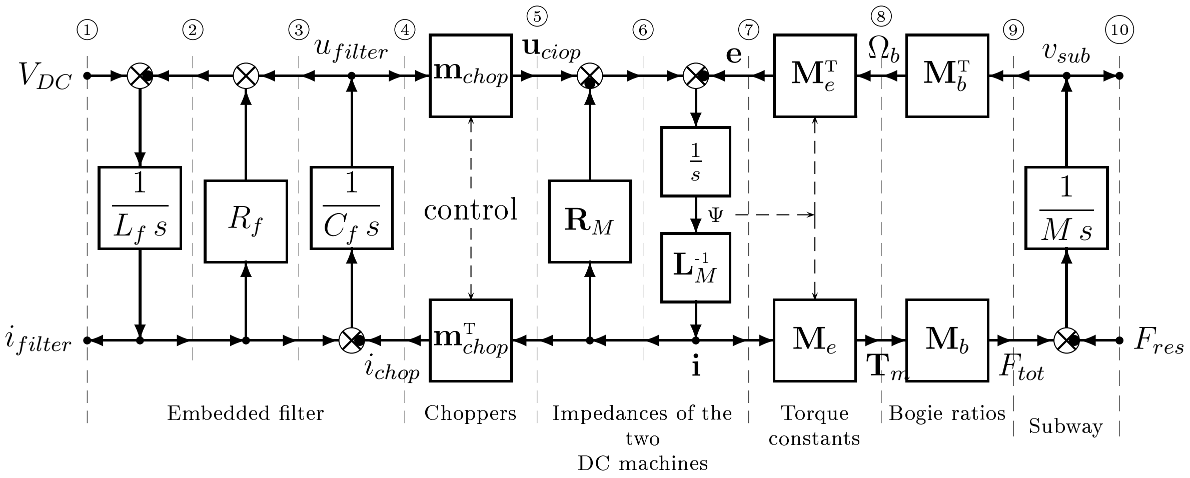

Figura 2: The POG scheme of the subway traction system shown in Fig. 1.

The POG scheme of the considered subway traction system is shown in Fig. 2. The scheme is characterized by the following matrices and vectors:

$$\mathbf{m}_{chop} = \begin{bmatrix} m_{chop1} \\ m_{chop2} \\ m_{chop3} \end{bmatrix}, \qquad \mathbf{R}_M = \begin{bmatrix} R_{field1} & 0 & 0 \\ 0 & R_{arm} & 0 \\ 0 & 0 & R_{field1} \end{bmatrix}, \qquad \mathbf{L}_M = \begin{bmatrix} L_{field1} & 0 & 0 \\ 0 & L_{arm} & 0 \\ 0 & 0 & L_{field1} \end{bmatrix},$$

$$\mathbf{M}_e = \begin{bmatrix} 0 & k_{mach1} i_{field1} & 0 \\ 0 & k_{mach2} i_{field2} & 0 \end{bmatrix}, \quad \mathbf{M}_b = \begin{bmatrix} m_{bog1} \; m_{bog2} \end{bmatrix}, \quad \mathbf{M}_T = \mathbf{M}_b \mathbf{M}_e, \quad \mathbf{i} = \begin{bmatrix} i_{field1} \\ i_{arm} \\ i_{field2} \end{bmatrix},$$

$$\mathbf{u}_{chop} = \begin{bmatrix} u_{chop1} \\ u_{chop2} \\ u_{chop3} \end{bmatrix}, \quad \Psi = \begin{bmatrix} \psi_{field1} \\ \psi_{arm} \\ \psi_{field2} \end{bmatrix}, \quad \mathbf{e} = \begin{bmatrix} 0 \\ e_{arm} \\ 0 \end{bmatrix}, \quad \mathbf{\Omega}_b = \begin{bmatrix} \Omega_{bog1} \\ \Omega_{bog2} \end{bmatrix}, \quad \mathbf{T}_m = \begin{bmatrix} T_{mach1} \\ T_{mach1} \end{bmatrix},$$

where $i_{field1} = \psi_{field1}/L_{field1}$, $i_{field2} = \psi_{field2}/L_{field2}$, $L_{arm} = L_{arm1} + L_{arm2}$, $R_{arm} = R_{arm1} + R_{arm2}$ and $e_{arm} = e_{arm1} + e_{arm2}$. Note that each dashed line present in the POG scheme (labeled with circled numbers) represents a power section which is in correspondence with a particular real section of the considered physical system.

From the POG scheme of Fig. 2 it is also easy to write down the dynamic equations of the system in the state space form:

$$\underbrace{\begin{bmatrix} L_f & 0 & 0 & 0 \\ 0 & C_f & & 0 \\ 0 & 0 & \mathbf{L}_M & 0 \\ 0 & 0 & 0 & M \end{bmatrix}}_{\mathbf{L}} \underbrace{\begin{bmatrix} \dot{i}_{filter} \\ \dot{u}_{filter} \\ \dot{\mathbf{i}} \\ \dot{v}_{sub} \end{bmatrix}}_{\dot{\mathbf{x}}} = \underbrace{\begin{bmatrix} -R_f & -1 & 0 & \\ 1 & 0 & -\mathbf{m}_{chop}^{\mathrm{T}} & \\ 0 & \mathbf{m}_{chop} & -\mathbf{R}_M & -\mathbf{M}_T^{\mathrm{T}} \\ 0 & 0 & \mathbf{M}_T & 0 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} i_{filter} \\ u_{filter} \\ \mathbf{i} \\ v_{sub} \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix}}_{\mathbf{B}} \underbrace{\begin{bmatrix} V_{DC} \\ F_{res} \end{bmatrix}}_{\mathbf{u}} \tag{1}$$

where $\mathbf{x}$ and $\mathbf{u}$ are the *state* and the *input vectors*, and $\mathbf{L}$, $\mathbf{A}$ and $\mathbf{B}$ are the *energy*, the *power* and the *input* matrices of the system. Equations (1) can be rewritten in a compact form as $\mathbf{L}\,\dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} + \mathbf{B}\,\mathbf{u}$. For linear system the following properties holds: 1) the quadratic function $E = \frac{1}{2}\mathbf{x}^{\mathrm{T}}\mathbf{L}\mathbf{x}$ represents the *total energy* stored in the system; 2) Let $\mathbf{A}_s = \frac{1}{2}(\mathbf{A} + \mathbf{A}^{\mathrm{T}})$ be the symmetric part of matrix A. The term $P_d = \mathbf{x}^{\mathrm{T}}\mathbf{A}_s\mathbf{x} =$ is the *dissipating power* in the system; 3) The skew-symmetric part of matrix A, that is $\mathbf{A}_w = \frac{1}{2}(\mathbf{A} - \mathbf{A}^{\mathrm{T}})$, represents all the physical elements associated with *pure energy redistribution* within the system, without losses and without energy storing (i.e. all types of connection blocks).

The Simulink block diagram corresponding to the considered POG scheme (see files "Traction_system.m" and "Traction_system_mdl.mdl") is shown in Fig. 3. Some simulation

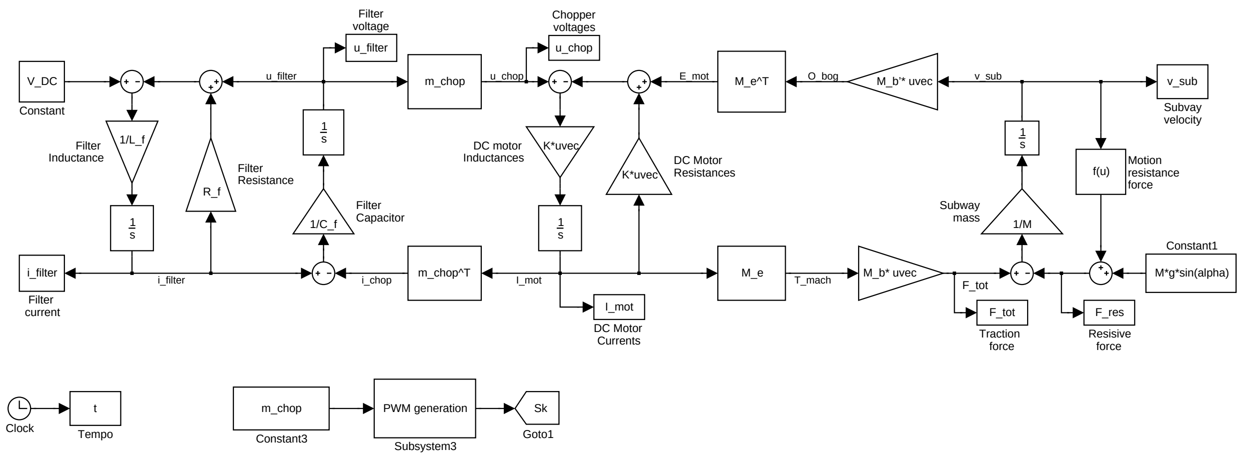

Figura 3: Simulink block diagram corresponding to the POG scheme shown in Fig. 2.

results are shown in Fig. 5-Fig. 9. These results have been obtained using the parameters and the initial conditions shown in Fig. 4. These parameters have been used just to test the correctness and the efficiency of the model: they do not have any specific realistic value. Note that the considered Simulink model simulates also the switching of the choppers. In particular, in the presented simulation constant duty cycles have been used ("m_chop = [0.2 0.9 0.1]"), but the models allows also the use of time-variable duty cycles values.

The current i_{filter} and the voltage u_{filter} of the embedded input filter are shown in Fig. 5. The field and armature currents i_{field1}, i_{field2} and i_{arm} are shown in Fig. 6. The subway velocity v_{sub} is reported in the upper part of Fig. 7. The traction force F_{tot} and the resistive force F_{res} are shown in the lower part of the same figure. Finally, details of the current ripples and chopper voltages are shown in Fig. 8 and Fig. 9.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%  System parameters
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
V_DC = 400*Volt;          %%%%% Input voltage
C_f  = 200*microF;        %%%%% Embedded Filter Capacitor
R_f  = 2*Ohm;             %%%%% Embedded Filter Resistance
L_f  = 20*mH;             %%%%% Embedded Filter Inductances Lf
%
L_field1 = 15*mH;         %%%%% Field winding Inductances
R_field1 = 2*Ohm;         %%%%% Field winding Resistances
L_arm1 = 30*mH;           %%%%% Armature winding Inductances
R_arm1 = 1.5*Ohm;         %%%%% Armature winding Resistances
L_field2 = 15*mH;         %%%%% Field winding Inductances
R_field2 = 2*Ohm;         %%%%% Field winding Resistances
L_arm2 = 30*mH;           %%%%% Armature winding Inductances
R_arm2 = 1.5*Ohm;         %%%%% Armature winding Resistances
%
k_mach1 = 0.1;            %%%%% First torque coefficient
k_mach2 = 0.1;            %%%%% Second torque coefficient
mbog1 = 2;                %%%%% First bugie ratio
mbog2 = 2;                %%%%% Second Bugie ratio
%
M = 10000*kg;             %%%%% Mass of the subway
%
alpha = 0;                %%%%% Slope angle
a_res = 1;                %%%%% Resistive force: linear coefficient
b_res = 2;                %%%%% Resistive force: quadratic coefficient
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%% Initial conditions
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
u_filter_0 = 0*Volt;      %%%%% Embedded Filter Initial Voltage
i_filter_0 = 0*Amp;       %%%%% Embedded Filter Initial current
i_field1_0 = 0*Amp;       %%%%% Field winding initial current
i_field2_0 = 0*Amp;       %%%%% Field winding initial current
i_arm = 0*Amp;            %%%%% Armature winding initial current
I_M_0=[i_field1_0 i_arm i_field2_0];  %%%%% DC motor currents
v_sub_0 = 30*km/ora;      %%%%% Subway initial velocity
%
m_chop = [0.3 0.9 0.3];            %%%%% Switching functions
m_chop_phase = [0 0.33 0.66];      %%%%% Phase of the switching functions
%
Ts=0.02*msec;      %%%%% Switching period
```

Figura 4: Parameters and the initial conditions used in simulation.

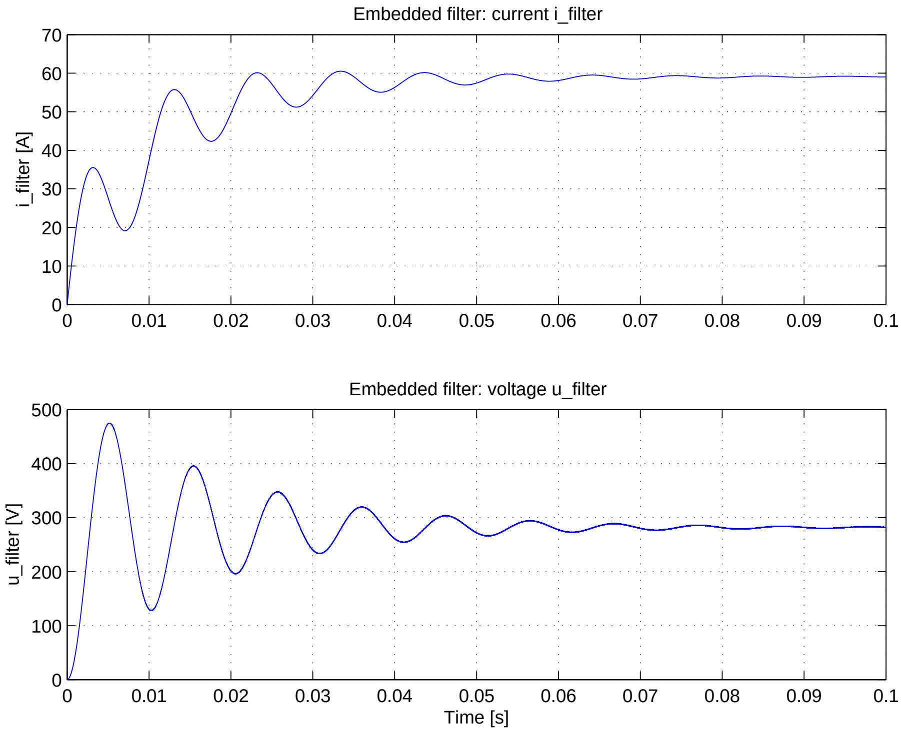

Figura 5: Embedded filter: current i_{filter} and voltage u_{filter}.

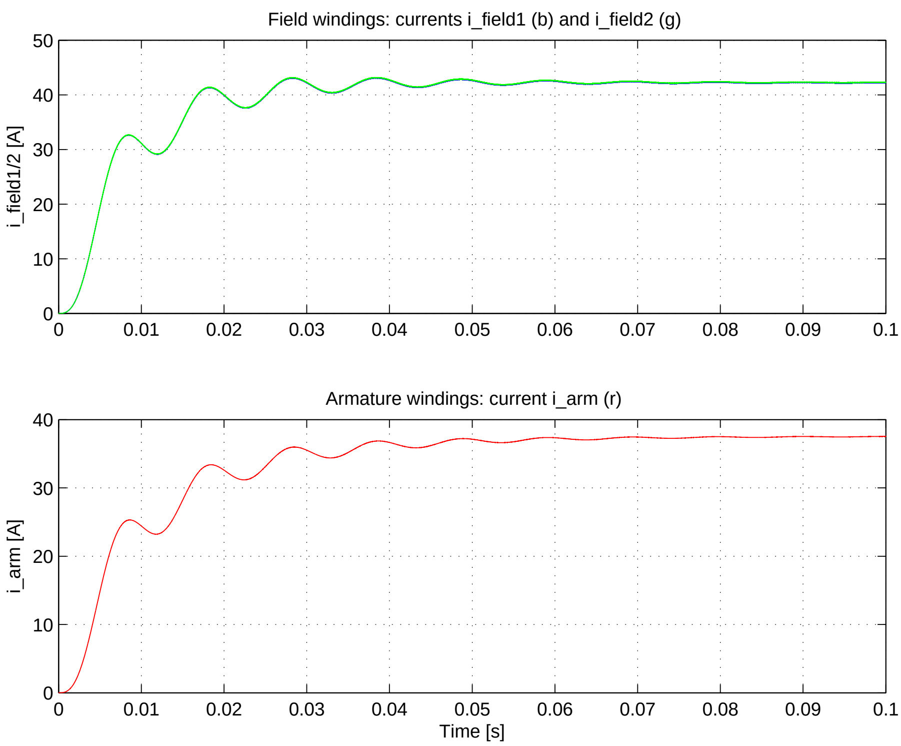

Figura 6: Field and armature windings: currents i_{field1} (blue), i_{field2} (green) and i_{arm} (red).

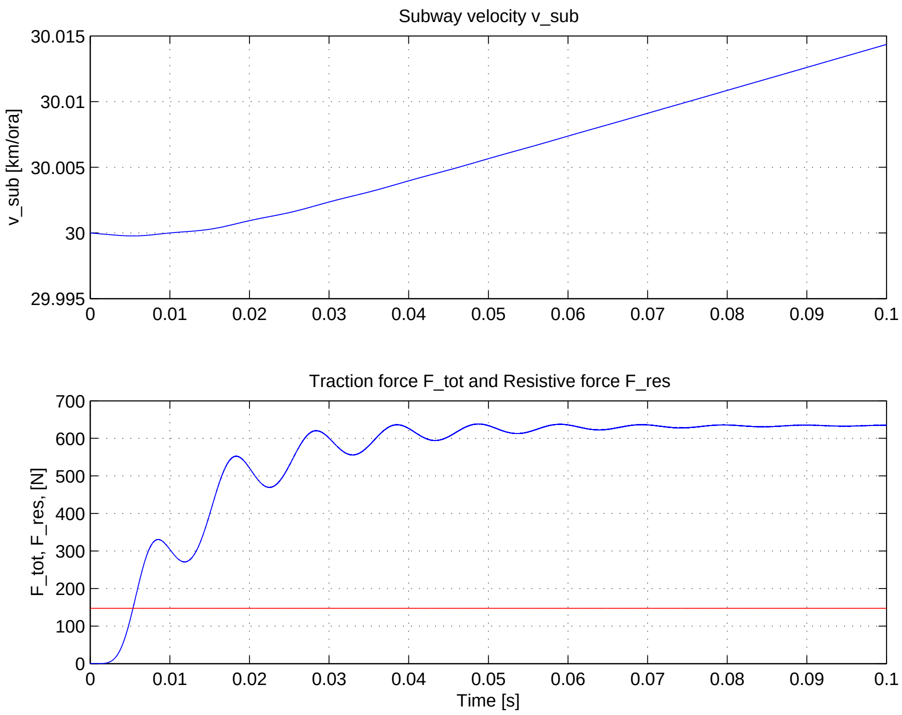

Figura 7: Subway velocity v_{sub}, traction force F_{tot} and resistive force F_{res}.

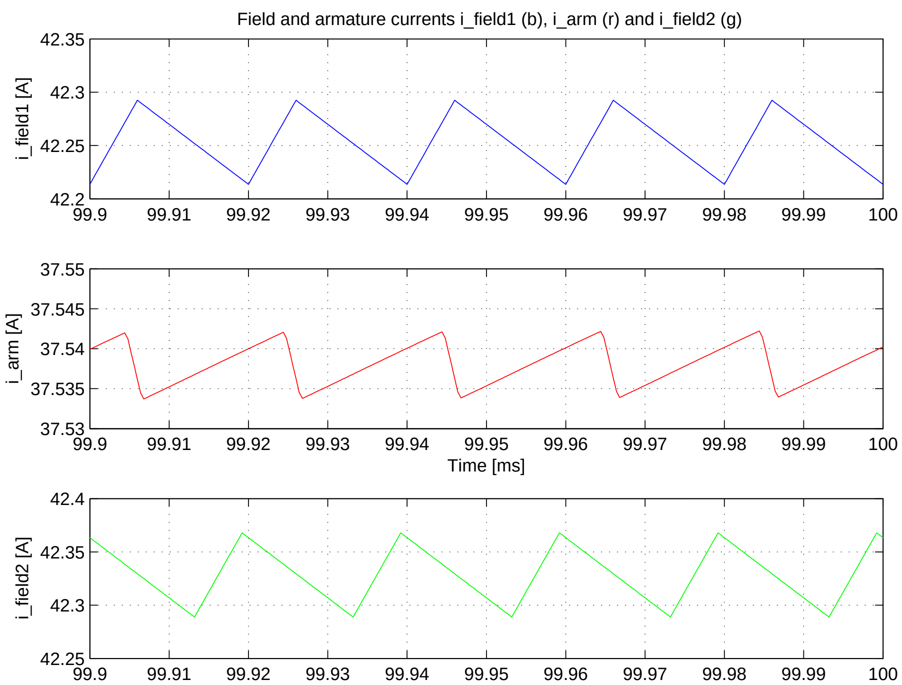

Figura 8: Field and armature currents i_{field1} (blue), i_{arm} (red) and i_{field2} (green).

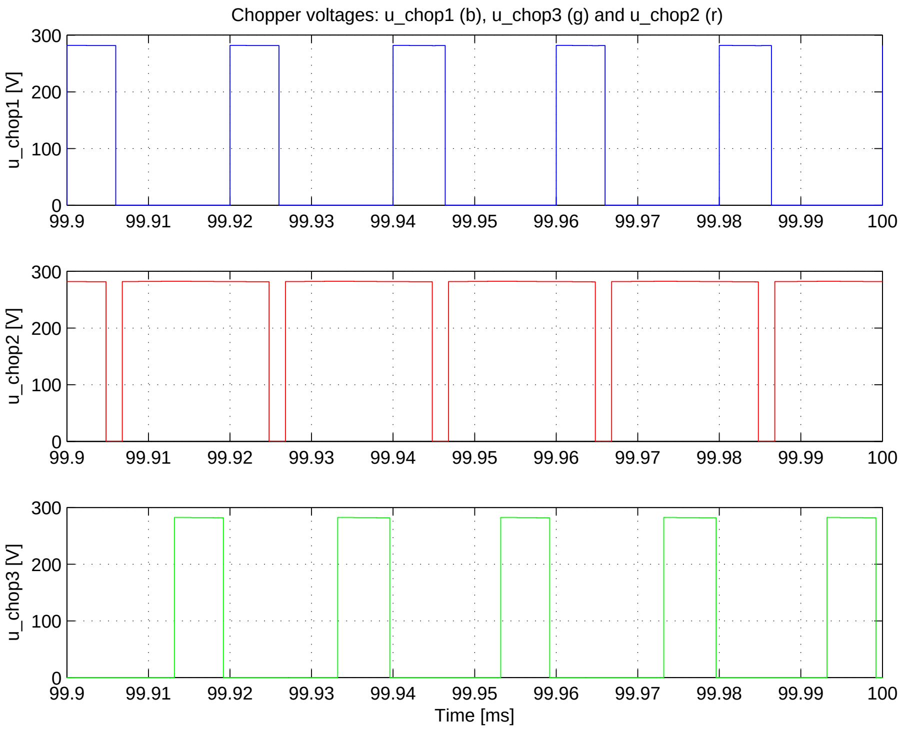

Figura 9: Chopper voltages: u_{chop1} (blue), u_{chop3} (green) and u_{chop2} (r).