Pole placement ($m = 1$)

- Property. *Let* $\mathcal{S} = (\mathbf{A}, \mathbf{b}, \mathbf{C}, \mathbf{d})$ *be a time-invariant linear system of dimension* n*, completely reachable and with only one input* ($m = 1$)*. For each monic polynomial* $p(\lambda)$ *of degree* n*, it exists a vector* $\mathbf{k}^T \in \mathcal{R}^{1\times n}$ *such that the characteristic polynomial of the matrix* $\mathbf{A} + \mathbf{b}\mathbf{k}^T$ *of the feedback system* $\mathcal{S}_\mathbf{k}$ *is equal to* $p(\lambda)$*.*

- Proof. Let α_i $(i = 0, \ldots, n-1)$ be the coefficients of the characteristic polynomial $\Delta_\mathbf{A}(\lambda)$ of matrix $\mathbf{A}$:

$$\Delta_\mathbf{A}(\lambda) = \lambda^n + \alpha_{n-1}\lambda^{n-1} + \ldots + \alpha_1\lambda + \alpha_0$$

and let d_i $(i = 0, \ldots, n-1)$ be the coefficients of the freely chosen polynomial $p(\lambda)$:

$$p(\lambda) = \lambda^n + d_{n-1}\lambda^{n-1} + \ldots + d_1\lambda + d_0.$$

Since the couple $(\mathbf{A}, \mathbf{b})$ is reachable, it exists a state space transformation $\mathbf{x} = \mathbf{T}_c\mathbf{x}_c$ which brings system $\mathcal{S}$ into the controllability canonical form:

$$\mathbf{A}_c = \mathbf{T}_c^{-1}\mathbf{A}\mathbf{T}_c = \begin{bmatrix} 0 & 1 & 0\ldots & 0 \\ 0 & 0 & 1\ldots & 0 \\ \vdots & \vdots & & \vdots \\ -\alpha_0 & -\alpha_1 & \ldots & -\alpha_{n-1} \end{bmatrix}, \qquad \mathbf{b}_c = \mathbf{T}_c^{-1}\mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$

where $\mathbf{T}_c = \mathcal{R}^+(\mathcal{R}_c^+)^{-1}$. Using the following state feedback control law:

$$u(t) = \mathbf{k}_c^T\mathbf{x}_c(t) + v(t)$$

where vector $\mathbf{k}_c^T$ has the following structure:

$$\mathbf{k}_c^T = [k_0, k_1, \ldots, k_{n-1}]$$

one obtains the following state equation:

$$\dot{\mathbf{x}}_c(t) = (\mathbf{A}_c + \mathbf{b}_c\mathbf{k}_c^T)\mathbf{x}_c(t) + \mathbf{b}_c\, v(t)$$

where

$$\mathbf{A}_c + \mathbf{b}_c\mathbf{k}_c^T = \begin{bmatrix} 0 & 1 & 0\ldots & 0 \\ 0 & 0 & 1\ldots & 0 \\ \vdots & \vdots & & \vdots \\ k_0-\alpha_0 & k_1-\alpha_1 & \ldots & k_{n-1}-\alpha_{n-1} \end{bmatrix}$$

- The characteristic polynomial of matrix $\mathbf{A}_c + \mathbf{b}_c\mathbf{k}_c^T$ is:

$$\Delta_{\mathbf{A}_c+\mathbf{b}_c\mathbf{k}_c^T}(\lambda) = \lambda^n + \lambda^{n-1}(\alpha_{n-1}-k_{n-1}) + \ldots + (\alpha_0 - k_0)$$

Imposing $\Delta_{\mathbf{A}_c+\mathbf{b}_c\mathbf{k}_c^T}(\lambda)$ equal to the polynomial $p(\lambda)$ we obtain:

$$d_i = \alpha_i - k_i \qquad \Rightarrow \qquad k_i = \alpha_i - d_i \qquad \text{for} \qquad i = 0,\ \ldots,\ n-1,$$

from which it follows:

$$\mathbf{k}_c^T = \left[\ \alpha_0 - d_0,\ \ \alpha_1 - d_1,\ \ \ldots,\ \ \alpha_{n-1} - d_{n-1}\ \right]$$

- The vector $\mathbf{k}_c^T$, in the controllability canonical form, forces the feedback system to have its eigenvalues equal to the zeros of polynomial $p(\lambda)$.

- Being:

$$u(t) = \mathbf{k}_c^T\mathbf{x}_c(t) + v(t) = \mathbf{k}^T\mathbf{x}(t) + v(t) = \mathbf{k}^T\mathbf{T}_c\mathbf{x}_c(t) + v(t)$$

it follows that:

$$\mathbf{k}_c^T = \mathbf{k}^T\,\mathbf{T}_c \qquad \Rightarrow \qquad \boxed{\mathbf{k}^T = \mathbf{k}_c^T\mathbf{T}_c^{-1} = \mathbf{k}_c^T[\mathcal{R}^+(\mathcal{R}_c^+)^{-1}]^{-1}}$$

So, vector $\mathbf{k}^T$ can be computed using the following formula:

$$\mathbf{k}^T = \mathbf{k}_c^T \left\{ \left[\ \mathbf{b},\ \ \mathbf{A}\mathbf{b},\ \ \ldots,\ \ \mathbf{A}^{n-1}\mathbf{b}\ \right] \begin{bmatrix} \alpha_1 & \alpha_2 & \ldots & \alpha_{n-1} & 1 \\ \alpha_2 & \ldots & \ldots & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \alpha_{n-1} & 1 & \ldots & 0 & 0 \\ 1 & 0 & \ldots & 0 & 0 \end{bmatrix} \right\}^{-1}$$

where $\mathbf{k}_c^T$ is the vector defined above.

- Note: the vector $\mathbf{k}^T$ can be computed only if the matrices $\mathbf{A}$, $\mathbf{b}$ and the polynomials $p(\lambda)$ and $\Delta_{\mathbf{A}}(\lambda)$ are known.

Example. Let us consider the following discrete linear system:

$$\mathbf{x}(k+1) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u(k)$$

Compute, if it is possible, the static feedback control law $u(k) = \mathbf{k}^T\mathbf{x}(k)$ such that the feedback system $\mathbf{A} + \mathbf{b}\mathbf{k}^T$ has all its eigenvalues in the origin ("dead-beat" behavior).

Solution. The reachability matrix of the system is:

$$\mathcal{R}^+ = \begin{bmatrix} 1 & 3 & 6 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix} \qquad \text{det}\mathcal{R}^+ = -1.$$

The system is completely reachable and therefore it exists a static feedback control law $u(k) = \mathbf{k}^T\mathbf{x}(k)$ such that all the poles of the feedback system can be located arbitrarily. The characteristic polynomial of matrix $\mathbf{A}$ is:

$$\Delta_{\mathbf{A}}(\lambda) = \det(\lambda\mathbf{I} - \mathbf{A}) = (\lambda - 1)^3 = \lambda^3 \underbrace{-3}_{\alpha_2} \lambda^2 \underbrace{+3}_{\alpha_1} \lambda \underbrace{-1}_{\alpha_0}$$

The desired characteristic polynomial is:

$$p(\lambda) = \lambda^3 = \lambda^3 + \underbrace{0}_{d_2} \lambda^2 + \underbrace{0}_{d_1} \lambda + \underbrace{0}_{d_0}$$

Vector $\mathbf{k}^T$ can be determined as follows:

$$\begin{aligned} \mathbf{k}^T &= \mathbf{k}_c^T[\mathcal{R}^+(\mathcal{R}_c^+)^{-1}]^{-1} = \mathbf{k}_c^T \left\{ \begin{bmatrix} 1 & 3 & 6 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & -3 & 1 \\ -3 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \right\}^{-1} \\ &= \mathbf{k}_c^T \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 1 \\ 1 & -2 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} \underbrace{-1}_{\alpha_0 - d_0} & \underbrace{3}_{\alpha_1 - d_1} & \underbrace{-3}_{\alpha_2 - d_2} \end{bmatrix} \begin{bmatrix} 1 & -2 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} -1 & -1 & -1 \end{bmatrix} \end{aligned}$$

Ackerman formula

- *Let us consider a continuous or discrete-time linear system with only one input ($m = 1$):*

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{b}\,u(t) \qquad \text{or} \qquad \mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{b}\,u(k)$$

  and let $p(\lambda)$ be a polynomial freely chosen:

$$p(\lambda) = \lambda^n + d_{n-1}\lambda^{n-1} + \ldots + d_1\lambda + d_0$$

  If the couple $(\mathbf{A},\, \mathbf{b})$ is reachable, then the gain vector $\mathbf{k}^T$ such that $\Delta_{\mathbf{A}+\mathbf{b}\mathbf{k}^T}(\lambda) = p(\lambda)$ can be computed using the following Ackerman formula*:*

$$\boxed{\mathbf{k}^T = -\mathbf{q}^T p(\mathbf{A})}$$

  where $\mathbf{q}^T$ is the last row of the inverse of the reachabilily matrix $\mathcal{R}^+$:

$$\mathbf{q}^T = \left[\begin{array}{cccc} 0 & \ldots & 0 & 1 \end{array}\right] (\mathcal{R}^+)^{-1}$$

  and $p(\mathbf{A})$ denotes the matrix obtained from polynomial $p(\lambda)$ when parameter λ is substituted by matrix $\mathbf{A}$.

- The advantage of using the Ackerman formula is that it does not require the knowledge of the characteristic polynomial $\Delta_{\mathbf{A}}(\lambda)$ of matrix $\mathbf{A}$.

Example. Let us refer to the previous example where it was $p(\lambda) = \lambda^3$. The gain vector $\mathbf{k}^T$ can also be computed as follows:

$$\begin{aligned}
\mathbf{k}^T &= -\mathbf{q}^T p(\mathbf{A}) = -\left[\begin{array}{ccc} 0 & 0 & 1 \end{array}\right] (\mathcal{R}^+)^{-1}\mathbf{A}^3 \\
&= -\left[\begin{array}{ccc} 0 & 0 & 1 \end{array}\right] \begin{bmatrix} 1 & 3 & 6 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}^3 \\
&= -\left[\begin{array}{ccc} 0 & 0 & 1 \end{array}\right] \begin{bmatrix} 1 & -3 & 3 \\ -2 & 5 & -3 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 6 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} = \left[\begin{array}{ccc} -1 & -1 & -1 \end{array}\right]
\end{aligned}$$

Example. Given the following system $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}\,u(t)$:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u(t)$$

compute the gain vector $\mathbf{k}^T$ of the static feedback control law $u(t) = \mathbf{k}^T\mathbf{x}$ which places in -1, -2 and -2 the three eigenvalues of the feedback system.

The system is completely reachable:

$$\mathcal{R}^+ = \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \qquad \mathbf{q}^T = -\frac{1}{2}\begin{bmatrix} -1 & 1 & 1 \end{bmatrix}$$

The desired polynomial is:

$$p(s) = (s+2)^2(s+1) = s^3 + 5s^2 + 8s + 4$$

Being:

$$\mathbf{A}^2 = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{A}^3 = \begin{bmatrix} 1 & 4 & 2 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

one obtains that:

$$\begin{aligned} p(\mathbf{A}) &= (\mathbf{A}+2\mathbf{I})^2(\mathbf{A}+\mathbf{I}) = \mathbf{A}^3 + 5\mathbf{A}^2 + 8\mathbf{A} + 4\mathbf{I} \\ &= \begin{bmatrix} 3 & 2 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}^2 \begin{bmatrix} 2 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 18 & 30 & 12 \\ 0 & 9 & 9 \\ 0 & 9 & 9 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 4 & 2 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} + 5\begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + 8\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} + 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

So, the following gain vector is obtained:

$$\mathbf{k}^T = -\mathbf{q}^T p(\mathbf{A}) = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 18 & 30 & 12 \\ 0 & 9 & 9 \\ 0 & 9 & 9 \end{bmatrix} = \begin{bmatrix} -9 & -6 & 3 \end{bmatrix}$$

The same result can also be obtained using the following formula:

$$\mathbf{k}^T = \mathbf{k}_c^T[\mathcal{R}^+(\mathcal{R}_c^+)^{-1}]^{-1}$$

In this case the characteristic polynomial of matrix $\mathbf{A}$ must be computed:

$$\det(s\mathbf{I} - \mathbf{A}) = \begin{vmatrix} s-1 & -2 & 0 \\ 0 & s & -1 \\ 0 & -1 & s \end{vmatrix} = (s+1)(s-1)^2 = s^3 - s^2 - s + 1$$

The desired polynomial is

$$p(s) = (s+2)^2(s+1) = s^3 + 5s^2 + 8s + 4$$

Vector $\mathbf{k}^T$ is obtained as follows

$$\begin{aligned}
\mathbf{k}^T &= \mathbf{k}_c^T\{\mathcal{R}^+(\mathcal{R}_c^+)^{-1}\}^{-1} \\
&= \begin{bmatrix} -3 & -9 & -6 \end{bmatrix} \left\{ \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \right\}^{-1} \\
&= \begin{bmatrix} -3 & -9 & -6 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}^{-1} \\
&= \begin{bmatrix} -3 & -9 & -6 \end{bmatrix} \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix} \frac{1}{2} \\
&= \begin{bmatrix} -9 & -6 & 3 \end{bmatrix}
\end{aligned}$$

Example. Let us consider the following continuous-time linear system $\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{b}u(t)$ and $y(t) = \mathbf{c}\mathbf{x}(t)$:

$$\begin{cases} \dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & -1 \\ -1 & -1 & 0 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} u(t) \\ y(t) = \begin{bmatrix} -1 & 1 & 0 \end{bmatrix} \mathbf{x}(t) \end{cases}$$

Compute, if it is possible, a static feedback control law $u(t) = \mathbf{k}^T\mathbf{x}(t)$ which places in -1 as many eigenvalues of the feedback systems as possible.

The reachability matrix of the system is:

$$\mathcal{R}^+ = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 0 & 0 \\ -1 & -2 & -2 \end{bmatrix}$$

Matrix $\mathcal{R}^+$ is not full rank and therefore the given system is not completely reachable. The reachability subspace $\mathcal{X}^+$ has the following structure:

$$\mathcal{X}^+ = \text{span}\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \right\} = \text{span}\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \right\}$$

The given system can be put in the reachability standard form using the following transformation matrix:

$$\mathbf{T}_s = \left[\begin{array}{cc|c} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -1 & 1 \end{array}\right] \qquad \rightarrow \qquad \mathbf{T}_s^{-1} = \left[\begin{array}{cc|c} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{array}\right]$$

Let $\mathbf{x} = \mathbf{T}_s\bar{\mathbf{x}}$. The reachability standard form of the system is the following:

$$\begin{cases} \dot{\bar{\mathbf{x}}}(t) & = \left[\begin{array}{cc|c} 0 & 0 & -1 \\ 1 & 1 & -1 \\ \hline 0 & 0 & -1 \end{array}\right] \bar{\mathbf{x}} + \left[\begin{array}{c} 1 \\ 1 \\ \hline 0 \end{array}\right] u(t) \\ y(t) & = \left[\begin{array}{cc|c} 1 & -1 & 0 \end{array}\right] \bar{\mathbf{x}} \end{cases}$$

The eigenvalues of the reachable subspace $\mathcal{X}^+$ are the eigenvalues of submatrix $\mathbf{A}_{11}$, that is $\lambda = 0$ and $\lambda = 1$. The eigenvalue of the not reachable part of the system is the eigenvalue of submatrix $\mathbf{A}_{22}$, that is $\lambda = -1$.

Since the not reachable part of the system is stable, then it exists a gain vector $\mathbf{k}^T = \left[\begin{array}{ccc} k_1 & k_2 & k_3 \end{array}\right]$ which stabilizes the feedback system and such that all the eigenvalues of the reachable part can be located in -1. Before computing vector $\mathbf{k}^T$ one has to compute the vector $\tilde{\mathbf{k}}^T$ which, in the reachability standard form of the system, locates in -1 all the eigenvalues of the reachable part of the system. The characteristic polynomials of matrix $\mathbf{A}_{11}$ and matrix $\mathbf{A}_{11} + \mathbf{b}_1\tilde{\mathbf{k}}^T$ are:

$$\Delta_{\mathbf{A}_{11}}(s) = s^2 - s, \qquad \Delta_{\mathbf{A}_{11}+\mathbf{b}_1\tilde{\mathbf{k}}^T}(s) = p(s) = (s+1)^2 = s^2 + 2s + 1.$$

Vector $\tilde{\mathbf{k}}^T$ can be computed, for example, using the following formula:

$$\begin{aligned} \tilde{\mathbf{k}}^T &= \mathbf{k}_c^T\mathbf{T}_c^{-1} = \left[\begin{array}{cc} -1 & -3 \end{array}\right] \left\{ \left[\begin{array}{cc} 1 & 0 \\ 1 & 2 \end{array}\right] \left[\begin{array}{cc} -1 & 1 \\ 1 & 0 \end{array}\right] \right\}^{-1} \\ &= \left[\begin{array}{cc} -1 & -3 \end{array}\right] \left[\begin{array}{cc} -1 & 1 \\ 1 & 1 \end{array}\right]^{-1} = \left[\begin{array}{cc} -1 & -3 \end{array}\right] \left[\begin{array}{cc} -0.5 & 0.5 \\ 0.5 & 0.5 \end{array}\right] = \left[\begin{array}{cc} -1 & -2 \end{array}\right] \end{aligned}$$

Finally, the gain vector $\mathbf{k}^T$ can be determined as follows:

$$\mathbf{k}^T = \left[\begin{array}{cc} \tilde{\mathbf{k}}^T & \alpha \end{array}\right] \mathbf{T}_s^{-1} = \left[\begin{array}{ccc} -1 & -2 & \alpha \end{array}\right] \left[\begin{array}{cc|c} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc} -2+\alpha & -1 & \alpha \end{array}\right]$$

where α is a free parameter.

Transfer function of a feedback system

- Let us consider a SISO linear system in the controllability canonical form:

$$\mathbf{A}_c = \begin{bmatrix} 0 & 1 & 0\ldots & 0 \\ 0 & 0 & 1\ldots & 0 \\ \vdots & \vdots & & \vdots \\ -\alpha_0 & -\alpha_1 & \ldots & -\alpha_{n-1} \end{bmatrix}, \quad \mathbf{b}_c = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}, \quad \mathbf{C}_c = \left[\begin{array}{cccc} \beta_0 & \beta_1 & \ldots & \beta_{n-1} \end{array}\right].$$

- The transfer function $\mathbf{H}(s)$ of the system is:

$$\mathbf{H}(s) = \mathbf{C}_c(s\mathbf{I} - \mathbf{A}_c)^{-1}\mathbf{b}_c = \mathbf{C}_c \frac{agg(s\mathbf{I} - \mathbf{A}_c)}{det(s\mathbf{I} - \mathbf{A}_c)}\mathbf{b}_c$$

where:

$$\det(s\mathbf{I} - \mathbf{A}_c) = s^n + \alpha_{n-1}s^{n-1} + \ldots + \alpha_0$$

$$agg(s\mathbf{I} - \mathbf{A}_c) = \begin{bmatrix} * & * & \ldots & 1 \\ ** & * & \ldots & s \\ \vdots & \vdots & & \vdots \\ ** & * & \ldots & s^{n-1} \end{bmatrix}$$

and therefore

$$\mathbf{H}(s) = \frac{\beta_{n-1}s^{n-1} + \beta_{n-2}s^{n-2} + \ldots + \beta_0}{s^n + \alpha_{n-1}s^{n-1} + \ldots + \alpha_0}$$

- It can be proved that the transfer function $\mathbf{H}_K$ of the feedback system is:

$$\mathbf{H}_K(s) = \frac{\beta_{n-1}s^{n-1} + \beta_{n-2}s^{n-2} + \ldots + \beta_0}{s^n + (\alpha_{n-1} - k_{n-1})s^{n-1} + \ldots + (\alpha_0 - k_0)}$$

- The two transfer functions $\mathbf{H}(s)$ and $\mathbf{H}_K(s)$ have the same polynomial at the numerator because "*the use of a static feedback control law does not modify the zeros of the system, it modifies only the poles*".