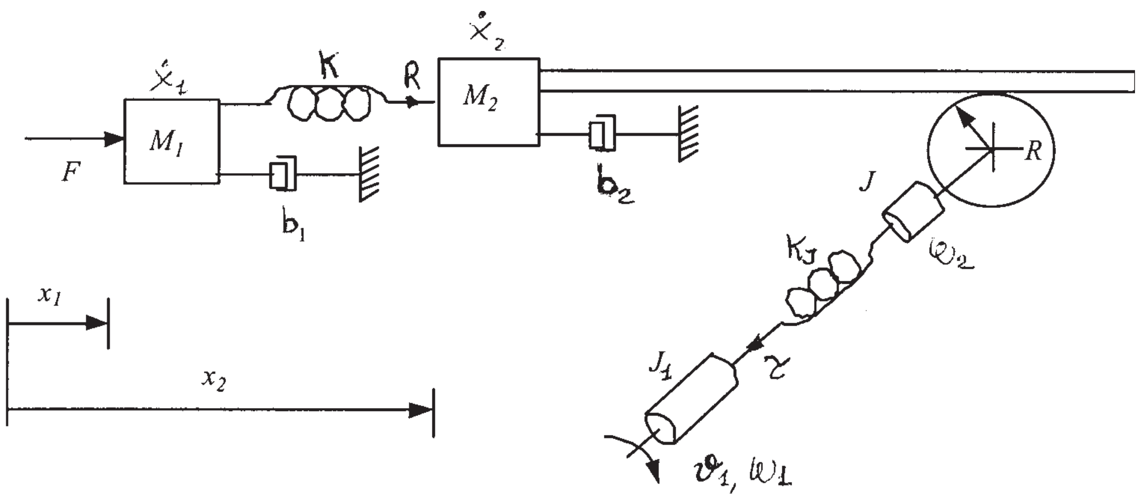
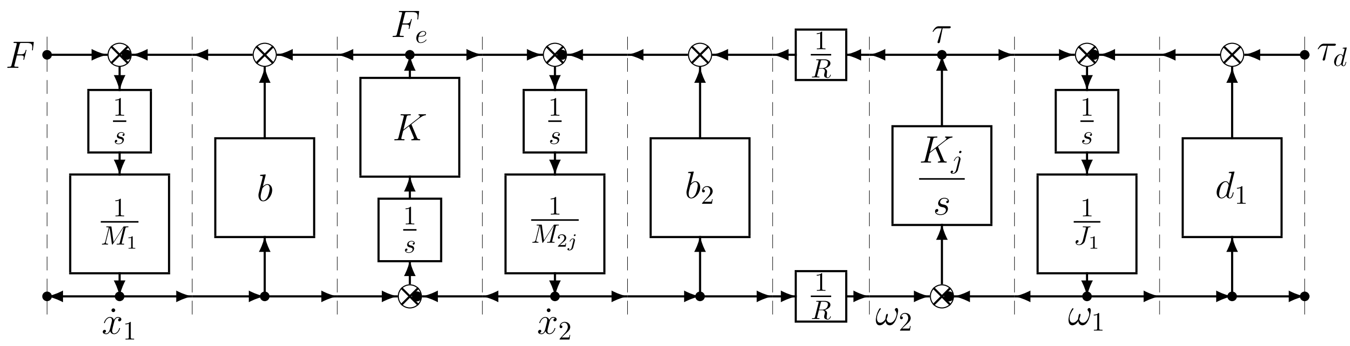
Mechanical transmission: state space model

- Mechanical transmission:



- POG dynamic model of the considered mechanical transmission:



- State vector (output power variables):

$$\mathbf{x} = \left[\begin{array}{ccccc} \dot{x}_1 & F_e & \dot{x}_2 & \tau & \omega_1 \end{array} \right]^T$$

- The force F is the system control input. The torque τ_d is an external disturbance input.
- The output variable is the angular velocity ω_1 of the last rotational element.

- The equivalent mass M_{2j} can be expressed as follows:

$$M_{2j} = M_2 + \frac{J_2}{R^2}$$

- State space equations:

$$\underbrace{\begin{bmatrix} M_1\,\ddot{x}_1 \\ \frac{1}{K}\,\dot{F}_e \\ M_{2j}\,\ddot{x}_2 \\ \frac{1}{K_j}\,\dot{\tau} \\ J_1\,\dot{\omega}_1 \end{bmatrix}}_{\mathbf{L}\dot{\mathbf{x}}} = \underbrace{\begin{bmatrix} -b_1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -b_2 & -\frac{1}{R} & 0 \\ 0 & 0 & \frac{1}{R} & 0 & -1 \\ 0 & 0 & 0 & 1 & -d_1 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} \dot{x}_1 \\ F_e \\ \dot{x}_2 \\ \tau \\ \omega_1 \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix}}_{\mathbf{B}} \underbrace{\begin{bmatrix} F \\ \tau_d \end{bmatrix}}_{\mathbf{u}}$$

$$\mathbf{y} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{\mathbf{C}} \mathbf{x}$$

that is

$$\begin{cases} \mathbf{L}\,\dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} + \mathbf{B}\,\mathbf{u} \\ \mathbf{y} = \mathbf{C}\,\mathbf{x} \end{cases} \quad \Downarrow \quad \begin{cases} \dot{\mathbf{x}} = \mathbf{L}^{-1}\mathbf{A}\,\mathbf{x} + \mathbf{L}^{-1}\mathbf{B}\,\mathbf{u} \\ \mathbf{y} = \mathbf{C}\,\mathbf{x} \end{cases}$$

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$$\mathbf{L} = \begin{bmatrix} M_1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{K} & 0 & 0 & 0 \\ 0 & 0 & M_{2j} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{K_j} & 0 \\ 0 & 0 & 0 & 0 & J_1 \end{bmatrix}$$

- The symmetric part of matrix $\mathbf{A}$ is a function of the dissipative parameters b_1, b_1, R and d_1. The skew-symmetric part of matrix $\mathbf{A}$ is a function only of the "connection" coefficients.
- Possible values for the parameters to be used for simulations in Matlab/Simulink environment:

```
M1 = 0.6*Kg;                  % First mass
b1 = 2*N/(40*m/sec);          % Linear friction coefficient of the first mass
K  = 100*N/(1*cm);            % Stiffness of the first spring
M2 = 1*Kg;                    % Second mass
b2 = 1*N/(50*m/sec);          % Linear friction coefficient of the first mass
R  = 10*cm;                   % Radius of the wheel
J2 = 150*gr*(12*cm)^2;        % Inerzia of element J2
Kj = 100*N/(0.1*rad);         % Stiffness of the torsional spring
J1 = 190*gr*(10*cm)^2;        % Inerzia of element J1
d1 = 10*N*m/(100*rad/sec);    % Linear friction coefficient of element J1
M2j = M2+J2/(R^2);            % Equivalent translational mass
```