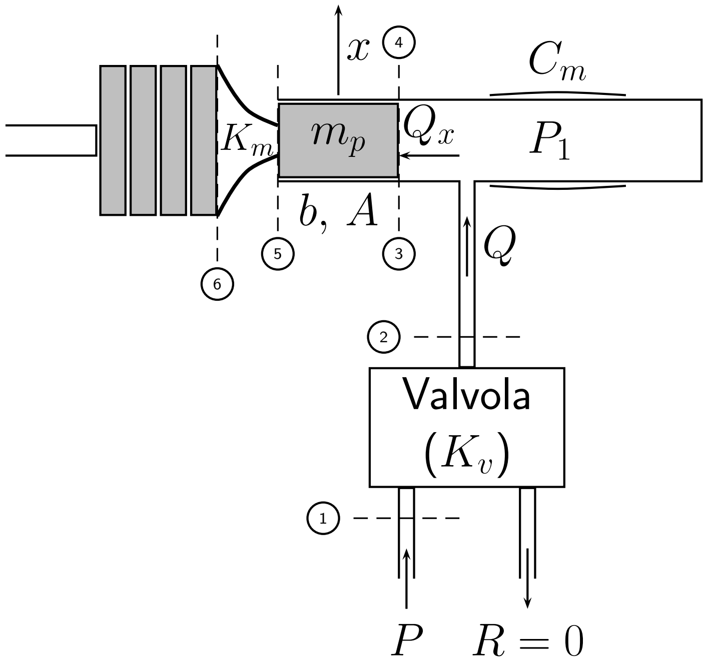
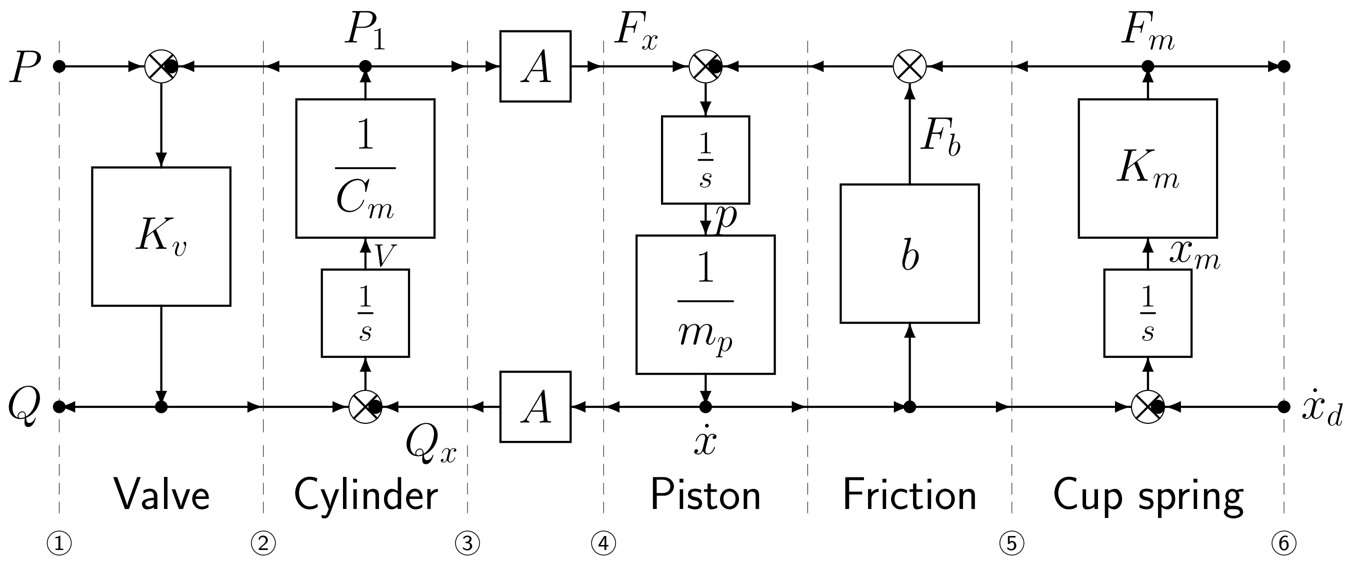
Hydraulic clutch: state space dynamic model

- Let us consider the following simplified dynamic model of an hydraulic clutch:

P	Supply pressure
Q	Volume flow rate
K_v	Valve proportional constant
C_m	Hydraulic capacity of the cylinder
P_1	Pressure within the cylinder
A	Section of the piston
x_m	squeezing of the cup spring
x	Position of the piston
$\dot{x}$	Velocity of the piston
m_p	Mass of the piston
b	Friction coefficient of the piston
K_m	Stiffness of the spring
F_m	Spring force against the piston
$\dot{x}_d$	Wear velocity of the clutch discs



- The corresponding POG dynamic model is:



- The state vector $\mathbf{x}$, the energy vector $\mathbf{q}$ and the input vector $\mathbf{u}$ are the following:

$$\mathbf{x} = \begin{bmatrix} P_1 \\ \dot{x} \\ F_m \end{bmatrix}^T, \qquad \mathbf{q} = \begin{bmatrix} V \\ p \\ x_m \end{bmatrix} \qquad \mathbf{u} = \begin{bmatrix} P \\ \dot{x}_d \end{bmatrix}^T$$

- The POG state space model is the following:

$$\underbrace{\begin{bmatrix} C_m & 0 & 0 \\ 0 & m_p & 0 \\ 0 & 0 & \frac{1}{K_m} \end{bmatrix}}_{\mathbf{L}} \underbrace{\begin{bmatrix} \dot{P}_1 \\ \ddot{x} \\ \dot{F}_m \end{bmatrix}}_{\dot{\mathbf{x}}} = \underbrace{\begin{bmatrix} -K_v & -A & 0 \\ A & -b & -1 \\ 0 & 1 & 0 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} P_1 \\ \dot{x} \\ F_m \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} K_v & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix}}_{\mathbf{B}} \underbrace{\begin{bmatrix} P \\ \dot{x}_d \end{bmatrix}}_{\mathbf{u}}$$

- Matrix $\mathbf{A}$ can be expressed as follows:

$$\mathbf{A} = \underbrace{\begin{bmatrix} -K_v & 0 & 0 \\ 0 & -b & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\mathbf{A}_s} + \underbrace{\begin{bmatrix} 0 & -A & 0 \\ A & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}}_{\mathbf{A}_w} = \mathbf{A}_s + \mathbf{A}_w$$

- Matrix $\mathbf{A}_s$ **is a function only of the dissipative static coefficients**" of the system: K_v e b.

- The skew-symmetric matrix $\mathbf{A}_w$ **is a function only of the "connection coefficients"** of the system: the connection coefficient A between sections 2 and 3, and the unitary coefficient in section 5.

- Left-multiplying this equation by matrix $\mathbf{L}^{-1}$ one obtains the following "classical" state space equation:

$$\dot{\mathbf{x}} = \underbrace{\mathbf{L}^{-1}\mathbf{A}}_{\overline{\mathbf{A}}}\,\mathbf{x} + \underbrace{\mathbf{L}^{-1}\mathbf{B}}_{\overline{\mathbf{B}}}\,\mathbf{u} \qquad \Leftrightarrow \qquad \dot{\mathbf{x}} = \overline{\mathbf{A}}\,\mathbf{x} + \overline{\mathbf{B}}\,\mathbf{u}$$

• Applying the state space transformation $\mathbf{x} = \mathbf{L}^{-1}\mathbf{q}$ to system (1) one obtains the following dynamic equation of the system:

$$\dot{\mathbf{q}} = \mathbf{A}\,\mathbf{L}^{-1}\mathbf{q} + \mathbf{B}\,\mathbf{u}$$

• The use of the energy vector $\mathbf{q}$ is quite suitable, but it is not the most frequently used.