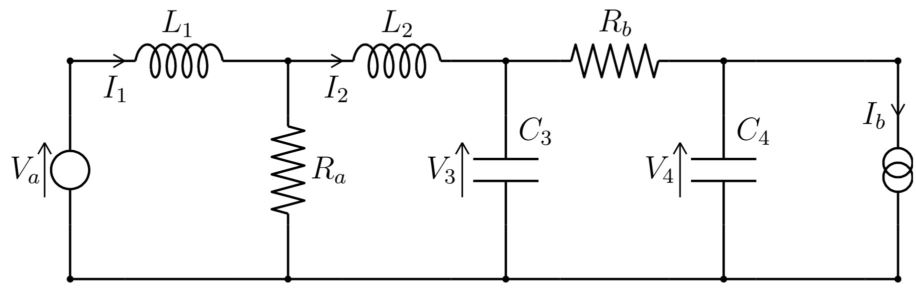
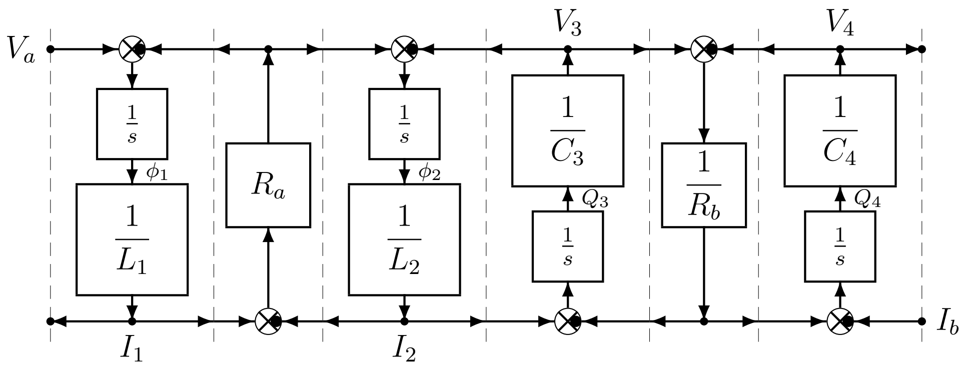
# Simple Examples of POG Modeling

1. Consider the following electric circuit composed by the inductances $L_1$, $L_2$, the capacities $C_3$, $C_3$, the resistances $R_a$, $R_b$, the input voltage $V_a$ and the output current $I_b$:



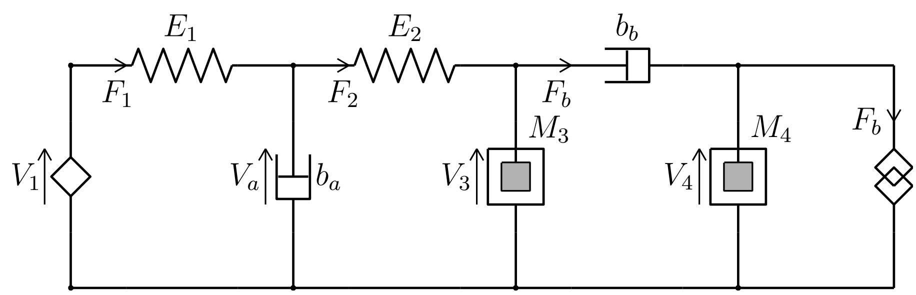

The POG model of the given electric circuit is the following:




Let $\mathbf{x} = \begin{bmatrix} I_1 & I_2 & V_3 & V_4 \end{bmatrix}^{\mathrm{T}}$ be the state vector (composed by the output power variables of the dynamic elements ) and let $\mathbf{u} = \begin{bmatrix} V_a & I_b \end{bmatrix}^{\mathrm{T}}$ be the input vector. Write the corresponding dynamic system $\overline{\mathbf{L}}\dot{\mathbf{x}} = \overline{\mathbf{A}}\mathbf{x} + \overline{\mathbf{B}}\mathbf{u}$ in the state space:

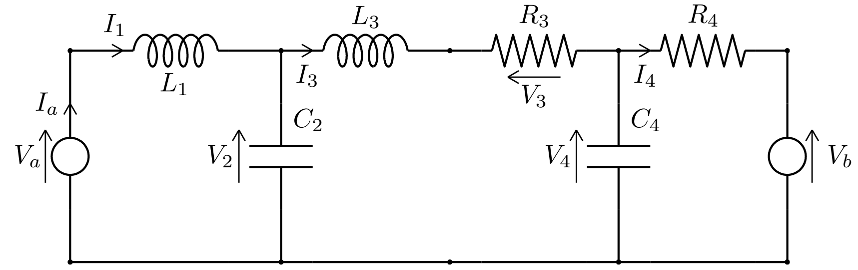
$$\underbrace{\begin{bmatrix} L_1 & 0 & 0 & 0 \\ 0 & L_2 & 0 & 0 \\ 0 & 0 & C_3 & 0 \\ 0 & 0 & 0 & C_4 \end{bmatrix}}_{\overline{\mathbf{L}}} \underbrace{\begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \\ \dot{V}_3 \\ \dot{V}_4 \end{bmatrix}}_{\dot{\mathbf{x}}} = \underbrace{\begin{bmatrix} -R_a & R_a & 0 & 0 \\ R_a & -R_a & -1 & 0 \\ 0 & 1 & -\frac{1}{R_b} & \frac{1}{R_b} \\ 0 & 0 & \frac{1}{R_b} & -\frac{1}{R_b} \end{bmatrix}}_{\overline{\mathbf{A}}} \underbrace{\begin{bmatrix} I_1 \\ I_2 \\ V_3 \\ V_4 \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix}}_{\overline{\mathbf{B}}} \underbrace{\begin{bmatrix} V_a \\ I_b \end{bmatrix}}_{\mathbf{u}}$$

- **Analogy**: direct correspondence between Electrical and Mechanical physical domains:

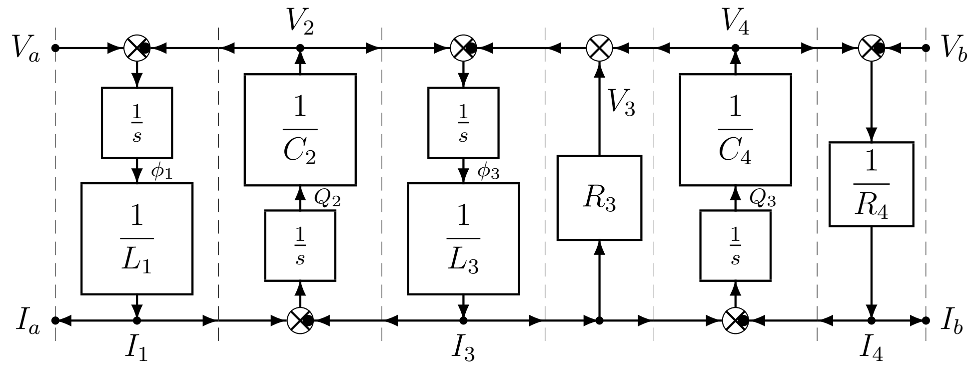



The physical domain is different (Mechanical), but the mathematical model is the same if $V_1 = Va$, $E_1 = L_1$, $E_2 = L_2$, $M_3 = C_3$, $M_4 = C_4$, $F_b = I_b$, $F_1 = I_1$, $F_2 = I_2$, $b_a = R_a$ and $b_b = R_b$.

2. Consider the following electric circuit composed by the inductances $L_1$, $L_3$, the capacities $C_2$, $C_3$ and the resistances $R_3$ and $R_4$. Two inputs act on the system: the voltage $V_a$ and the voltage $V_b$. The outputs of the system are: the current $I_a$ and the current $I_b = I_4$.




The POG model of the given electric circuit is the following:

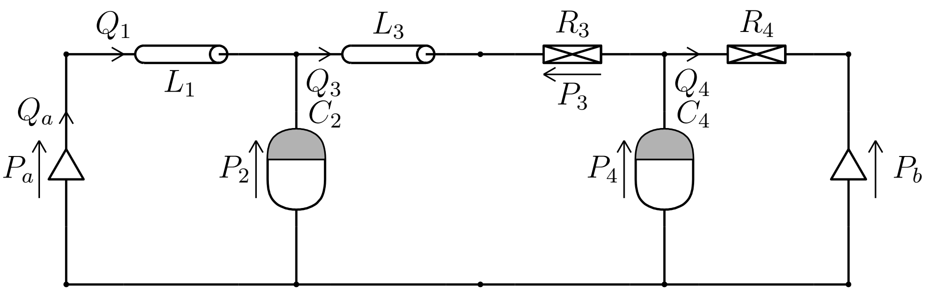



Let $\mathbf{x} = \begin{bmatrix} I_1 & V_2 & I_3 & V_4 \end{bmatrix}^{\mathrm{T}}$ be the state vector, $\mathbf{u} = \begin{bmatrix} V_a & V_b \end{bmatrix}^{\mathrm{T}}$ the input vector and $\mathbf{y} = \begin{bmatrix} I_a & I_b \end{bmatrix}^{\mathrm{T}}$ the output vector. Write the corresponding dynamic system $\overline{\mathbf{L}}\dot{\mathbf{x}} = \overline{\mathbf{A}}\mathbf{x} + \overline{\mathbf{B}}\mathbf{u}$ and $\mathbf{y} = \overline{\mathbf{C}}\mathbf{x} + \overline{\mathbf{D}}\mathbf{u}$ in the state space:

$$\underbrace{\begin{bmatrix} L_1 & 0 & 0 & 0 \\ 0 & C_2 & 0 & 0 \\ 0 & 0 & L_3 & 0 \\ 0 & 0 & 0 & C_4 \end{bmatrix}}_{\overline{\mathbf{L}}} \underbrace{\begin{bmatrix} \dot{I}_1 \\ \dot{V}_2 \\ \dot{I}_3 \\ \dot{V}_4 \end{bmatrix}}_{\dot{\mathbf{x}}} = \underbrace{\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & -R_3 & -1 \\ 0 & 0 & 1 & -\frac{1}{R_4} \end{bmatrix}}_{\overline{\mathbf{A}}} \underbrace{\begin{bmatrix} I_1 \\ V_2 \\ I_3 \\ V_4 \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{1}{R_4} \end{bmatrix}}_{\overline{\mathbf{B}}} \underbrace{\begin{bmatrix} V_a \\ V_b \end{bmatrix}}_{\mathbf{u}}$$

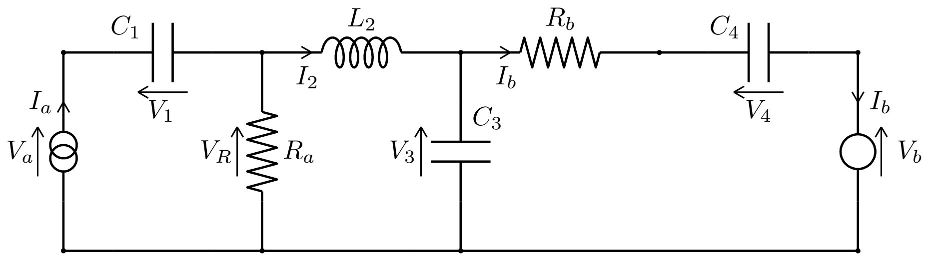
$$\underbrace{\begin{bmatrix} I_a \\ I_b \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{R_4} \end{bmatrix}}_{\overline{\mathbf{C}}} \mathbf{x} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{R_4} \end{bmatrix}}_{\overline{\mathbf{D}}} \underbrace{\begin{bmatrix} V_a \\ V_b \end{bmatrix}}_{\mathbf{u}}$$

- **Analogy**: direct correspondence between Electrical and Hydraulic physical domains:

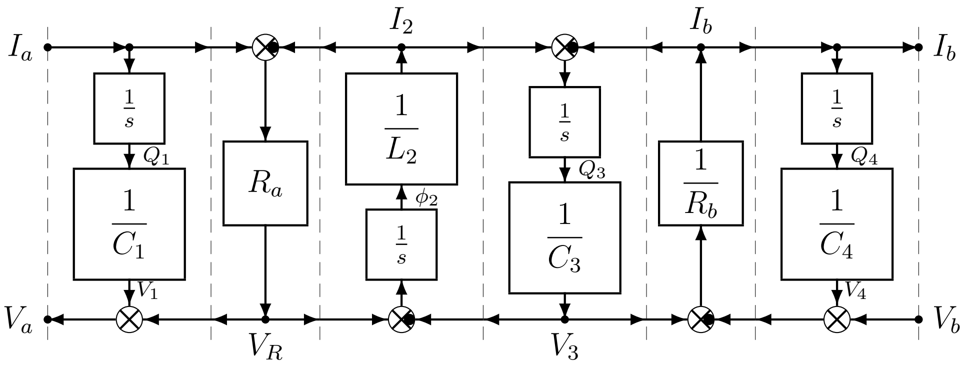



The physical domain is different (Hydraulic), but the mathematical model is the same if $P_a = V_a$, $P_b = V_b$, $P_2 = V_2$, $P_4 = V_4$, $Q_a = I_a$, $Q_1 = I_1$, $Q_3 = I_3$, $Q_4 = I_4$, .

3. Consider the following electric circuit composed by the capacities $C_1$, $C_3$, $C_4$, the inductance $L_2$ and the resistances $R_a$ and $R_b$. Two inputs act on the system: the current $I_a$ and the voltage $V_b$. The outputs of the system are: the voltage $V_a$ and the current $I_b$.




The POG model of the given electric circuit is the following:

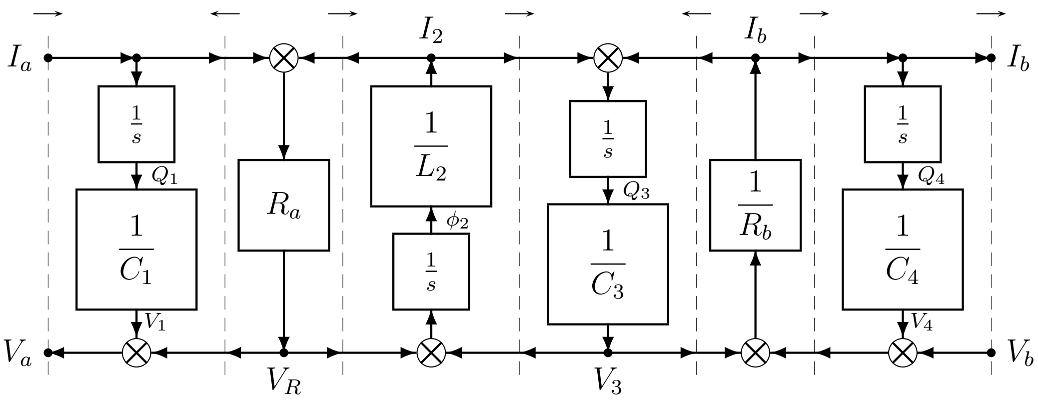



Let $\mathbf{x} = \begin{bmatrix} V_1 & I_2 & V_3 & V_4 \end{bmatrix}^{\mathrm{T}}$ be the state vector, $\mathbf{u} = \begin{bmatrix} I_a & V_b \end{bmatrix}^{\mathrm{T}}$ the input vector and $\mathbf{y} = \begin{bmatrix} V_a & I_b \end{bmatrix}^{\mathrm{T}}$ the output vector. Write the corresponding dynamic system $\overline{\mathbf{L}}\dot{\mathbf{x}} = \overline{\mathbf{A}}\mathbf{x} + \overline{\mathbf{B}}\mathbf{u}$ and $\mathbf{y} = \overline{\mathbf{C}}\mathbf{x} + \overline{\mathbf{D}}\mathbf{u}$ in the state space:

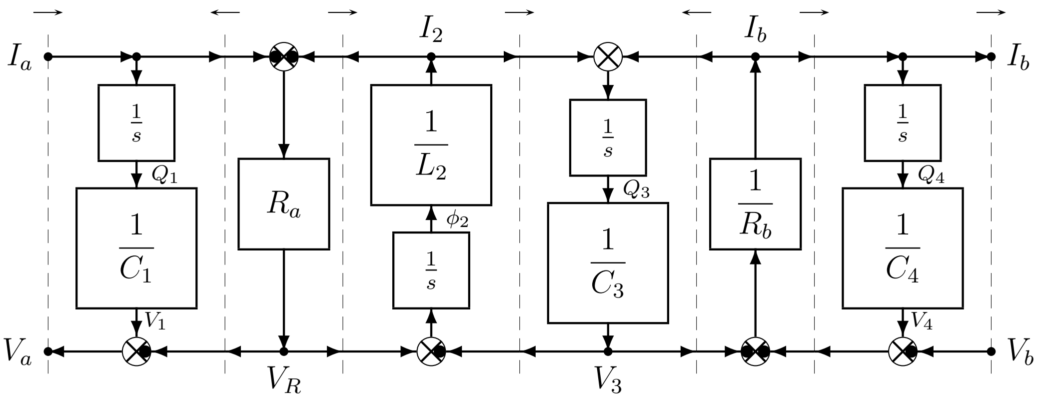
$$\underbrace{\begin{bmatrix} C_1 & 0 & 0 & 0 \\ 0 & L_2 & 0 & 0 \\ 0 & 0 & C_3 & 0 \\ 0 & 0 & 0 & C_4 \end{bmatrix}}_{\overline{\mathbf{L}}} \underbrace{\begin{bmatrix} \dot{V}_1 \\ \dot{I}_2 \\ \dot{V}_3 \\ \dot{V}_4 \end{bmatrix}}_{\dot{\mathbf{x}}} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -R_a & -1 & 0 \\ 0 & 1 & -\frac{1}{R_b} & \frac{1}{R_b} \\ 0 & 0 & \frac{1}{R_b} & -\frac{1}{R_b} \end{bmatrix}}_{\overline{\mathbf{A}}} \underbrace{\begin{bmatrix} V_1 \\ I_2 \\ V_3 \\ V_4 \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} 1 & 0 \\ R_a & 0 \\ 0 & \frac{1}{R_b} \\ 0 & -\frac{1}{R_b} \end{bmatrix}}_{\overline{\mathbf{B}}} \underbrace{\begin{bmatrix} I_a \\ V_b \end{bmatrix}}_{\mathbf{u}}$$

$$\underbrace{\begin{bmatrix} V_a \\ I_b \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} 1 & -R_a & 0 & 0 \\ 0 & 0 & \frac{1}{R_b} & -\frac{1}{R_b} \end{bmatrix}}_{\overline{\mathbf{C}}} \mathbf{x} + \underbrace{\begin{bmatrix} R_a & 0 \\ 0 & -\frac{1}{R_b} \end{bmatrix}}_{\overline{\mathbf{D}}} \underbrace{\begin{bmatrix} I_a \\ V_b \end{bmatrix}}_{\mathbf{u}}$$

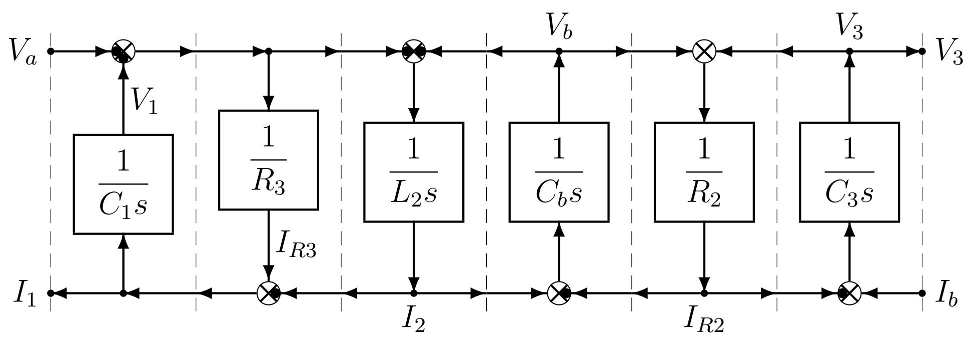
• Correct the following POG scheme adding minus signs in the summation elements:



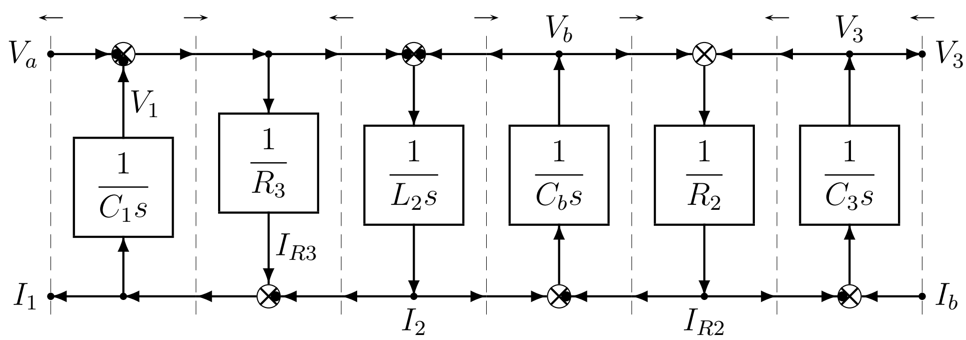

Solution:



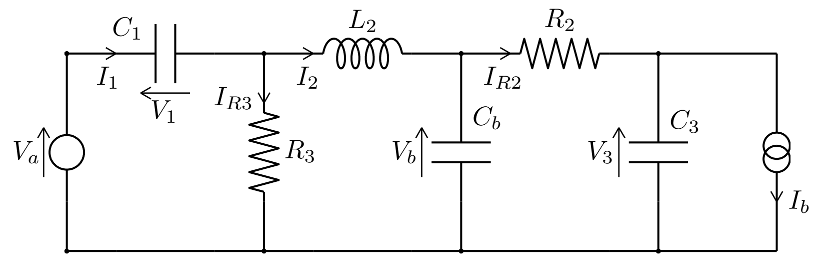

• Add the positive directions of the power flows to the following POG scheme:



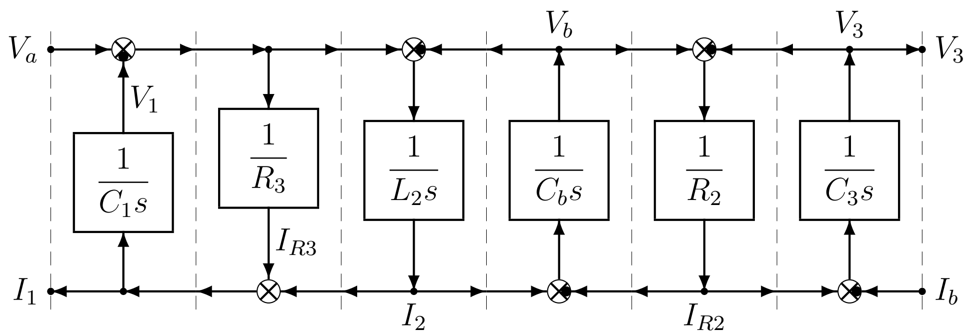

Solution:



4. Consider the following electric circuit composed by the capacities $C_1$, $C_b$, $C_3$ and the resistances $R_2$ and $R_3$. Two inputs act on the system: the voltage $V_a$ and the current $I_b$. The outputs of the system are: the current $I_1$ and the voltage $V_3$.




The POG model of the given electric circuit is the following:

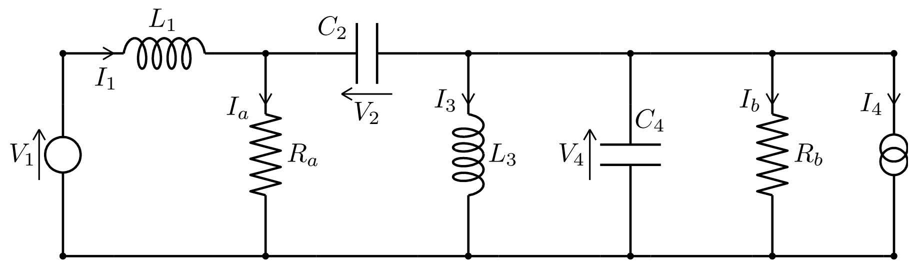



Let $\mathbf{x} = \begin{bmatrix} V_1 & I_2 & V_b & V_3 \end{bmatrix}^{\mathrm{T}}$ be the state vector, $\mathbf{u} = \begin{bmatrix} V_a & I_b \end{bmatrix}^{\mathrm{T}}$ the input vector and $\mathbf{y} = \begin{bmatrix} I_1 & V_3 \end{bmatrix}^{\mathrm{T}}$ the output vector. Write the corresponding dynamic system $\overline{\mathbf{L}}\dot{\mathbf{x}} = \overline{\mathbf{A}}\mathbf{x} + \overline{\mathbf{B}}\mathbf{u}$ and $\mathbf{y} = \overline{\mathbf{C}}\mathbf{x} + \overline{\mathbf{D}}\mathbf{u}$ in the state space:

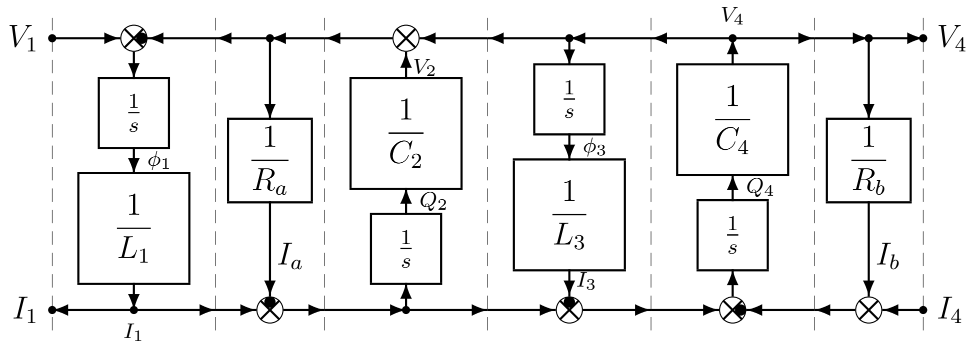
$$\underbrace{\begin{bmatrix} C_1 & 0 & 0 & 0 \\ 0 & L_2 & 0 & 0 \\ 0 & 0 & C_b & 0 \\ 0 & 0 & 0 & C_3 \end{bmatrix}}_{\overline{\mathbf{L}}} \underbrace{\begin{bmatrix} \dot{V}_1 \\ \dot{I}_2 \\ \dot{V}_b \\ \dot{V}_3 \end{bmatrix}}_{\dot{\mathbf{x}}} = \underbrace{\begin{bmatrix} -\frac{1}{R_3} & 1 & 0 & 0 \\ -1 & 0 & -1 & 0 \\ 0 & 1 & -\frac{1}{R_2} & \frac{1}{R_2} \\ 0 & 0 & \frac{1}{R_2} & -\frac{1}{R_2} \end{bmatrix}}_{\overline{\mathbf{A}}} \underbrace{\begin{bmatrix} V_1 \\ I_2 \\ V_b \\ V_3 \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} \frac{1}{R_3} & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix}}_{\overline{\mathbf{B}}} \underbrace{\begin{bmatrix} V_a \\ I_b \end{bmatrix}}_{\mathbf{u}}$$

$$\underbrace{\begin{bmatrix} I_1 \\ V_3 \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} -\frac{1}{R_3} & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}}_{\overline{\mathbf{C}}} \mathbf{x} + \underbrace{\begin{bmatrix} \frac{1}{R_3} & 0 \\ 0 & 0 \end{bmatrix}}_{\overline{\mathbf{D}}} \underbrace{\begin{bmatrix} V_a \\ I_b \end{bmatrix}}_{\mathbf{u}}$$

5. Consider the following electric circuit composed by the inductances $L_1$, $L_3$, the capacities $C_2$, $C_4$ and the resistances $R_a$ and $R_b$. Two inputs act on the system: the voltage $V_1$ and the current $I_4$. The outputs of the system are: the current $I_1$ and the voltage $V_4$.




The POG model of the given electric circuit has the following structure:

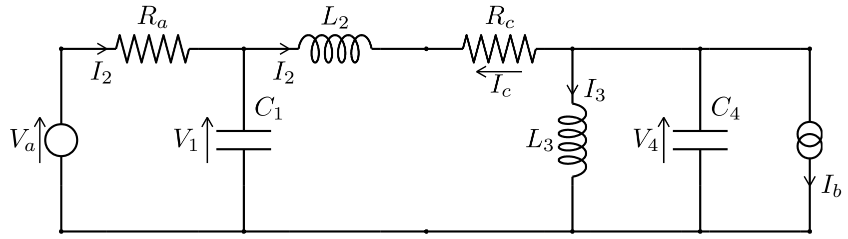



Let $\mathbf{x} = \begin{bmatrix} I_1 & V_2 & I_3 & V_4 \end{bmatrix}^{\mathrm{T}}$ be the state vector, $\mathbf{u} = \begin{bmatrix} V_1 & I_4 \end{bmatrix}^{\mathrm{T}}$ the input vector and $\mathbf{y} = \begin{bmatrix} I_1 & V_4 \end{bmatrix}^{\mathrm{T}}$ the output vector. Write the corresponding dynamic system $\overline{\mathbf{L}}\dot{\mathbf{x}} = \overline{\mathbf{A}}\mathbf{x} + \overline{\mathbf{B}}\mathbf{u}$ and $\mathbf{y} = \overline{\mathbf{C}}\mathbf{x} + \overline{\mathbf{D}}\mathbf{u}$ in the state space:

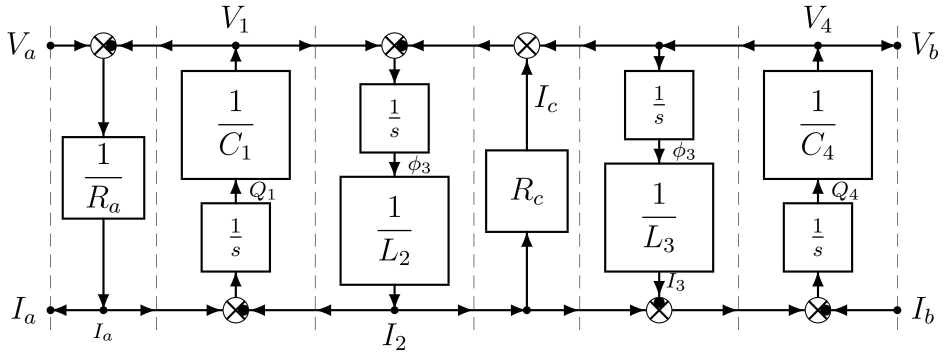
$$
\underbrace{\begin{bmatrix} L_1 & 0 & 0 & 0 \\ 0 & C_2 & 0 & 0 \\ 0 & 0 & L_3 & 0 \\ 0 & 0 & 0 & C_4 \end{bmatrix}}_{\overline{\mathbf{L}}}
\underbrace{\begin{bmatrix} \dot{I}_1 \\ \dot{V}_2 \\ \dot{I}_3 \\ \dot{V}_4 \end{bmatrix}}_{\dot{\mathbf{x}}}
=
\underbrace{\begin{bmatrix} 0 & -1 & 0 & -1 \\ 1 & -\frac{1}{R_a} & 0 & -\frac{1}{R_a} \\ 0 & 0 & 0 & 1 \\ 1 & -\frac{1}{R_a} & -1 & -\frac{1}{R_a} - \frac{1}{R_b} \end{bmatrix}}_{\overline{\mathbf{A}}}
\underbrace{\begin{bmatrix} I_1 \\ V_2 \\ I_3 \\ V_4 \end{bmatrix}}_{\mathbf{x}}
+
\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix}}_{\overline{\mathbf{B}}}
\underbrace{\begin{bmatrix} V_1 \\ I_4 \end{bmatrix}}_{\mathbf{u}}
$$

$$
\underbrace{\begin{bmatrix} I_1 \\ V_4 \end{bmatrix}}_{\mathbf{y}}
=
\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\overline{\mathbf{C}}}
\mathbf{x} +
\underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}_{\overline{\mathbf{D}}}
\underbrace{\begin{bmatrix} V_1 \\ I_4 \end{bmatrix}}_{\mathbf{u}}
$$

6. Consider the following electric circuit composed by the capacities $C_1$, $C_4$, the inductances $L_2$, $L_3$ and the resistances $R_a$, $R_b$ and $R_c$. Two inputs act on the system: the voltage $V_a$ and the current $I_b$. The outputs of the system are: the current $I_a$ and the voltage $V_b$.




The POG model of the given electric circuit has the following structure:

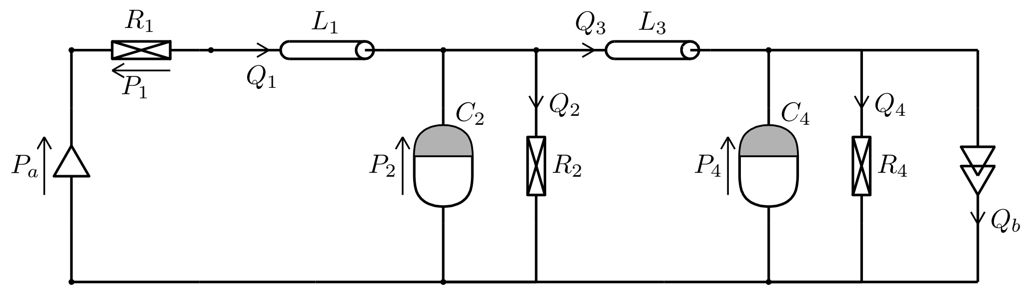



Let $\mathbf{x} = \begin{bmatrix} V_1 & I_2 & I_3 & V_4 \end{bmatrix}^{\mathrm{T}}$ be the state vector, $\mathbf{u} = \begin{bmatrix} V_a & I_b \end{bmatrix}^{\mathrm{T}}$ the input vector and $\mathbf{y} = \begin{bmatrix} I_a & V_b \end{bmatrix}^{\mathrm{T}}$ the output vector. Write the corresponding dynamic system $\overline{\mathbf{L}}\dot{\mathbf{x}} = \overline{\mathbf{A}}\mathbf{x} + \overline{\mathbf{B}}\mathbf{u}$ and $\mathbf{y} = \overline{\mathbf{C}}\mathbf{x} + \overline{\mathbf{D}}\mathbf{u}$ in the state space:

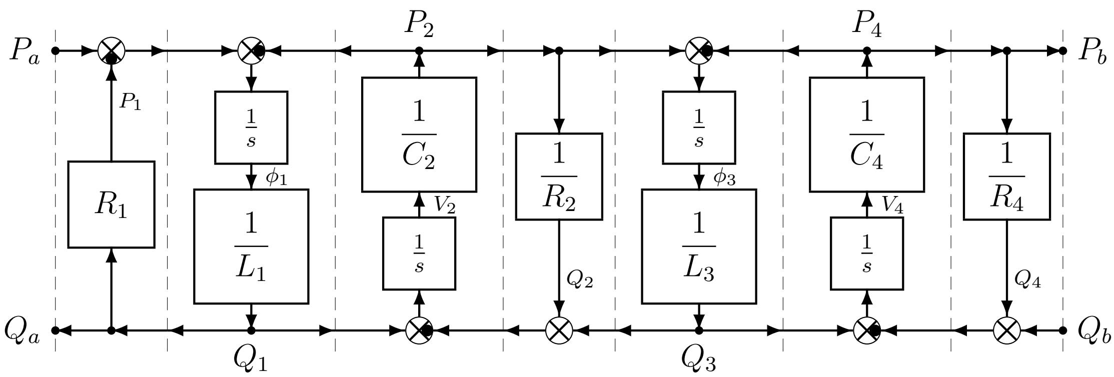
$$\underbrace{\begin{bmatrix} C_1 & 0 & 0 & 0 \\ 0 & L_2 & 0 & 0 \\ 0 & 0 & L_3 & 0 \\ 0 & 0 & 0 & C_4 \end{bmatrix}}_{\overline{\mathbf{L}}} \underbrace{\begin{bmatrix} \dot{V}_1 \\ \dot{I}_2 \\ \dot{I}_3 \\ \dot{V}_4 \end{bmatrix}}_{\dot{\mathbf{x}}} = \underbrace{\begin{bmatrix} -\frac{1}{R_a} & -1 & 0 & 0 \\ 1 & -R_c & 0 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \end{bmatrix}}_{\overline{\mathbf{A}}} \underbrace{\begin{bmatrix} V_1 \\ I_2 \\ I_3 \\ V_4 \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} \frac{1}{R_a} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix}}_{\overline{\mathbf{B}}} \underbrace{\begin{bmatrix} V_a \\ I_b \end{bmatrix}}_{\mathbf{u}}$$

$$\underbrace{\begin{bmatrix} I_a \\ V_b \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} -\frac{1}{R_a} & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}}_{\overline{\mathbf{C}}} \mathbf{x} + \underbrace{\begin{bmatrix} \frac{1}{R_a} & 0 \\ 0 & 0 \end{bmatrix}}_{\overline{\mathbf{D}}} \underbrace{\begin{bmatrix} V_a \\ I_b \end{bmatrix}}_{\mathbf{u}}$$

7. Consider the following hydraulic circuit composed by the hydraulic inductances $L_1$, $L_3$, the hydraulic capacities $C_2$, $C_4$ and the hydraulic resistances $R_1$, $R_2$, $R_4$, $R_5$ and $R_6$. Two inputs act on the system: the pressione $P_a$ and the volume flow rate $Q_b$. The outputs of the system are: the volume flow rate $Q_a = Q_1 + Q_5$ and the pressione $P_b = P_4$.




The POG model of the given hydraulic circuit has the following structure:

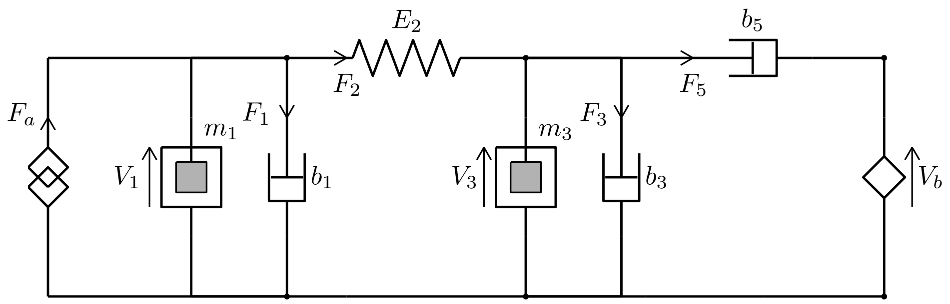



Let $\mathbf{x} = \begin{bmatrix} Q_1 & P_2 & Q_3 & P_4 \end{bmatrix}^{\mathrm{T}}$ be the state vector, $\mathbf{u} = \begin{bmatrix} P_a & Q_b \end{bmatrix}^{\mathrm{T}}$ the input vector and $\mathbf{y} = \begin{bmatrix} Q_a & P_b \end{bmatrix}^{\mathrm{T}}$ the output vector. Write the corresponding dynamic system $\overline{\mathbf{L}}\dot{\mathbf{x}} = \overline{\mathbf{A}}\mathbf{x} + \overline{\mathbf{B}}\mathbf{u}$ and $\mathbf{y} = \overline{\mathbf{C}}\mathbf{x} + \overline{\mathbf{D}}\mathbf{u}$ in the state space:

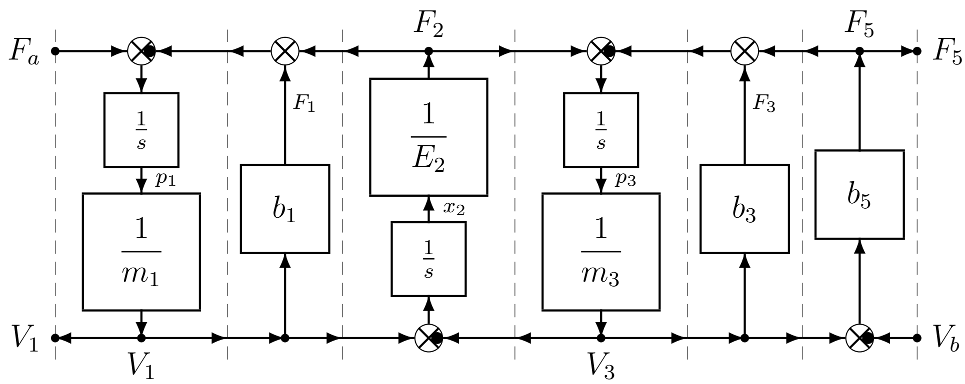
$$\underbrace{\begin{bmatrix} L_1 & 0 & 0 & 0 \\ 0 & C_2 & 0 & 0 \\ 0 & 0 & L_3 & 0 \\ 0 & 0 & 0 & C_4 \end{bmatrix}}_{\overline{\mathbf{L}}} \underbrace{\begin{bmatrix} \dot{Q}_1 \\ \dot{P}_2 \\ \dot{Q}_3 \\ \dot{P}_4 \end{bmatrix}}_{\dot{\mathbf{x}}} = \underbrace{\begin{bmatrix} -R_1 & -1 & 0 & 0 \\ 1 & -\frac{1}{R_2} & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -\frac{1}{R_4} \end{bmatrix}}_{\overline{\mathbf{A}}} \underbrace{\begin{bmatrix} Q_1 \\ P_2 \\ Q_3 \\ P_4 \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix}}_{\overline{\mathbf{B}}} \underbrace{\begin{bmatrix} P_a \\ Q_b \end{bmatrix}}_{\mathbf{u}}$$

$$\underbrace{\begin{bmatrix} Q_a \\ P_b \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\overline{\mathbf{C}}} \mathbf{x} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}_{\overline{\mathbf{D}}} \underbrace{\begin{bmatrix} P_a \\ Q_b \end{bmatrix}}_{\mathbf{u}}$$

8. Consider the following mechanical system composed by masses $m_1$, $m_3$, elasticity $E_4$ and dampers $b_1$, $b_2$ and $b_5$. Two inputs act on the system: the force $F_a$ and the velocity $V_b$. The outputs of the system are: velocity $V_1$ and the force $F_5$.




The POG model of the given mechanical system has the following structure:

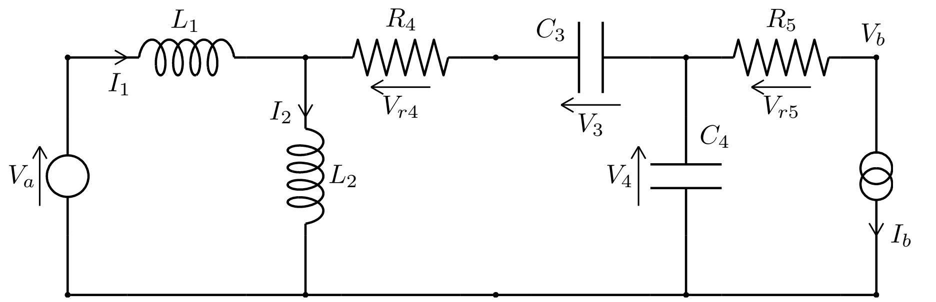



Let $\mathbf{x} = \begin{bmatrix} V_1 & F_2 & V_3 \end{bmatrix}^{\mathrm{T}}$ be the state vector, $\mathbf{u} = \begin{bmatrix} F_a & V_b \end{bmatrix}^{\mathrm{T}}$ the input vector and $\mathbf{y} = \begin{bmatrix} V_1 & F_5 \end{bmatrix}^{\mathrm{T}}$ the output vector. Write the corresponding dynamic system $\overline{\mathbf{L}}\dot{\mathbf{x}} = \overline{\mathbf{A}}\mathbf{x} + \overline{\mathbf{B}}\mathbf{u}$ and $\mathbf{y} = \overline{\mathbf{C}}\mathbf{x} + \overline{\mathbf{D}}\mathbf{u}$ in the state space:

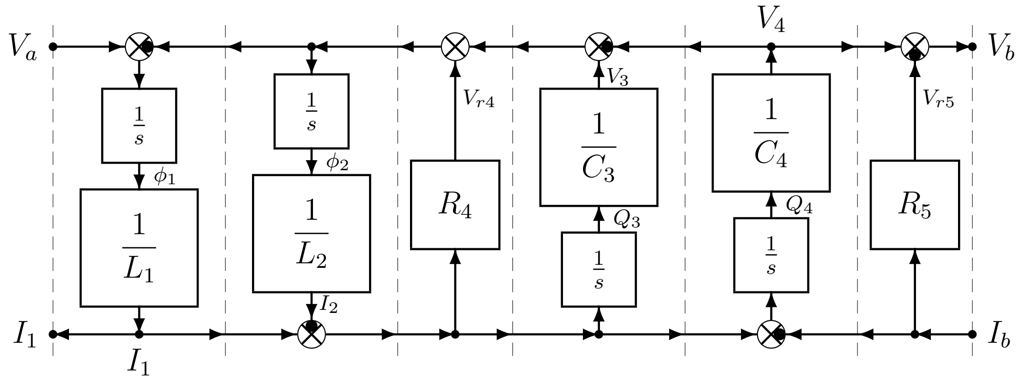
$$\underbrace{\begin{bmatrix} m_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}}_{\overline{\mathbf{L}}} \underbrace{\begin{bmatrix} \dot{V}_1 \\ \dot{F}_2 \\ \dot{V}_3 \end{bmatrix}}_{\dot{\mathbf{x}}} = \underbrace{\begin{bmatrix} -b_1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -b_3-b_5 \end{bmatrix}}_{\overline{\mathbf{A}}} \underbrace{\begin{bmatrix} V_1 \\ F_2 \\ V_3 \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & b_5 \end{bmatrix}}_{\overline{\mathbf{B}}} \underbrace{\begin{bmatrix} F_a \\ V_b \end{bmatrix}}_{\mathbf{u}}$$

$$\underbrace{\begin{bmatrix} V_1 \\ F_5 \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & b_5 \end{bmatrix}}_{\overline{\mathbf{C}}} \mathbf{x} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & -b_5 \end{bmatrix}}_{\overline{\mathbf{D}}} \underbrace{\begin{bmatrix} F_a \\ V_b \end{bmatrix}}_{\mathbf{u}}$$

9. Consider the following electric circuit composed by the inductances $L_1$, $L_2$, the capacities $C_3$, $C_4$ and the resistances $R_1$, $R_4$ and $R_5$. Two inputs act on the system: the voltage $V_a$ and the current $I_b$. The outputs of the system are: the current $I_1$ and the voltage $V_b$.




The POG model of the given electric circuit has the following structure:




Let $\mathbf{x} = \begin{bmatrix} I_1 & I_2 & V_3 & V_4 \end{bmatrix}^{\mathrm{T}}$ be the state vector, $\mathbf{u} = \begin{bmatrix} V_a & I_b \end{bmatrix}^{\mathrm{T}}$ the input vector and $\mathbf{y} = \begin{bmatrix} I_1 & V_b \end{bmatrix}^{\mathrm{T}}$ the output vector. Write the corresponding dynamic system $\overline{\mathbf{L}}\dot{\mathbf{x}} = \overline{\mathbf{A}}\mathbf{x} + \overline{\mathbf{B}}\mathbf{u}$ and $\mathbf{y} = \overline{\mathbf{C}}\mathbf{x} + \overline{\mathbf{D}}\mathbf{u}$ in the state space:

$$\underbrace{\begin{bmatrix} L_1 & 0 & 0 & 0 \\ 0 & L_2 & 0 & 0 \\ 0 & 0 & C_3 & 0 \\ 0 & 0 & 0 & C_4 \end{bmatrix}}_{\overline{\mathbf{L}}} \underbrace{\begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \\ \dot{V}_3 \\ \dot{V}_4 \end{bmatrix}}_{\dot{\mathbf{x}}} = \underbrace{\begin{bmatrix} -R_4 & R_4 & -1 & -1 \\ R_4 & -R_4 & 1 & 1 \\ 1 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix}}_{\overline{\mathbf{A}}} \underbrace{\begin{bmatrix} I_1 \\ I_2 \\ V_3 \\ V_4 \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix}}_{\overline{\mathbf{B}}} \underbrace{\begin{bmatrix} V_a \\ I_b \end{bmatrix}}_{\mathbf{u}}$$

$$\underbrace{\begin{bmatrix} I_1 \\ V_b \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\overline{\mathbf{C}}} \mathbf{x} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & -R_5 \end{bmatrix}}_{\overline{\mathbf{D}}} \underbrace{\begin{bmatrix} V_a \\ I_b \end{bmatrix}}_{\mathbf{u}}$$