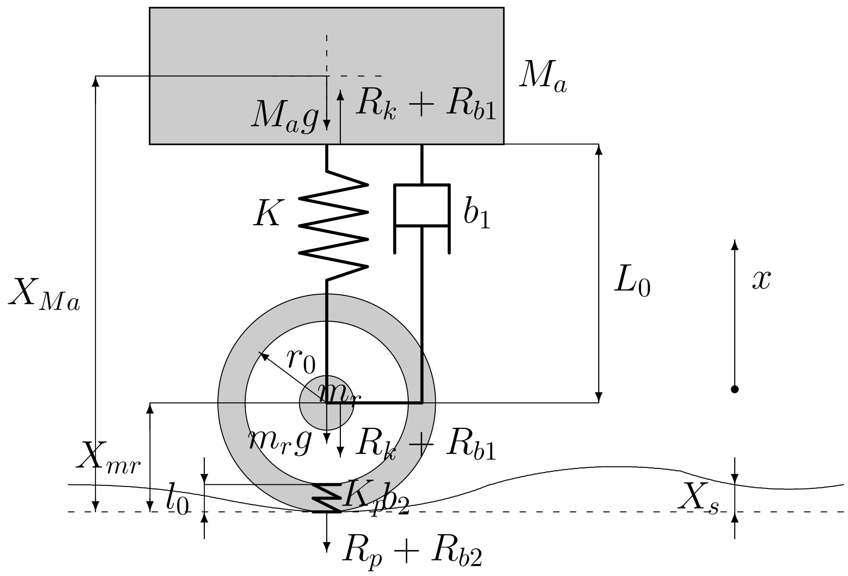
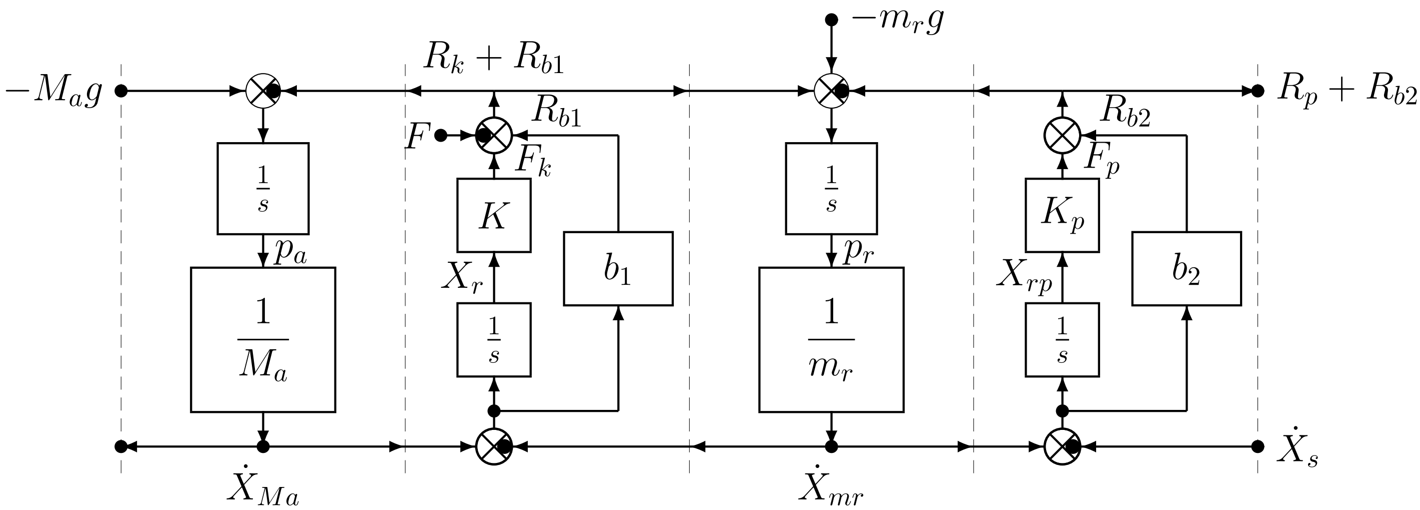
Car dumper: state space dynamic model

- Physical model of a car dumper:



- POG block scheme of the car dumper:



- Energy vector $\mathbf{q}$, state vector $\mathbf{x}$ and input vector $\mathbf{u}$:

$$\mathbf{q} = \begin{bmatrix} p_a \\ X_r \\ p_r \\ X_{rp} \end{bmatrix}, \qquad \mathbf{x} = \begin{bmatrix} \dot{X}_{Ma} \\ F_k \\ \dot{X}_{mr} \\ F_p \end{bmatrix}, \qquad \mathbf{u} = \begin{bmatrix} g \\ F \\ \dot{X}_s \end{bmatrix}$$

- The POG state space dynamic model of the system is:

$$\mathbf{L}\,\dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} + \mathbf{B}\,\mathbf{u}$$

- The energy matrix $\mathbf{L}$ and the input matrix $\mathbf{B}$ have the following form:

$$\mathbf{L} = \begin{bmatrix} M_a & 0 & 0 & 0 \\ 0 & \frac{1}{K} & 0 & 0 \\ 0 & 0 & m_r & 0 \\ 0 & 0 & 0 & \frac{1}{K_p} \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} -M_a & 1 & 0 \\ 0 & 0 & 0 \\ -m_r & -1 & b_2 \\ 0 & 0 & -1 \end{bmatrix}$$

- The symmetric part $\mathbf{A}_s$ and the skew symmetric part $\mathbf{A}_w$ af matrix $\mathbf{A}$ are:

$$\mathbf{A}_s = \begin{bmatrix} -b_1 & 0 & b_1 & 0 \\ 0 & 0 & 0 & 0 \\ b_1 & 0 & -b_1 - b_2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \mathbf{A}_w = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

- So, the POG dynamic model of the system can be expressed as follows:

$$\begin{bmatrix} M_a & & & \\ & \frac{1}{K} & & \\ & & m_r & \\ & & & \frac{1}{K_p} \end{bmatrix} \begin{bmatrix} \ddot{X}_{Ma} \\ \dot{F}_k \\ \ddot{X}_{mr} \\ \dot{F}_r \end{bmatrix} = \begin{bmatrix} -b_1 & -1 & b_1 & 0 \\ 1 & 0 & -1 & 0 \\ b_1 & 1 & -b_1 - b_2 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{X}_{Ma} \\ F_k \\ \dot{X}_{mr} \\ F_r \end{bmatrix} + \begin{bmatrix} -M_a & 1 & 0 \\ 0 & 0 & 0 \\ -m_r & -1 & b_2 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} g \\ F \\ \dot{X}_s \end{bmatrix}$$