System and Control Theory
Test of Dicember 23, 2015
Questions and Exercises

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1. Write the general solution of the following time-varying difference equation $\mathbf{x}(k+1) = \mathbf{A}(k)\mathbf{x}(k) + \mathbf{B}(k)\mathbf{u}(k)$, being $\mathbf{x}(h)$ the state at time h:

$$\mathbf{x}(k) = \mathbf{\Phi}(k,h)\mathbf{x}(h) + \sum_{j=h}^{k-1} \mathbf{\Phi}(k,j+1)\mathbf{B}(j)\mathbf{u}(j)$$

where $\mathbf{\Phi}(k,h)$ denotes the state transition matrix of the system.

2. Write the explicit form of the *transition matrix of the system* $\mathbf{\Phi}(t,t_0)$ for continuous-time invariant linear system:

$$\mathbf{\Phi}(t,t_0) = e^{A(t-t_0)}$$

3. Write the general solution of the differential equation $\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$ starting from the initial condition $\mathbf{x}(0)$ at time $t_0 = 0$:

$$\mathbf{x}(t) = e^{\mathbf{A}t}\mathbf{x}(0) + \int_0^t e^{\mathbf{A}(t-\tau)}\mathbf{B}\mathbf{u}(\tau)d\tau$$

4. Write the symbol used for describing "*the set of the reachable states at time t_1, starting from* $\{t_0, \mathbf{x}(t_0)\}$":

$$\mathcal{X}^+(t_0, t_1, \mathbf{x}(t_0))$$

5. Compute the reachability matrix $\mathcal{R}^+$ and the observability matrix $\mathcal{O}^-$ of the following system:

$$\begin{cases} \dot{\mathbf{x}}(t) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u(t) \\ y(t) = \begin{bmatrix} 1 & -1 & 1 \end{bmatrix} \mathbf{x}(t) \end{cases} \qquad \mathcal{R}^+ = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}, \quad \mathcal{O}^- = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

The system is: ○ reachable? ⊗ not-reachable? ⊗ observable? ○ not-observable?

6. The following symbolic representation:

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), t) \\ \mathbf{y}(t) = \mathbf{g}(\mathbf{x}(t), \mathbf{u}(t), t) \end{cases} \qquad \mathbf{x}(t) \in \mathbf{R}^n$$

is used for describing a system with the following characteristics:

⊗ a dynamic system; ⊗ a continuous-time system;
○ a linear system; ⊗ a lumped system;
⊗ a time-varying system; ○ a system without inputs;

7. Compute, as function of the initial condition $\mathbf{x}(0) = [x_1(0),\ x_2(0),\ x_3(0),\ x_4(0)]^{\mathrm{T}}$, the free evolution of the following continuous-time autonomous system:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{x}(t) \qquad \mathbf{x}(t) = \begin{bmatrix} e^{-t} & t\,e^{-t} & \frac{t^2}{2}\,e^{-t} & 0 \\ 0 & e^{-t} & t\,e^{-t} & 0 \\ 0 & 0 & e^{-t} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \\ x_4(0) \end{bmatrix}$$

8. Let $\mathbf{A}$ and $\bar{\mathbf{A}}$ be two similar matrices $\mathbf{A} = \mathbf{T}\bar{\mathbf{A}}\mathbf{T}^{-1}$. The matrix function $\mathbf{A}^k$ satisfies the following property:

○ $\mathbf{A}^k = \mathbf{T}^{-1}\,\bar{\mathbf{A}}^k\,\mathbf{T}$; ○ $\mathbf{A}^k = \mathbf{T}^{-k}\,\bar{\mathbf{A}}^k\,\mathbf{T}^k$; ⊗ $\mathbf{A}^k = \mathbf{T}\,\bar{\mathbf{A}}^k\,\mathbf{T}^{-1}$; ○ $\mathbf{A}^k = \mathbf{T}^k\,\bar{\mathbf{A}}^k\,\mathbf{T}^{-k}$;

9. Write the formula for computing the state transition matrix $e^{\mathbf{A}t}$ of a continuous-time linear system using the Laplace transform:

$$e^{\mathbf{A}t} = \mathcal{L}^{-1}[(s\mathbf{I} - \mathbf{A})^{-1}]$$

10. Consider the following linear dynamic system:

$$\dot{\mathbf{x}} = \begin{bmatrix} -6 & -1 \\ -4 & -6 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 2 \\ 2 \end{bmatrix} \mathbf{u} = \mathbf{A}\,\mathbf{x} + \mathbf{B}\,\mathbf{u}, \qquad \mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 2.5 & 1 \\ -1 & 2.5 \end{bmatrix} \mathbf{x} = \begin{bmatrix} \mathbf{c}_1 \\ \mathbf{c}_2 \end{bmatrix} \mathbf{x} = \mathbf{C}\,\mathbf{x}.$$

a) Write the formula used for computing the equilibrium point $\mathbf{x}_0$ corresponding to the constant input $\mathbf{u} = \mathbf{u}_0$:

$$\mathbf{x}_0 = -\mathbf{A}^{-1}\,\mathbf{B}\,\mathbf{u}_0.$$

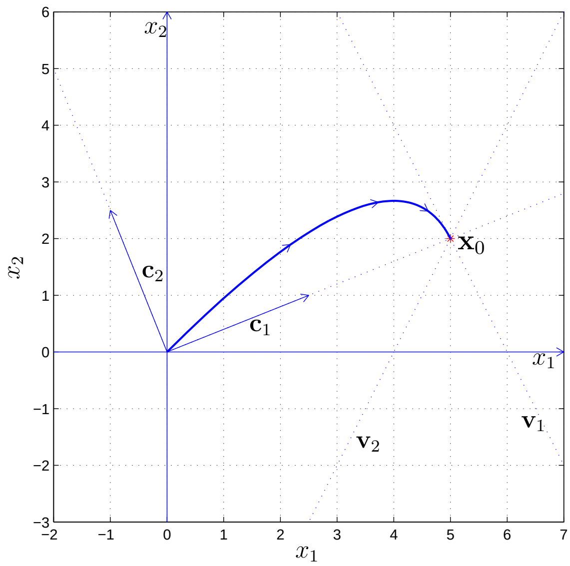
It is easy to verify that for constant input $\mathbf{u}_0 = 16$ the equilibrium point of the given system is $\mathbf{x}_0 = [5,\ 2]^{\mathrm{T}}$. The eigenvalues of matrix A are:

$$\lambda_1 = -4, \qquad \lambda_2 = -8$$

The corresponding eigenvectors are:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix} \qquad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

b) In the box on the right draw the qualitative trajectory of the dynamic system $\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}$ starting from the initial condition $\mathbf{x}(0) = 0$ and when the constant input is $\mathbf{u}_0 = 16$.



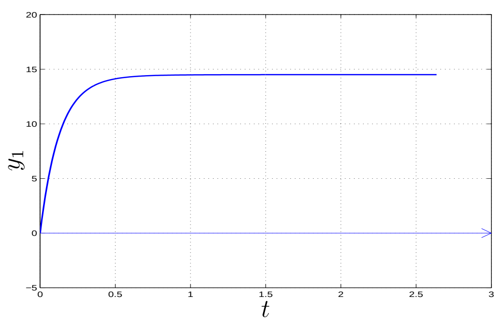
c) Which type of trajectory characterizes the time response of the system when $\mathbf{u}_0 = 16$:

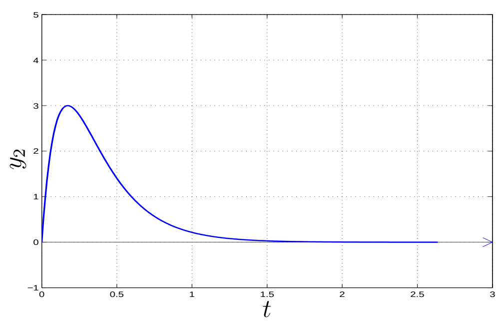
⊗ Node? ○ Degen. Node? ○ Focus? ○ Saddle? ⊗ Stable? ○ Unstable?

d) Compute the settling time T_a and the period T_ω of the oscillation of the time response $\mathbf{x}(t)$:

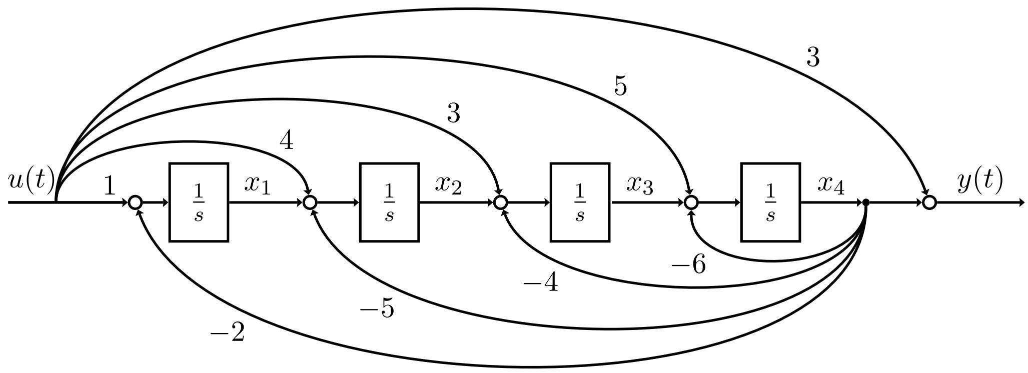
$$T_a = \frac{3}{|\lambda_1|} = \frac{3}{4} = 0.75 \text{ s}, \qquad T_\omega = \infty.$$

e) Draw the qualitative behavior of the time response of the two outputs $y_1(t)$ and $y_2(t)$:





11. Given following block scheme:



Compute the transfer function of the system:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{5\,s^3 + 3s^2 + 4\,s + 1}{s^4 + 6\,s^3 + 4\,s^2 + 5\,s + 2} + 3$$

Without further calculations it is possible to state that the given system:

- ○ is surely stable;
- ⊗ is completely observable;
- ○ is completely reachable;
- ⊗ is in the observability canonical form;
- ○ is in the reachability standard form;
- ⊗ is a system constructable;

12. Given a linear system $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}u$ completely reachable and with only one input. Let $\Delta_{\mathbf{A}}(\lambda) = \lambda^n + \alpha_{n-1}\lambda^{n-1} + \ldots + \alpha_1\lambda + \alpha_0$ be the characteristic polynomial of matrix $\mathbf{A}$ and let $p(\lambda) = \lambda^n + d_{n-1}\lambda^{n-1} + \ldots + d_1\lambda + d_0$ be a monic polynomial freely chosen. Write the expression of vector $\mathbf{k}^{\mathrm{T}}$ which using the static feedback $\mathbf{u} = \mathbf{k}^{\mathrm{T}}\mathbf{x}$ is able to match the eigenvalues of matrix $\mathbf{A} + \mathbf{b}\mathbf{k}^{\mathrm{T}}$ with the roots of polynomial $p(\lambda)$:

$$\mathbf{k}^{\mathrm{T}} = \mathbf{k}_c^{\mathrm{T}} \left\{ \begin{bmatrix} \mathbf{b}, & \mathbf{A}\mathbf{b}, & \ldots, & \mathbf{A}^{n-1}\mathbf{b} \end{bmatrix} \begin{bmatrix} \alpha_1 & \alpha_2 & \ldots & \alpha_{n-1} & 1 \\ \alpha_2 & \ldots & \ldots & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \alpha_{n-1} & 1 & \ldots & 0 & 0 \\ 1 & 0 & \ldots & 0 & 0 \end{bmatrix} \right\}^{-1}$$

where $\mathbf{k}_c^{\mathrm{T}} = \begin{bmatrix} \alpha_0 - d_0, & \alpha_1 - d_1, & \ldots, & \alpha_{n-1} - d_{n-1} \end{bmatrix}$.

13. Given the following discrete-time linear system:

$$\begin{cases} \mathbf{x}(k+1) = \left[\begin{array}{c|c|c} 1 & \mathbf{0} & \mathbf{0} \\ \hline 2 & -1 & 1 \\ \hline \mathbf{0} & \mathbf{0} & 0.5 \end{array}\right] \mathbf{x}(k) + \left[\begin{array}{c} 2 \\ 1 \\ \hline \mathbf{0} \end{array}\right] u(k) \\ y(k) = \left[\begin{array}{c|cc} 1 & \mathbf{0} & \mathbf{0} \end{array}\right] \mathbf{x}(k) \end{cases}$$

Thinking to the block structure of the systems in standard form it is possible to state that:

- ⊗ the system is in the standard observability form;
- ⊗ the system is in the reachability standard form;
- ○ for this system it is possibile to build an asymptotic state observer;
- ⊗ the system can be stabilized using a static state feedback;

14. Consider the point-to-point control problem for a discrete-time linear system. Among the infinite solutions **u** which move the system from the initial state $\mathbf{x}(0)$ to the final state $\mathbf{x}(k)$ in the time interval $[0,\ k]$ write the solution **u** which minimizes the Euclidean norm:

$$\mathbf{u} = (\mathcal{R}_k^+)^T[\mathcal{R}_k^+(\mathcal{R}_k^+)^T]^{-1}[\mathbf{x}(k) - \mathbf{A}^k\mathbf{x}(0)]$$

15. Write the "*separation property*" of the regulator:

The design of feedback block $(\mathbf{A}+\mathbf{BK})$ *and the estimation block* $(\mathbf{A}+\mathbf{LC})$ *can be done independently:*

$$\det[z\mathbf{I} - \bar{\mathbf{A}}] = \det[z\mathbf{I} - (\mathbf{A}+\mathbf{BK})]\,\det[z\mathbf{I} - (\mathbf{A}+\mathbf{LC})]$$

16. The Ackermann formula for computing the gain vector **l** of an asymptotic state observer which freely places the eigenvalues of matrix $\mathbf{A}+\mathbf{lc}$ can be used:

○ only if the system is reachable;	⊗ only if the system is observable;
⊗ only for sistemi ad una sola uscita;	○ only for systems with one input;
○ only if polynomial $\det(s\mathbf{I}-\mathbf{A})$ is known;	⊗ only if polynomial $p(\lambda)$ is known;

17. Given the continuous-time linear system $\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$, write the structure of:

a) an **open loop** state estimator and the time evolution of the estimation error $\mathbf{e}(t) = \mathbf{x}(t) - \hat{\mathbf{x}}(t)$ obtained starting from the initial condition $\mathbf{e}(0)$:

$$\dot{\hat{\mathbf{x}}}(t) = \mathbf{A}\hat{\mathbf{x}}(t) + \mathbf{B}\mathbf{u}(t), \qquad \mathbf{e}(t) = e^{\mathbf{A}t}\,\mathbf{e}(0)$$

b) uno stimatore asintotico of the system **in catena chiusa of order pieno** and the time evolution of the estimation error $\mathbf{e}(t) = \mathbf{x}(t) - \hat{\mathbf{x}}(t)$ obtained starting from the initial condition $\mathbf{e}(0)$:

$$\dot{\hat{\mathbf{x}}}(t) = (\mathbf{A}+\mathbf{LC})\hat{\mathbf{x}}(t) + \mathbf{B}\mathbf{u}(t) - \mathbf{L}\mathbf{y}(t), \qquad \mathbf{e}(t) = e^{(\mathbf{A}+\mathbf{LC})t}\,\mathbf{e}(0)$$

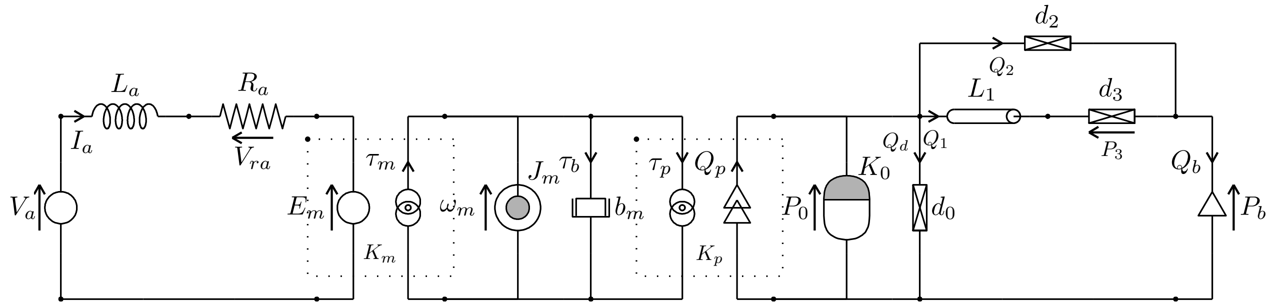
18. Write, within the following table, the symbols and the names of the energy variables and power variables that characterize the Energetic Domain: *Mechanical Rotational*. Moreover, write the constitutive relation (both linear and nonlinear) and the differential equation which characterize the physical elements:

	Symbols		Constitutive Rel.	Linear Case	Differential Eq.
$\mathcal{D}_1$	J	Inertia			
q_1	P	Ang. Momentum	$P = \Phi_J(\omega)$	$P = J\,\omega$	$\frac{dP}{dt} = \tau$
v_1	ω	Ang. Velocity			
$\mathcal{D}_2$	E	Tors. Elasticity			
q_2	θ	Ang. Displacement	$\theta = \Phi_E(\tau)$	$\theta = E\,\tau$	$\frac{d\theta}{dt} = \omega$
v_2	τ	Torque			
$\mathcal{R}$	b	Friction	$\tau = \Phi_b(\omega)$	$\tau = b\,\omega$	

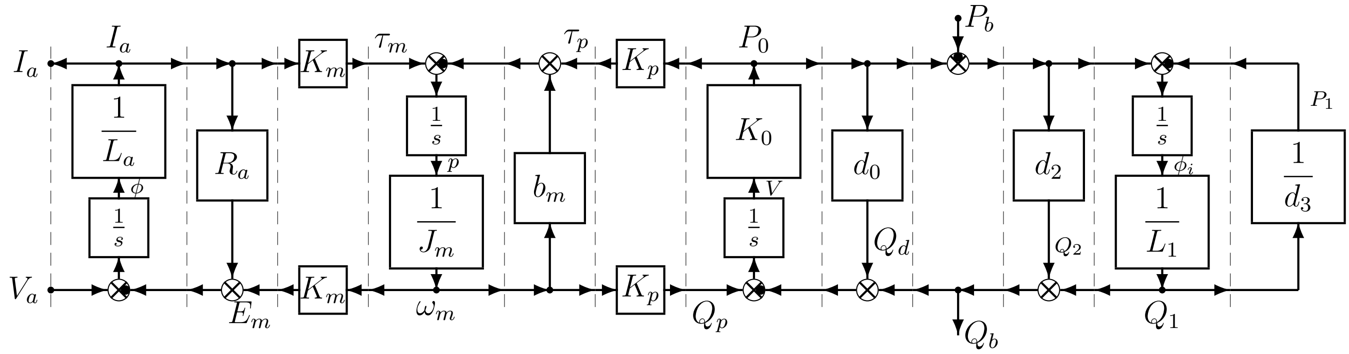
19. Which of the following functions $V(x_1, x_2)$ are positive definite in the neighborhood **of point (1, 0)**:

○ $V(x_1,x_2) = x_1^2 + (x_2-1)^2$;	○ $V(x_1,x_2) = (x_1^2-1)(x_2^2+1) + x_2^2$;
⊗ $V(x_1,x_2) = (x_1-1)^2 + x_2^2$;	○ $V(x_1,x_2) = (x_1^2+1)(x_2^2-1) + x_1^2$;

20. Consider the following dynamic system composed by a DC electric motor which moves an hydraulic pump which in turn feeds an hydraulic load: L_a, R_a, K_m, J_m and b_m are the parameters of the DC electric motor; K_p, K_0, L_1, d_0, d_2 and d_3 are the parameters of the gear pump and the hydraulic load. Two inputs act on the system: the voltage V_a and the pressure P_b. The outputs of the system are: the current I_a and the volume flow rate Q_b.



The POG model of the given dynamic system has the following structure:



Let $\mathbf{x} = \begin{bmatrix} I_a & \omega_m & P_0 & Q_1 \end{bmatrix}^{\mathrm{T}}$ be the state vector, $\mathbf{u} = \begin{bmatrix} V_a & P_b \end{bmatrix}^{\mathrm{T}}$ the input vector and $\mathbf{y} = \begin{bmatrix} I_a & Q_b \end{bmatrix}^{\mathrm{T}}$ the output vector. Write the corresponding dynamic system $\overline{\mathbf{L}}\dot{\mathbf{x}} = \overline{\mathbf{A}}\mathbf{x} + \overline{\mathbf{B}}\mathbf{u}$ and $\mathbf{y} = \overline{\mathbf{C}}\mathbf{x} + \overline{\mathbf{D}}\mathbf{u}$ in the state space:

$$\underbrace{\begin{bmatrix} L_a & 0 & 0 & 0 \\ 0 & J_m & 0 & 0 \\ 0 & 0 & \frac{1}{K_0} & 0 \\ 0 & 0 & 0 & L_1 \end{bmatrix}}_{\overline{\mathbf{L}}} \underbrace{\begin{bmatrix} \dot{I}_a \\ \dot{\omega}_m \\ \dot{P}_0 \\ \dot{Q}_1 \end{bmatrix}}_{\dot{\mathbf{x}}} = \underbrace{\begin{bmatrix} -R_a & -K_m & 0 & 0 \\ K_m & -b_m & -K_p & 0 \\ 0 & K_p & -d_0 - d_2 & -1 \\ 0 & 0 & 1 & -\frac{1}{d_3} \end{bmatrix}}_{\overline{\mathbf{A}}} \underbrace{\begin{bmatrix} I_a \\ \omega_m \\ P_0 \\ Q_1 \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & d_2 \\ 0 & -1 \end{bmatrix}}_{\overline{\mathbf{B}}} \underbrace{\begin{bmatrix} V_a \\ P_b \end{bmatrix}}_{\mathbf{u}}$$

$$\underbrace{\begin{bmatrix} I_a \\ Q_b \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & d_2 & 1 \end{bmatrix}}_{\overline{\mathbf{C}}} \mathbf{x} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & -d_2 \end{bmatrix}}_{\overline{\mathbf{D}}} \underbrace{\begin{bmatrix} V_a \\ P_b \end{bmatrix}}_{\mathbf{u}}$$

21. Write the La Salle - Krasowskii stability criterion for continuous-time systems.

Consider the nonlinear continuous-time system $\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}_0)$ and let $\mathbf{x}_0$ an equilibrium point corresponding to the constant input $\mathbf{u}_0$.
If in a neighborhood W of point $\mathbf{x}_0$ it exists a continuous function $V(\mathbf{x}) : W \to \mathcal{R}$ with continuous first time-derivatives and positive definite, *if the function $\dot{V}(\mathbf{x})$ is* negative semidefinite *and if the set $\mathcal{N} = \{\mathbf{x} \in W | \dot{V}(\mathbf{x}) = 0\}$ does not contain perturbed trajectories of the given system, then $\mathbf{x}_0$ is an equilibrium point* asymptotically stable *for the nonlinear given system.*

22. *Given the following nonlinear system $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$, continuous-time and autonomous:*

$$\begin{cases} \dot{x}_1 &=& \alpha x_1 - x_1 x_2 \\ \dot{x}_2 &=& x_1^2 - \beta x_2 \end{cases}$$

a) Compute, as a function of α and β, the 3 equilibrium points $\overline{\mathbf{x}}_1$, $\overline{\mathbf{x}}_2$ and $\overline{\mathbf{x}}_3$ of the system:

The equilibrium points of the system can be determined imposing $\dot{x}_1 = 0$ and $\dot{x}_2 = 0$:

$$x_1(\alpha - x_2) = 0, \qquad x_1^2 - \beta x_2 = 0.$$

From the first relation it follows that $x_1 = 0$ and $x_2 = \alpha$. Substituting in the second relation one obtains:

$$\overline{\mathbf{x}}_1 = (0,\ 0), \qquad \overline{\mathbf{x}}_2 = (\sqrt{\alpha\beta},\ \alpha), \qquad \overline{\mathbf{x}}_3 = (-\sqrt{\alpha\beta},\ \alpha).$$

b) Compute the Jacobian $\mathbf{A}(\mathbf{x}) = \frac{\partial \mathbf{f}(\mathbf{x})}{\partial \mathbf{x}}$ of the nonlinear system $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$:

The Jacobian of the nonlinear system is:

$$\mathbf{A}(\mathbf{x}) = \frac{\partial \mathbf{f}(\mathbf{x})}{\partial \mathbf{x}} = \begin{bmatrix} \alpha - x_2 & -x_1 \\ 2x_1 & -\beta \end{bmatrix}$$

c) Compute the matrices $\mathbf{A}_1$, $\mathbf{A}_2$ and $\mathbf{A}_3$ of the linearized system in the neighborhood of the equilibrium points:

The matrices $\mathbf{A}_1$, $\mathbf{A}_2$ and $\mathbf{A}_3$ of the linearized system have the following structure:

$$\mathbf{A}_1 = \begin{bmatrix} \alpha & 0 \\ 0 & -\beta \end{bmatrix}, \qquad \mathbf{A}_2 = \begin{bmatrix} 0 & -\sqrt{\alpha\beta} \\ 2\sqrt{\alpha\beta} & -\beta \end{bmatrix}, \qquad \mathbf{A}_3 = \begin{bmatrix} 0 & \sqrt{\alpha\beta} \\ -2\sqrt{\alpha\beta} & -\beta \end{bmatrix}.$$

d) Study, for varying α and β, the stability of the nonlinear system in the neighborhood of the 3 equilibrium points $\overline{\mathbf{x}}_1$, $\overline{\mathbf{x}}_2$ and $\overline{\mathbf{x}}_3$ using the reduced Lyapunov criterion:

The eigenvalues of matrix $\mathbf{A}_1$ are:

$$\lambda_1 = \alpha, \qquad \lambda_2 = -\beta.$$

Using the reduced Lyapunov criterion it follows that: when $\alpha < 0$ and $\beta > 0$ the equilibrium point $\overline{\mathbf{x}}_1 = (0,0)$ of the nonlinear system is asymptotically stable. When $\alpha > 0$ or $\beta < 0$ the equilibrium point $\mathbf{x}_1$ is unstable. In all the other cases the criterion cannot be used. The characteristic polynomial of the matrices $\mathbf{A}_2$ and $\mathbf{A}_3$ is the following:

$$\Delta_{\mathbf{A}_2}(s) = \Delta_{\mathbf{A}_3}(s) = s^2 + \beta s + 2\alpha\beta = 0$$

Using the reduced Lyapunov criterion it can be stated that the points an equilibrium point $\overline{\mathbf{x}}_2$ and $\overline{\mathbf{x}}_3$ of the nonlinear system: a) are asymptotically stable if $\alpha > 0$ and $\beta > 0$; b) are instabili if $\alpha < 0$ o $\beta < 0$; the criterion cannot be used per $\beta = 0$ or for $\alpha = 0$ and $\beta > 0$.

e) For $\alpha = 0$ and $\beta > 0$ study the stability of the nonlinear system in the neighborhood of point $\overline{\mathbf{x}} = (0,0)$ using the "direct" Lyapunov criterion and the following Lyapunov function: $V(\mathbf{x}) = x_1^2 + x_2^2$. Eventually, use the La Salle - Krasowskii criterion.

In the neighborhood of point $\overline{\mathbf{x}} = (0,0)$ *the function* $V(\mathbf{x}) = x_1^2 + x_2^2$ *is positive definite. Computing the time derivative of* $V(\mathbf{x})$ *along the system's trajectories when* $\alpha = 0$*:*

$$\dot{V} = 2x_1\dot{x}_1 + 2x_2\dot{x}_2 = 2x_1(-x_1x_2) + 2x_2(x_1^2 - \beta x_2) = -2\beta x_2^2 \leq 0$$

Applying the "direct" Lyapunov criterion it can be stated that in the neighborhood of point $\overline{\mathbf{x}} = (0,0)$ *the nonlinear system is stable. The set* $\mathcal{N} = \{(x_1, 0), x_1 \in R\}$ *of all the points where* $\dot{V} = 0$ *does not contain perturbed trajectories and therefore, using the La Salle - Krasowskii criterion, it can be stated that for* $\alpha = 0$ *and* $\beta > 0$ *the nonlinear system is asymptotically stable in the neighborhood of point* $\mathbf{x} = (0,0)$*. For* $\alpha = 0$ *and* $\beta = 0$ *the function* $\dot{V}$ *is zero and so the nonlinear system is simply stable in the neighborhood of* $\overline{\mathbf{x}}$*.*