

System and Control Theory
Test of December 20, 2012
Questions and Exercises

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1. Write the general solution of the following time-varying difference equation $\mathbf{x}(k+1) = \mathbf{A}(k)\mathbf{x}(k) + \mathbf{B}(k)\mathbf{u}(k)$, being $\mathbf{x}(h)$ the state at time h :

$$\mathbf{x}(k) = \Phi(k, h)\mathbf{x}(h) + \sum_{j=h}^{k-1} \Phi(k, j+1)\mathbf{B}(j)\mathbf{u}(j)$$

where $\Phi(k, h)$ is the state transition matrix of the system.

2. Write the discrete time behavior of the output function $\mathbf{y}(t)$, solution of the differential equation $\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$ and the static equation $\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)$ starting from the initial condition $\mathbf{x}(0)$ at time $t_0 = 0$:

$$\mathbf{y}(t) = \mathbf{C} e^{\mathbf{A}t}\mathbf{x}(0) + \mathbf{C} \int_0^t e^{\mathbf{A}(t-\tau)}\mathbf{B}\mathbf{u}(\tau)d\tau + \mathbf{D}\mathbf{u}(t)$$

3. Compute the reachability matrix \mathcal{R}^+ and the observability matrix \mathcal{O}^- of the following system:

$$\begin{cases} \dot{\mathbf{x}}(t) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 1 & 1 \\ 0 & -1 \\ 0 & 0 \end{bmatrix} \mathbf{u}(t) \\ \mathbf{y}(t) = \begin{bmatrix} 1 & -1 & 0 \end{bmatrix} \mathbf{x}(t) \end{cases} \quad \mathcal{R}^+ = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & -1 \\ 0 & -1 & 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathcal{O}^- = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 1 & 1 & -1 \end{bmatrix}$$

The system is: reachable? not-reachable? observable? not-observable?

Provide a base \mathcal{B}_R of the reachable subspace \mathcal{X}^+ and a base \mathcal{B}_O of the not-observable subspace \mathcal{E}^- :

$$\mathcal{X}^+ = \text{Im}[\mathcal{B}_R] = \text{Im} \begin{bmatrix} 1 & 1 \\ 0 & -1 \\ 0 & 0 \end{bmatrix}, \quad \mathcal{E}^- = \text{Im}[\mathcal{B}_O] = \text{Im} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

4. The following symbolic representation:

$$\begin{cases} \mathbf{x}(k+1) = \mathbf{A}(k)\mathbf{x}(k) + \mathbf{B}(k)\mathbf{u}(k) \\ \mathbf{y}(k) = \mathbf{C}(k)\mathbf{x}(k) + \mathbf{D}(k)\mathbf{u}(k) \end{cases}$$

is used for describing a system with the following characteristics:

- a linear system; a time-varying system ;
 a dynamic system; a lumped system;
 a continuous-time system; an autonomous system;

5. Apply the \mathcal{Z} -transform to the following discrete system:

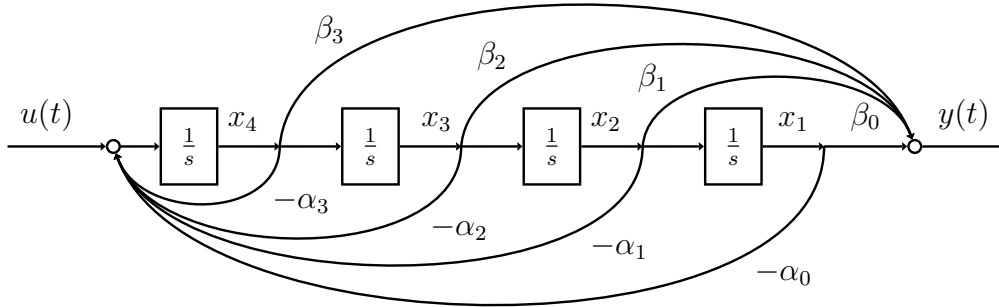
$$\mathcal{Z}[\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k)], \quad \mathcal{Z}[\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) + \mathbf{D}\mathbf{u}(k)]$$

and provides the expression of the transformed function $\mathbf{y}(z)$ of the output vector $\mathbf{y}(k)$ as a function of the initial state \mathbf{x}_0 and of the transform $\mathbf{u}(z)$ of the input signal $\mathbf{u}(k)$:

$$\mathbf{y}(z) = \mathbf{C}(z\mathbf{I} - \mathbf{A})^{-1}z\mathbf{x}_0 + [\mathbf{C}(z\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D}]\mathbf{u}(z)$$

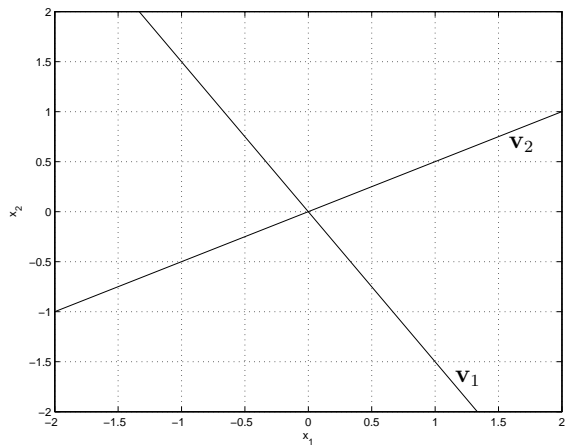
6. Draw the block scheme of the following continuous-time system where with \mathbf{x}_c denotes the vector $\mathbf{x}_c = [x_1 \ x_2 \ x_3 \ x_4]^T$.

$$\begin{cases} \dot{\mathbf{x}}_c(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\alpha_0 & -\alpha_1 & -\alpha_2 & -\alpha_3 \end{bmatrix} \mathbf{x}_c(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u(t) \\ y(t) = [\beta_0 \ \beta_1 \ \beta_2 \ \beta_3] \mathbf{x}_c(t) \end{cases}$$



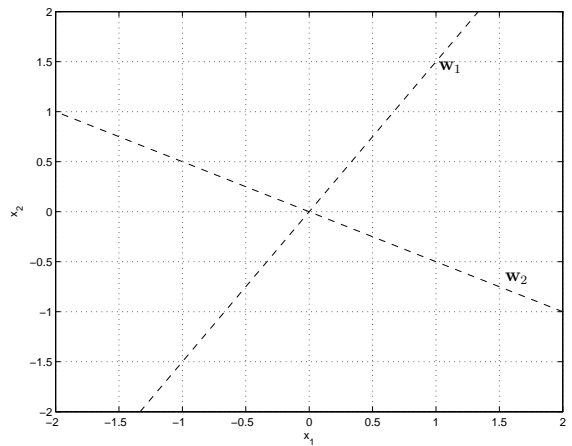
7. Draw qualitatively the trajectories of a second order dynamic system $\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t)$ characterized by the eigenvalues λ_i and the eigenvectors \mathbf{v}_i shown in the two following boxes.

1) Eigenvalues: $\lambda_1 = -1$ and $\lambda_2 = -3$. The corresponding real eigenvectors \mathbf{v}_1 and \mathbf{v}_2 are shown in figure.



Node? Focus? Saddle?
 Stable? Unstable?

2) Eigenvalues: $\lambda_{1,2} = 2 \pm 3j$. Eigenvectors: $\mathbf{v}_1 = \mathbf{w}_1 + \mathbf{w}_2j$ and $\mathbf{v}_2 = \mathbf{v}_1^*$. The real vectors \mathbf{w}_1 and \mathbf{w}_2 are shown in figure.



Node? Focus? Saddle?
 Stable? Unstable?

8. Given the transfer function $G(z)$, write the structure of corresponding dynamic system in the reachability canonical form denoting with $u(k)$ the input and with $y(k)$ the output:

$$G(z) = \frac{z^4 + 6z^3 + 4z^2 + 7z + 4}{z^4 + 5z^3 + 2z^2 + 3z + 1} = 1 + \frac{z^3 + 2z^2 + 4z + 3}{z^4 + 5z^3 + 2z^2 + 3z + 1}$$

$$\begin{cases} \mathbf{x}(k+1) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & -3 & -2 & -5 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u(k) \\ y(k) = [3 \ 4 \ 2 \ 1] \mathbf{x}(k) + [1] u(k) \end{cases}$$

9. Consider the point-to-point control problem for a continuous-time linear system. Write the necessary condition that must be verified such that it exists a control law \mathbf{u} that moves the system from the initial state $\mathbf{x}(0)$ to the final state $\mathbf{x}(t)$ in the time interval $[0, t]$:

$$\mathbf{x}(t) - e^{\mathbf{A}t}\mathbf{x}(0) \in \mathcal{X}^+$$

10. Compute the k -th power of the following Jordan miniblock:

$$\begin{bmatrix} \lambda & 1 & 0 & 0 \\ 0 & \lambda & 1 & 0 \\ 0 & 0 & \lambda & 1 \\ 0 & 0 & 0 & \lambda \end{bmatrix}^k = \begin{bmatrix} \lambda^k & k\lambda^{k-1} & \frac{k(k-1)}{2}\lambda^{k-2} & \frac{k(k-1)(k-2)}{6}\lambda^{k-3} \\ 0 & \lambda^k & k\lambda^{k-1} & \frac{k(k-1)}{2}\lambda^{k-2} \\ 0 & 0 & \lambda^k & k\lambda^{k-1} \\ 0 & 0 & 0 & \lambda^k \end{bmatrix}$$

11. Given a SISO linear system of the fourth order, completely observable, characterized by matrices \mathbf{A} , \mathbf{b} and \mathbf{c} . Let $p(\lambda) = \lambda^4 + \alpha_3\lambda^3 + \alpha_2\lambda^2 + \alpha_1\lambda + \alpha_0$ be the characteristic polynomial of matrix \mathbf{A} . Write the structure of matrix \mathbf{P} which, together with the space transformation $\mathbf{x} = \mathbf{P}\mathbf{x}_o$, brings the system in the observability canonical form.

$$\mathbf{P} = [(\mathcal{O}_c^-)^{-1}\mathcal{O}^-]^{-1} = \left(\begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & 1 \\ \alpha_2 & \alpha_3 & 1 & 0 \\ \alpha_3 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{c} \\ \mathbf{c}\mathbf{A} \\ \mathbf{c}\mathbf{A}^2 \\ \mathbf{c}\mathbf{A}^3 \end{bmatrix} \right)^{-1}$$

12. Given the continuous-time linear system $\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$, write the structure of:
- a) an **open loop** state estimator and the time evolution of the corresponding estimation error $\mathbf{e}(t) = \mathbf{x}(t) - \hat{\mathbf{x}}(t)$ starting from the initial condition $\mathbf{e}(0)$:

$$\dot{\hat{\mathbf{x}}}(t) = \mathbf{A}\hat{\mathbf{x}}(t) + \mathbf{B}\mathbf{u}(t), \quad \mathbf{e}(t) = e^{\mathbf{A}t}\mathbf{e}(0)$$

- b) a **in full order closed loop** state estimator and the time evolution of the corresponding estimation error $\mathbf{e}(t) = \mathbf{x}(t) - \hat{\mathbf{x}}(t)$ starting from the initial condition $\mathbf{e}(0)$:

$$\dot{\hat{\mathbf{x}}}(t) = (\mathbf{A} + \mathbf{L}\mathbf{C})\hat{\mathbf{x}}(t) + \mathbf{B}\mathbf{u}(t) - \mathbf{L}\mathbf{y}(t), \quad \mathbf{e}(t) = e^{(\mathbf{A} + \mathbf{L}\mathbf{C})t}\mathbf{e}(0)$$

13. Relatively to the linear discrete system $\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k)$, $\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k)$, write a necessary and sufficient condition for the complete “constructability” of the system:

$$\mathcal{E}^- = \ker \begin{bmatrix} \mathbf{C} \\ \mathbf{C}\mathbf{A} \\ \vdots \\ \mathbf{C}\mathbf{A}^{n-1} \end{bmatrix} \subseteq \ker \mathbf{A}^n$$

14. Given the following continuous-time linear system $\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$, $\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t)$. Write the expression of the matrices \mathbf{F} , \mathbf{G} and \mathbf{H} that characterize the corresponding sampled system $\mathbf{x}(k+1) = \mathbf{F}\mathbf{x}(k) + \mathbf{G}\mathbf{u}(k)$, $\mathbf{y}(k) = \mathbf{H}\mathbf{x}(k)$ with period T :

$$\mathbf{F} = e^{\mathbf{A}T}, \quad \mathbf{G} = \int_0^T e^{\mathbf{A}\sigma}\mathbf{B}d\sigma, \quad \mathbf{H} = \mathbf{C}$$

The corresponding sampled system is reachable if and only if for each couple λ_i, λ_j of eigenvalues of \mathbf{A} having the same real part it is:

$$\text{Im}(\lambda_i - \lambda_j) \neq \frac{2k\pi}{T} = k\omega_s \quad k = \pm 1, \pm 2, \pm 3, \dots$$

15. Write the Ackermann formula for computing the gain vector \mathbf{l} of an asymptotic state observer which freely places the eigenvalues of matrix $\mathbf{A} + \mathbf{l}\mathbf{c}$:

$$\mathbf{l} = -p(\mathbf{A})(\mathcal{O}^-)^{-1} \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} = -p(\mathbf{A})\mathbf{q}$$

This formula can be used

- only for systems with one input;
- only for systems which are completely observable;
- for any type of time-invariant linear system;

16. Write the block matrices $\bar{\mathbf{A}}$, $\bar{\mathbf{B}}$ and $\bar{\mathbf{C}}$ of a system in the reachability standard form:

$$\bar{\mathbf{A}} = \begin{bmatrix} \mathbf{A}_{1,1} & \mathbf{A}_{1,2} \\ 0 & \mathbf{A}_{2,2} \end{bmatrix} \quad \bar{\mathbf{B}} = \begin{bmatrix} \mathbf{B}_1 \\ 0 \end{bmatrix}$$

$$\bar{\mathbf{C}} = [\mathbf{C}_1 \quad \mathbf{C}_2]$$

17. Given a discrete-time nonlinear system $\mathbf{x}(k+1) = \mathbf{f}(\mathbf{x}(k))$. The equilibrium points of this system can be obtained :

- computing the points $\mathbf{x}_e(k)$ tali that $\mathbf{x}_e(k) = \mathbf{f}(\mathbf{x}_e(k))$.
- solving the equation $0 = \mathbf{f}(\mathbf{x}_e(k))$ with respect to the unknown $\mathbf{x}_e(k)$.
- computing the points of the state space where $\mathbf{x}_e(k+1) = \mathbf{x}_e(k)$.

18. Write the structure of the dual system \mathcal{S}_D corresponding to a given system $\mathcal{S} = (\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D})$:

$$\mathcal{S}_D = (\mathbf{A}^T, \mathbf{C}^T, \mathbf{B}^T, \mathbf{D}^T)$$

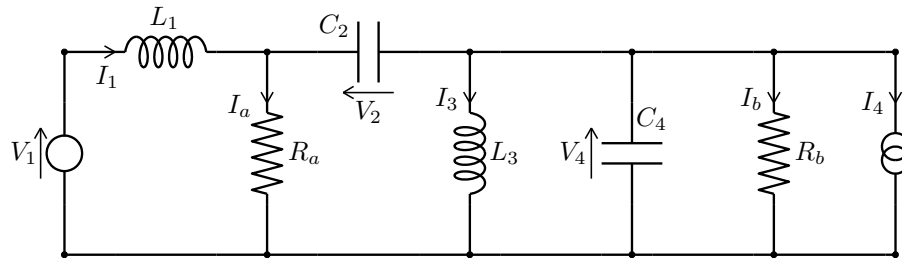
19. Consider a continuous-time nonlinear system $\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t))$, $\mathbf{y}(t) = \mathbf{g}(\mathbf{x}(t), \mathbf{u}(t))$ and let \mathbf{x}_0 be an equilibrium point of the system for constant input \mathbf{u}_0 . Write the definition of the matrices \mathbf{A} , \mathbf{B} , \mathbf{C} , \mathbf{D} of the linearized system:

$$\mathbf{A} = \left. \frac{\partial \mathbf{f}(\mathbf{x}, \mathbf{u})}{\partial \mathbf{x}} \right|_{(\mathbf{x}_0, \mathbf{u}_0)}, \quad \mathbf{B} = \left. \frac{\partial \mathbf{f}(\mathbf{x}, \mathbf{u})}{\partial \mathbf{u}} \right|_{(\mathbf{x}_0, \mathbf{u}_0)}, \quad \mathbf{C} = \left. \frac{\partial \mathbf{g}(\mathbf{x}, \mathbf{u})}{\partial \mathbf{x}} \right|_{(\mathbf{x}_0, \mathbf{u}_0)}, \quad \mathbf{D} = \left. \frac{\partial \mathbf{g}(\mathbf{x}, \mathbf{u})}{\partial \mathbf{u}} \right|_{(\mathbf{x}_0, \mathbf{u}_0)}$$

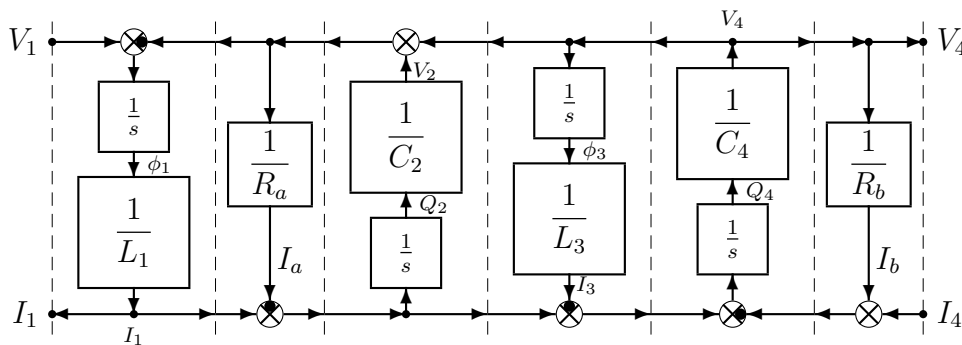
20. Write, within the following table, the symbols and the names of the energy variables and the power variables that characterize the Energetic Domain: *Mechanical Rotational*. Moreover, write the constitutive relation (both linear and non linear) and the differential equation which characterize the physical elements:

	Symbols	Constitutive Rel.	Linear Case	Differential Eq.
\mathcal{D}_1	J Inertia			
q_1	P Ang. Momentum	$P = \Phi_J(\omega)$	$P = J \omega$	$\frac{dP}{dt} = \tau$
v_1	ω Ang. Velocity			
\mathcal{D}_2	E Tors. Elasticity			
q_2	θ Ang. Displacement	$\theta = \Phi_E(\tau)$	$\theta = E \tau$	$\frac{d\theta}{dt} = \omega$
v_2	τ Torque			
\mathcal{R}	b Friction	$\tau = \Phi_b(\omega)$	$\tau = b \omega$	

21. Consider the following electric circuit composed by the inductances L_1 , L_3 , the capacities C_2 , C_4 and the resistances R_a and R_b . Two inputs act on the system: the voltage V_1 and the current I_4 . The outputs of the system are: the current I_1 and the voltage V_4 .



The POG model of the given electric circuit has the following structure:



Let $\mathbf{x} = [I_1 \ V_2 \ I_3 \ V_4]^T$ be the state vector, $\mathbf{u} = [V_1 \ I_4]^T$ the input vector and $\mathbf{y} = [I_1 \ V_4]^T$ the output vector. Write the corresponding dynamic system $\bar{\mathbf{L}}\dot{\mathbf{x}} = \bar{\mathbf{A}}\mathbf{x} + \bar{\mathbf{B}}\mathbf{u}$ and $\mathbf{y} = \bar{\mathbf{C}}\mathbf{x} + \bar{\mathbf{D}}\mathbf{u}$ in the state space:

$$\underbrace{\begin{bmatrix} L_1 & 0 & 0 & 0 \\ 0 & C_2 & 0 & 0 \\ 0 & 0 & L_3 & 0 \\ 0 & 0 & 0 & C_4 \end{bmatrix}}_{\bar{\mathbf{L}}} \underbrace{\begin{bmatrix} \dot{I}_1 \\ \dot{V}_2 \\ \dot{I}_3 \\ \dot{V}_4 \end{bmatrix}}_{\dot{\mathbf{x}}} = \underbrace{\begin{bmatrix} 0 & -1 & 0 & -1 \\ 1 & -\frac{1}{R_a} & 0 & -\frac{1}{R_a} \\ 0 & 0 & 0 & 1 \\ 1 & -\frac{1}{R_a} & -1 & -\frac{1}{R_a} - \frac{1}{R_b} \end{bmatrix}}_{\bar{\mathbf{A}}} \underbrace{\begin{bmatrix} I_1 \\ V_2 \\ I_3 \\ V_4 \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix}}_{\bar{\mathbf{B}}} \underbrace{\begin{bmatrix} V_1 \\ I_4 \end{bmatrix}}_{\mathbf{u}}$$

$$\underbrace{\begin{bmatrix} I_1 \\ V_4 \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\bar{\mathbf{C}}} \mathbf{x} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}_{\bar{\mathbf{D}}} \underbrace{\begin{bmatrix} V_1 \\ I_4 \end{bmatrix}}_{\mathbf{u}}$$

22. Write the La Salle - Krasowskii stability criterion for discrete-time nonlinear systems.

Consider the discrete-time nonlinear system $\mathbf{x}(k+1) = \mathbf{f}(\mathbf{x}(k), \mathbf{u}_0)$ and let \mathbf{x}_0 be an equilibrium point corresponding to the constant input \mathbf{u}_0 .

If in a neighborhood W of point \mathbf{x}_0 it exists a continuous function $V(\mathbf{x}) : W \rightarrow \mathcal{R}$ positive definite, if the function $\Delta V(\mathbf{x})$ is negative semidefinite and if the set $\mathcal{N} = \{\mathbf{x} \in W | \Delta V(\mathbf{x}) = 0\}$ does not contain perturbed trajectories of the given system, then \mathbf{x}_0 is an asymptotically stable equilibrium point.

23. Given the following continuous-time nonlinear system $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, u)$:

$$\begin{cases} \dot{x}_1 = x_2^2 - x_1^3 - x_1 \\ \dot{x}_2 = \alpha x_2 - x_1^3 x_2 + u \end{cases}$$

Set $u = 0$, it is easy to verify that the origin $\mathbf{x}_1 = (0, 0)$ is an equilibrium point for the system.

a) Compute the matrix $\mathbf{A}(\mathbf{x}) = \frac{\partial \mathbf{f}(\mathbf{x}, u)}{\partial \mathbf{x}}$ and the vector $\mathbf{b} = \frac{\partial \mathbf{f}(\mathbf{x}, u)}{\partial u}$ of the nonlinear system:

The matrix $\mathbf{A}(\mathbf{x})$ and the vector \mathbf{b} have the following structure:

$$\mathbf{A}(\mathbf{x}) = \begin{bmatrix} -3x_1^2 - 1 & 2x_2 \\ -3x_1^2 x_2 & \alpha - x_1^3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

b) Compute the matrix \mathbf{A}_1 and the vector \mathbf{b}_1 of the linearized system at the point $\mathbf{x}_1 = (0, 0)$, $u = 0$:

The matrix \mathbf{A}_1 and the vector \mathbf{b}_1 of the linearized system have the following structure:

$$\mathbf{A}_1 = \begin{bmatrix} -1 & 0 \\ 0 & \alpha \end{bmatrix}, \quad \mathbf{b}_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

c) Study, for varying parameter α , the stability of the nonlinear system in the neighborhood of point $\mathbf{x}_1 = (0, 0)$, $u = 0$ using the reduced Lyapunov criterion:

The characteristic polynomial and the eigenvalues of matrix \mathbf{A}_1 are:

$$\Delta_{\mathbf{A}_1}(s) = (s + 1)(s - \alpha) = 0 \quad \rightarrow \quad s = -1, \quad s = \alpha$$

Using the reduced Lyapunov criterion it can be stated that for $\alpha > 0$ the equilibrium point $\mathbf{x}_1 = (0, 0)$ of the nonlinear system is unstable, while for $\alpha < 0$ the equilibrium point \mathbf{x}_1 is asymptotically stable. For $\alpha = 0$ the criterion cannot be used.

d) For $\alpha = 0$ and $u = 0$, study the stability of the nonlinear system in the vicinity of the origin $\mathbf{x}_1 = (0, 0)$ using the “direct” Lyapunov criterion and the function: $V(\mathbf{x}) = x_1^4 + 2x_2^2$. Eventually, use the La Salle - Krasowskii criterion.

In the neighbourhood of the origin the function $V(\mathbf{x}) = x_1^4 + 2x_2^2$ is surely positive definite. La time derivative of the function $V(\mathbf{x})$ along the system's trajectories for $\alpha = 0$ and $u = 0$ is the following:

$$\dot{V} = 4x_1^3(x_2^2 - x_1^3 - x_1) + 4x_2(-x_1^3 x_2) = -4x_1^6 - 4x_1^4 \leq 0$$

The function \dot{V} is negative semidefinite and therefore, using the “direct” Lyapunov criterion, it can be stated that in the vicinity of the origin the nonlinear system is simply stable. The set $\mathcal{N} = \{(0, x_2), x_2 \in \mathbb{R}\}$ of all the points that nullify the \dot{V} does not contain perturbed trajectories of the system and therefore, using the La Salle - Krasowskii criterion, it can be stated that for $\alpha = 0$ the nonlinear system is asymptotically stable in the vicinity of the origin.

e) Set $\alpha = 0$ and $u = k_1 x_1 + k_2 x_2$, compute for which values of the parameters k_1 and k_2 the linearized system in the neighborhood of point $\mathbf{x}_1 = (0, 0)$ is asymptotically stable.

Set $\mathbf{k}^T = [k_1 \ k_2]$, the matrix \mathbf{A}_k of the linearized system in the vicinity of the origin has the following structure:

$$\mathbf{A}_k = \mathbf{A}_1 + \mathbf{b}_1 \mathbf{k}^T = \begin{bmatrix} -1 & 0 \\ k_1 & k_2 \end{bmatrix}$$

The linearized system is asymptotically stable for $k_2 < 0$ and $\forall k_1$.