Rappresentazione grafica - caso discreto

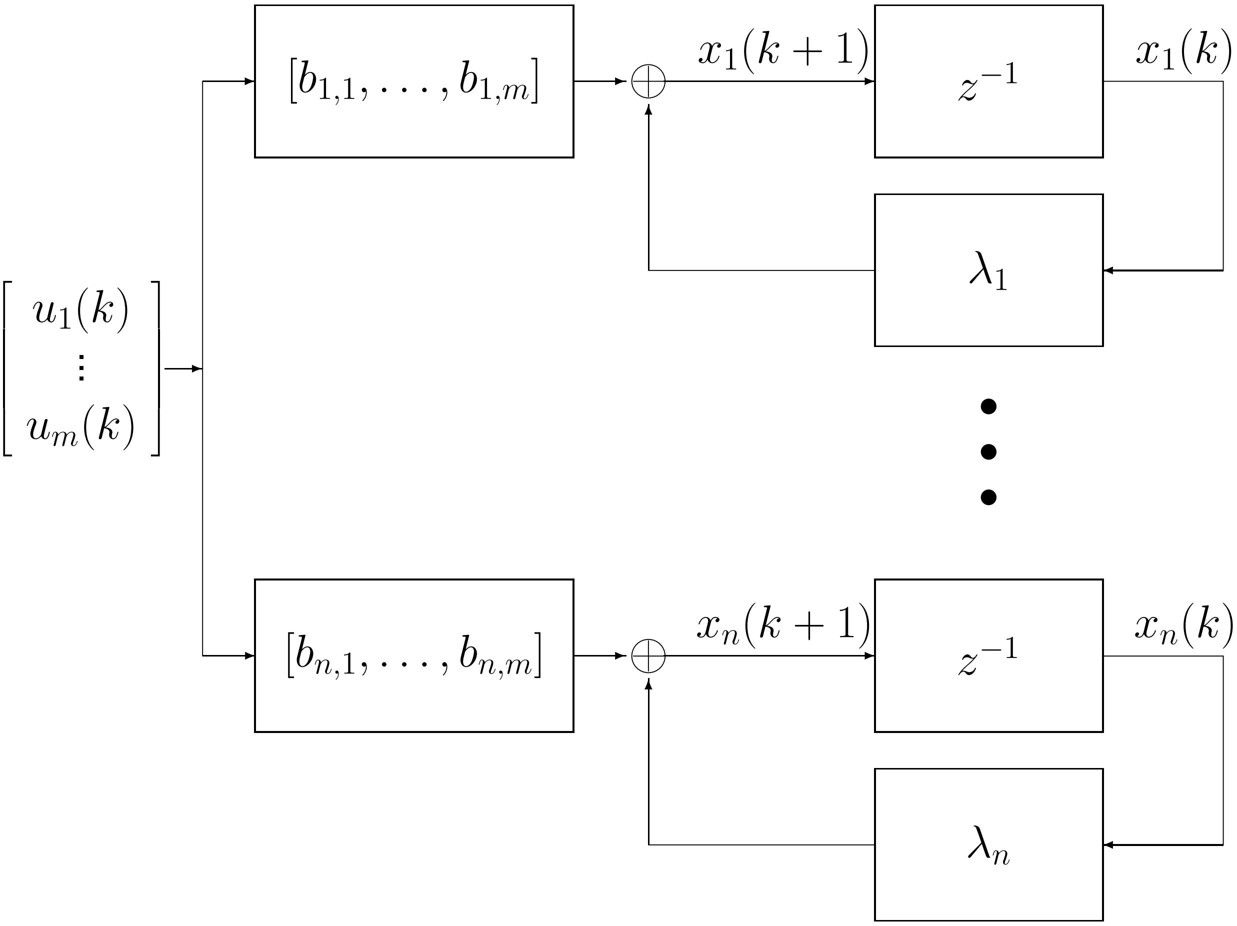
L'equazione alle differenze di tipo matriciale

$$\overline{\mathbf{x}}(k+1) = \overline{\mathbf{A}}\overline{\mathbf{x}}(k) + \overline{\mathbf{B}}u(k)$$

cioè

$$\begin{bmatrix} x_1(k+1) \\ \vdots \\ x_n(k+1) \end{bmatrix} = \begin{bmatrix} \lambda_1 & \dots & 0 \\ \vdots & & \vdots \\ 0 & \dots & \lambda_n \end{bmatrix} \begin{bmatrix} x_1(k) \\ \vdots \\ x_n(k) \end{bmatrix} + \begin{bmatrix} b_{1,1} & \dots & b_{1,m} \\ \vdots & & \vdots \\ b_{n,1} & \dots & b_{n,m} \end{bmatrix} \begin{bmatrix} u_1(k) \\ \vdots \\ u_m(k) \end{bmatrix}$$

descrive n sistemi lineari del primo ordine non interagenti tra di loro:



Rappresentazione grafica - caso continuo

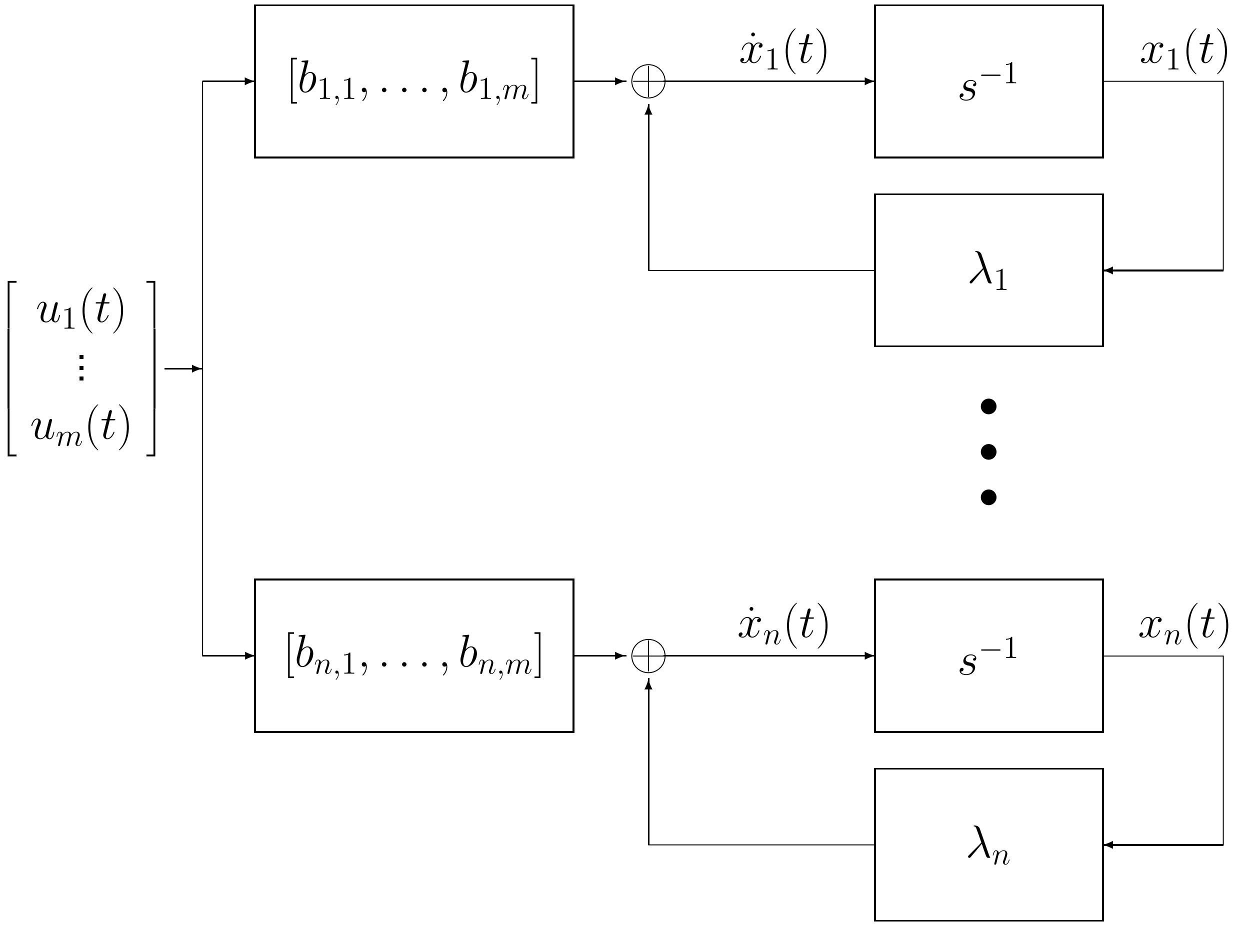
L'equazione differenziale di tipo matriciale:

$$\dot{\mathbf{x}}(t) = \overline{\mathbf{A}}\,\mathbf{x}(t) + \overline{\mathbf{B}}u(t)$$

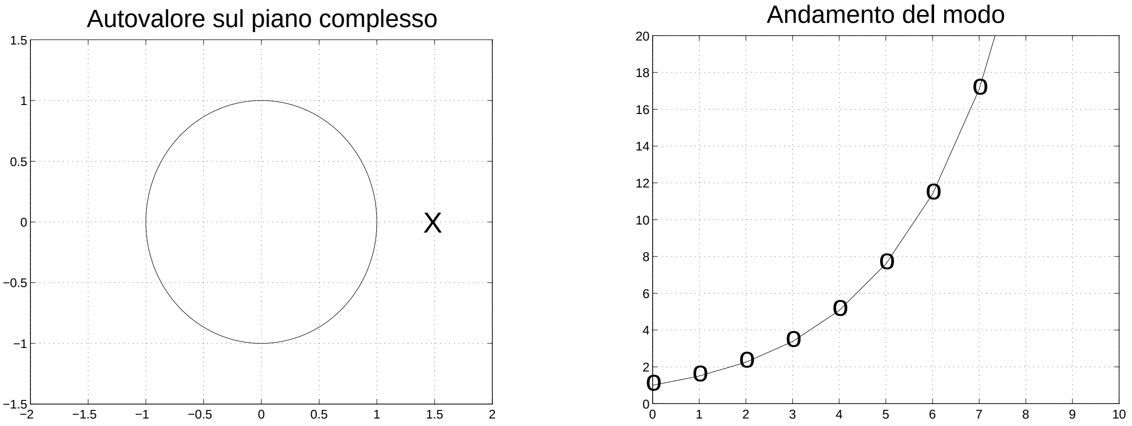
cioè

$$\begin{bmatrix} \dot{x}_1(t) \\ \vdots \\ \dot{x}_n(t) \end{bmatrix} = \begin{bmatrix} \lambda_1 & \dots & 0 \\ \vdots & & \vdots \\ 0 & \dots & \lambda_n \end{bmatrix} \begin{bmatrix} x_1(t) \\ \vdots \\ x_n(t) \end{bmatrix} + \begin{bmatrix} b_{1,1} & \dots & b_{1,m} \\ \vdots & & \vdots \\ b_{n,1} & \dots & b_{n,m} \end{bmatrix} \begin{bmatrix} u_1(t) \\ \vdots \\ u_m(t) \end{bmatrix}$$

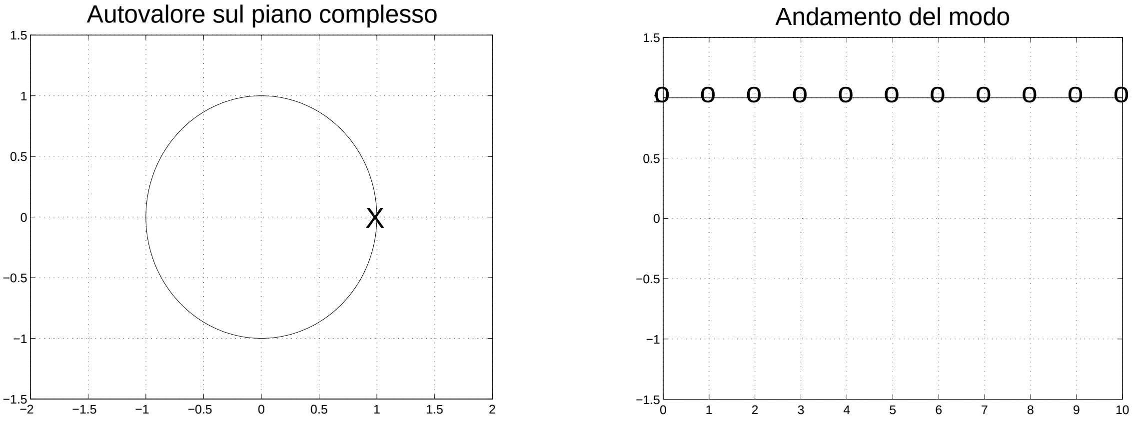
descrive n sistemi lineari del primo ordine non interagenti tra di loro:



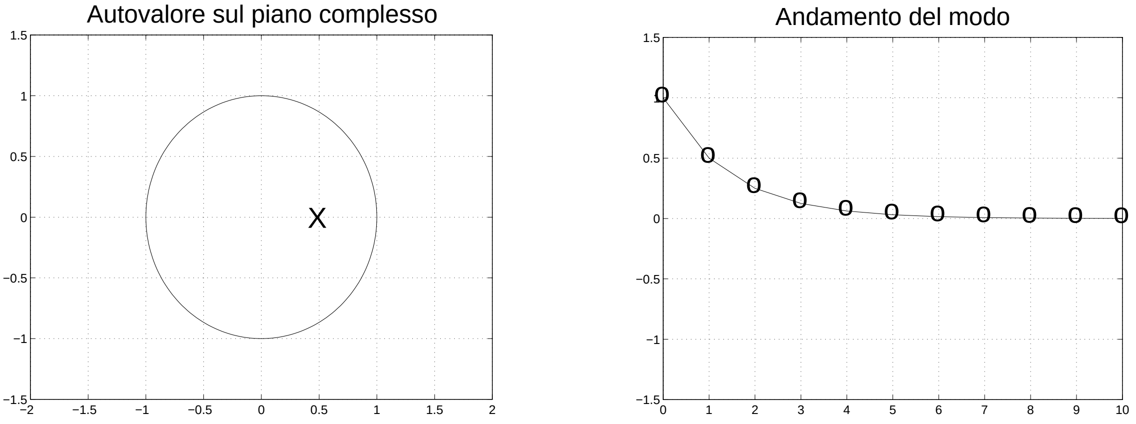
Autovalori reali - caso discreto I



$$\lambda^k \quad \text{dove} \quad \lambda = 1.5$$

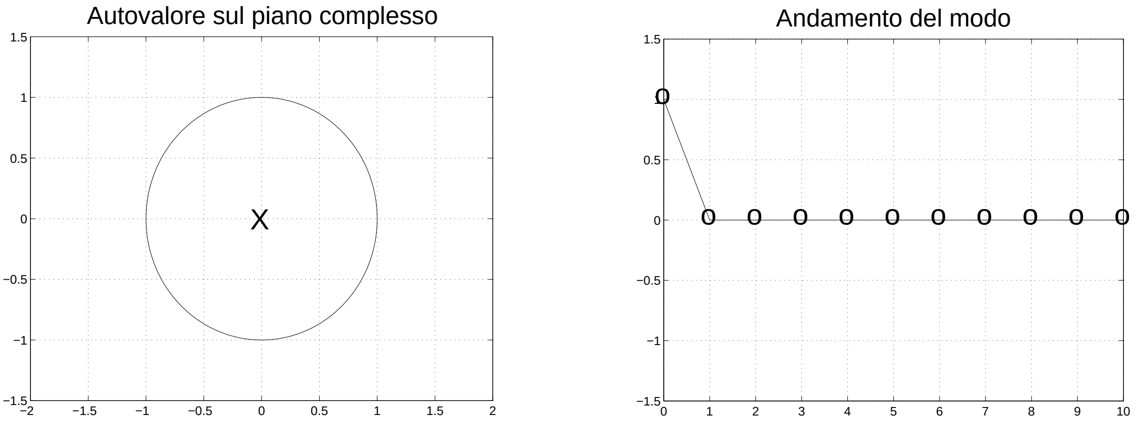


$$\lambda^k \quad \text{dove} \quad \lambda = 1$$

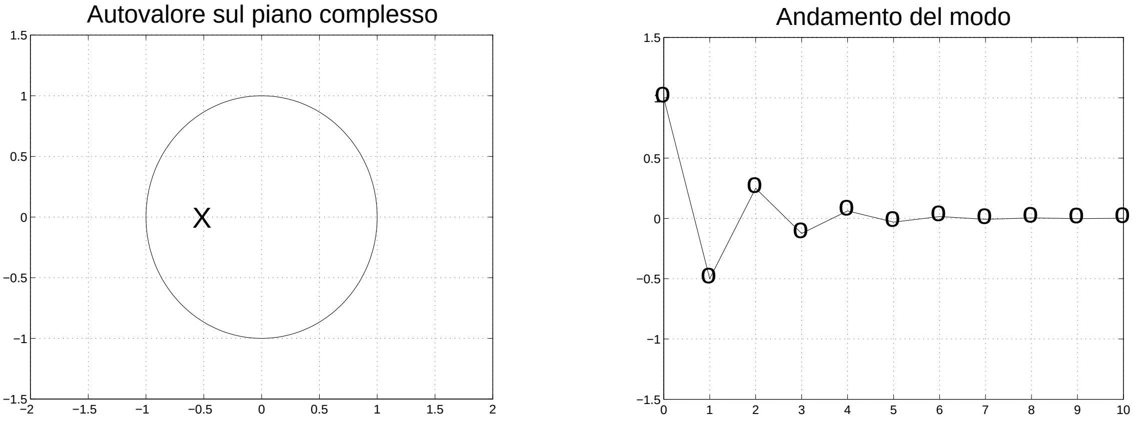


$$\lambda^k \quad \text{dove} \quad \lambda = 0.5$$

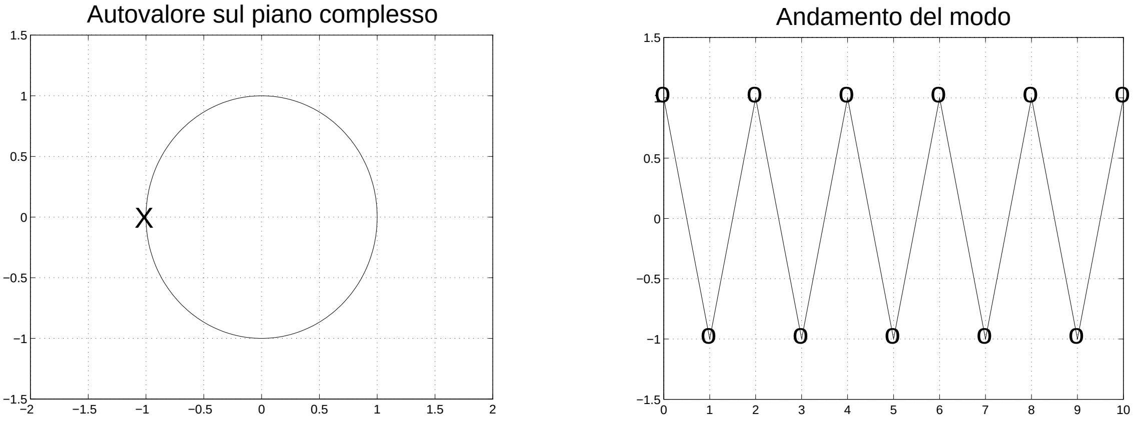
Autovalori reali - caso discreto II



λ^k dove $\lambda = 0$

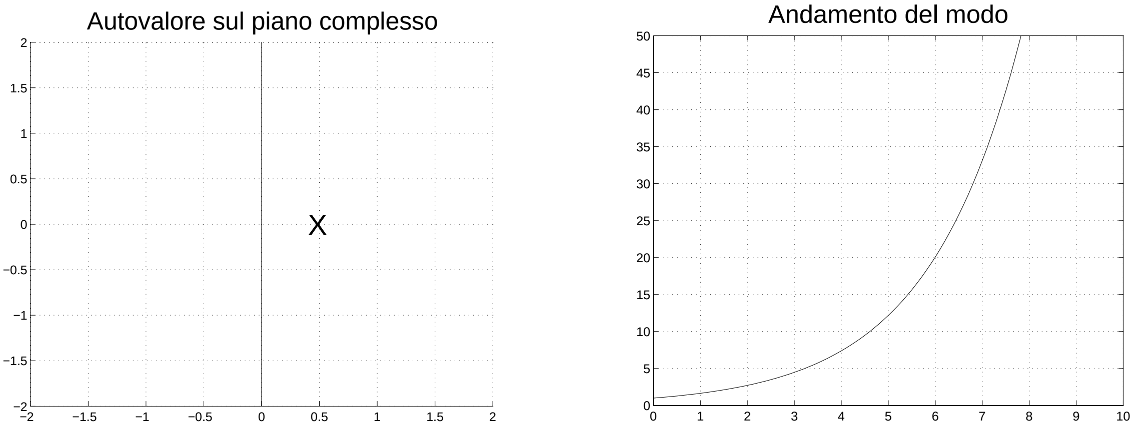


λ^k dove $\lambda = -0.5$

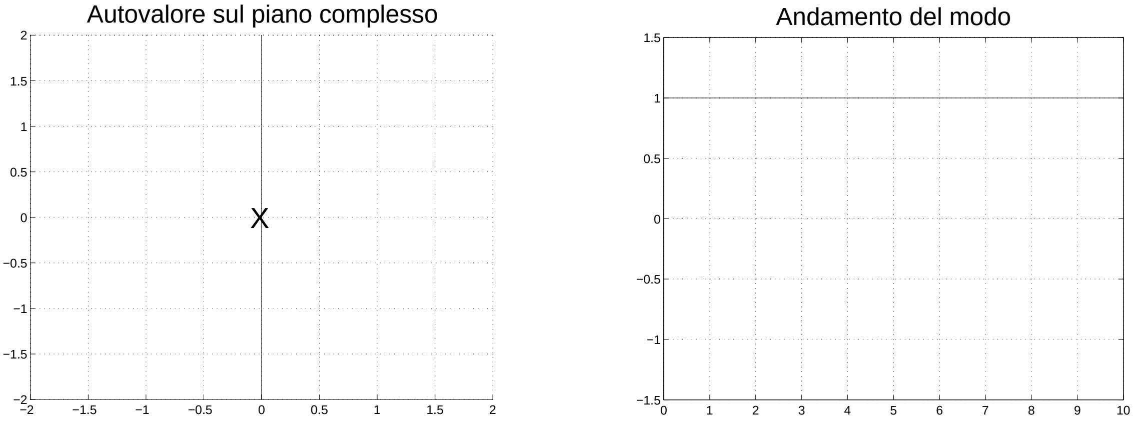


λ^k dove $\lambda = -1$

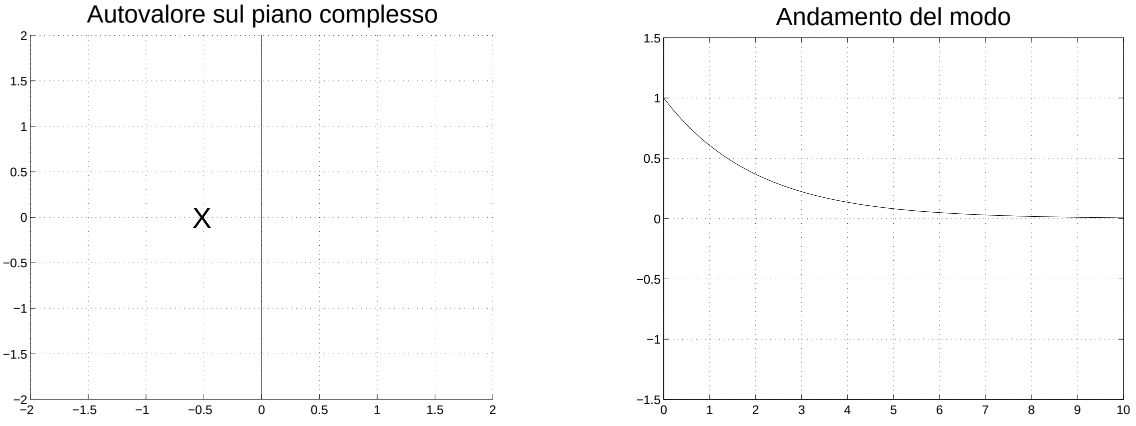
Autovalori reali - caso continuo



$$e^{\lambda t} \qquad \text{dove} \qquad \lambda = 0.5$$

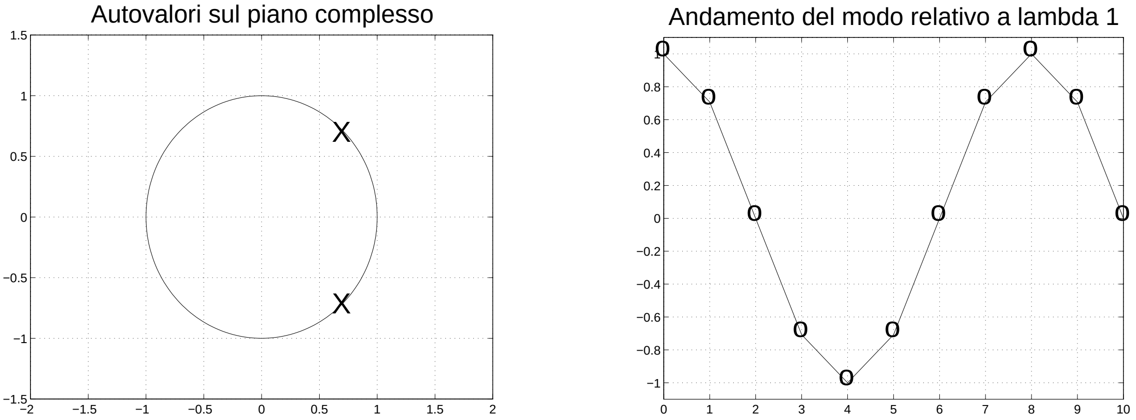


$$e^{\lambda t} \qquad \text{dove} \qquad \lambda = 0$$

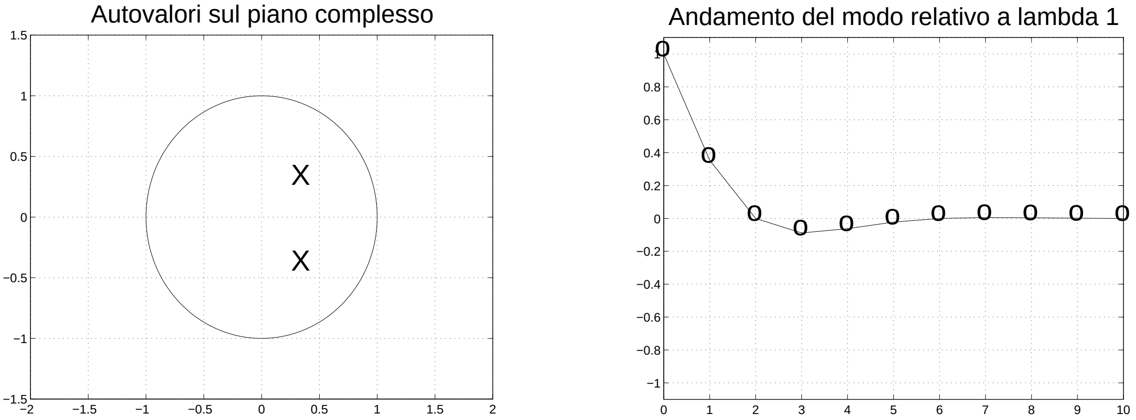


$$e^{\lambda t} \qquad \text{dove} \qquad \lambda = -0.5$$

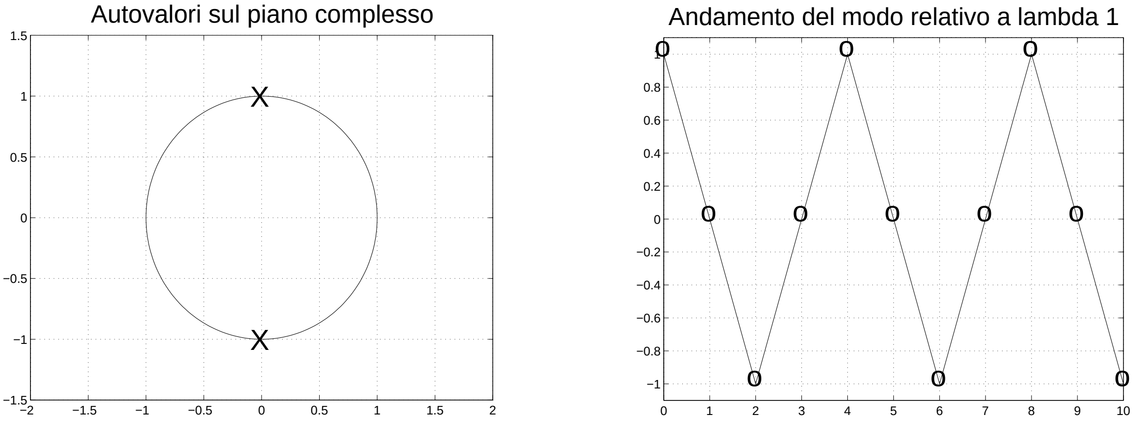
Autovalori complessi - caso discreto



$$\lambda_{1,2} = e^{\pm j\frac{\pi}{4}} \qquad \rightarrow \qquad \cos\frac{k\pi}{4}$$

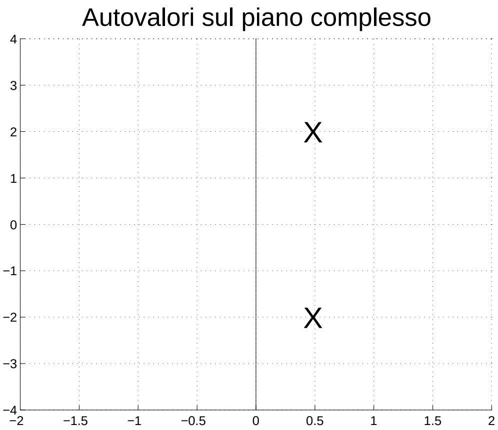


$$\lambda_{1,2} = 0.5e^{\pm j\frac{\pi}{4}} \qquad \rightarrow \qquad (0.5)^k\cos\frac{k\pi}{4}$$

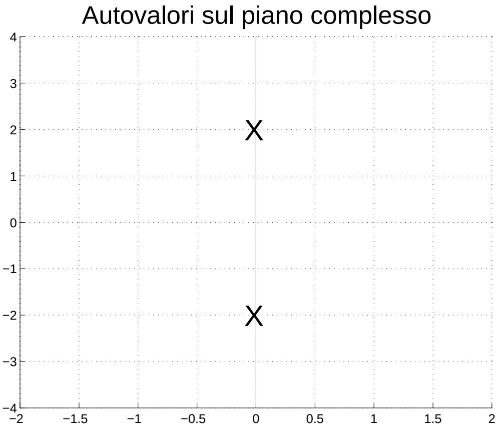


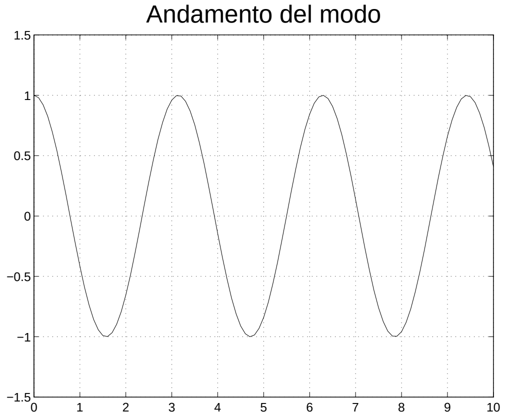
$$\lambda_{1,2} = e^{\pm j\frac{\pi}{2}} = \pm j \qquad \rightarrow \qquad \cos\frac{k\pi}{2}$$

Autovalori complessi - caso continuo

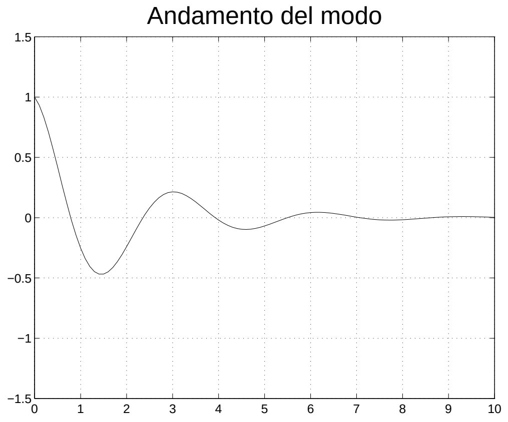


$$e^{\sigma t}\cos(\omega t) \qquad \textsf{dove} \qquad \lambda_{1,2} = 0.5 \pm j2$$





$$e^{\sigma t}\cos(\omega t) \qquad \textsf{dove} \qquad \lambda_{1,2} = \pm j2$$



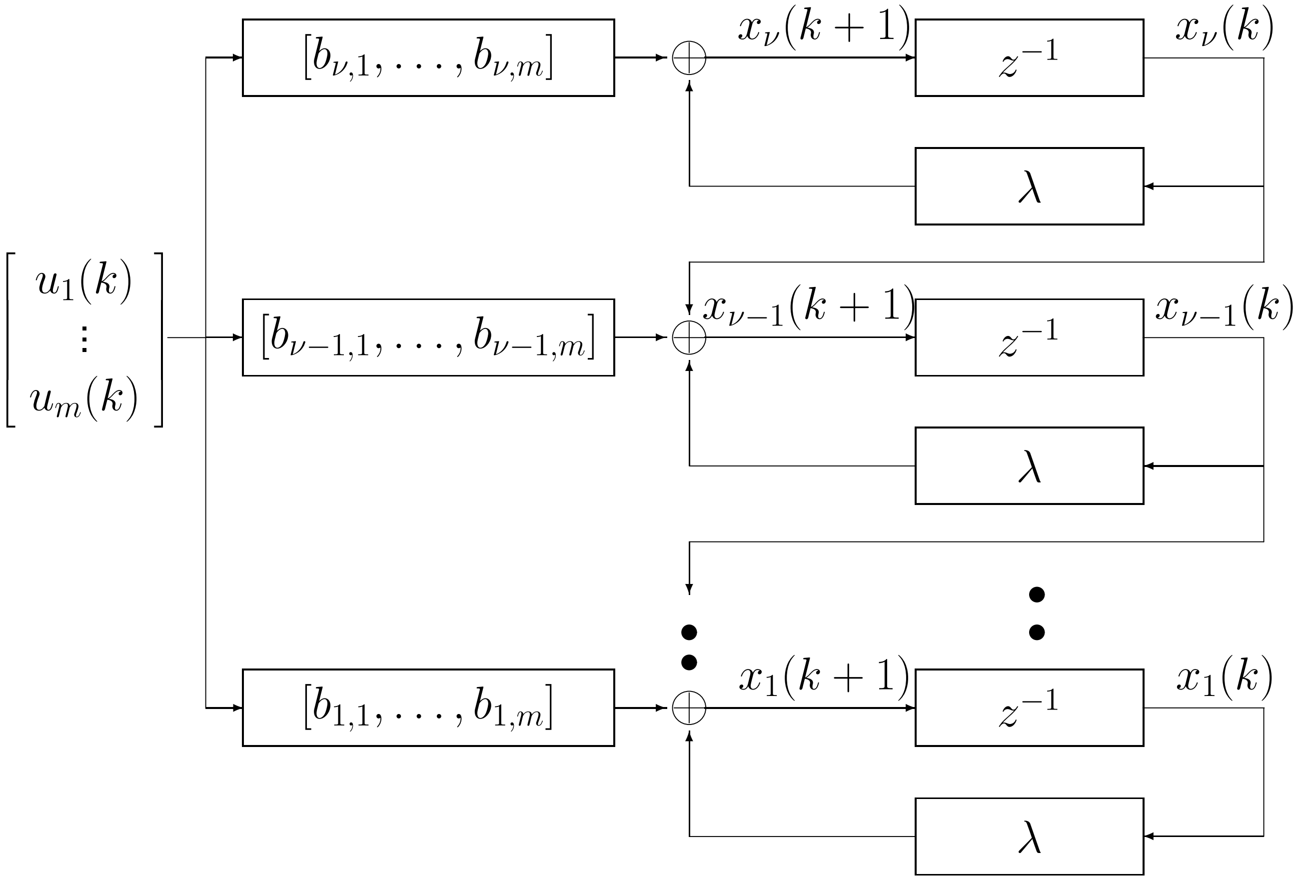
$$e^{\sigma t}\cos(\omega t) \qquad \textsf{dove} \qquad \lambda_{1,2} = -0.5 \pm j2$$

Rappresentazione grafica - caso discreto

L'interazione tra i modi corrispondenti ad un miniblocco di Jordan di dimensione ν, può essere evidenziata mediante uno schema a blocchi corrispondente alla relazione matriciale:

$$\begin{bmatrix} x_1(k+1) \\ \vdots \\ x_{\nu-1}(k+1) \\ x_\nu(k+1) \end{bmatrix} = \begin{bmatrix} \lambda & 1 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & 1 \\ 0 & 0 & \dots & \lambda \end{bmatrix} \begin{bmatrix} x_1(k) \\ \vdots \\ x_{\nu-1}(k) \\ x_\nu(k) \end{bmatrix} + \begin{bmatrix} b_{1,1} & \dots & b_{1,m} \\ \vdots & & \vdots \\ b_{\nu-1,1} & \dots & b_{\nu-1,m} \\ b_{\nu,1} & \dots & b_{\nu,m} \end{bmatrix} \begin{bmatrix} u_1(k) \\ \vdots \\ u_m(k) \end{bmatrix}$$

descrive ν sistemi lineari del primo ordine interagenti tra di loro:

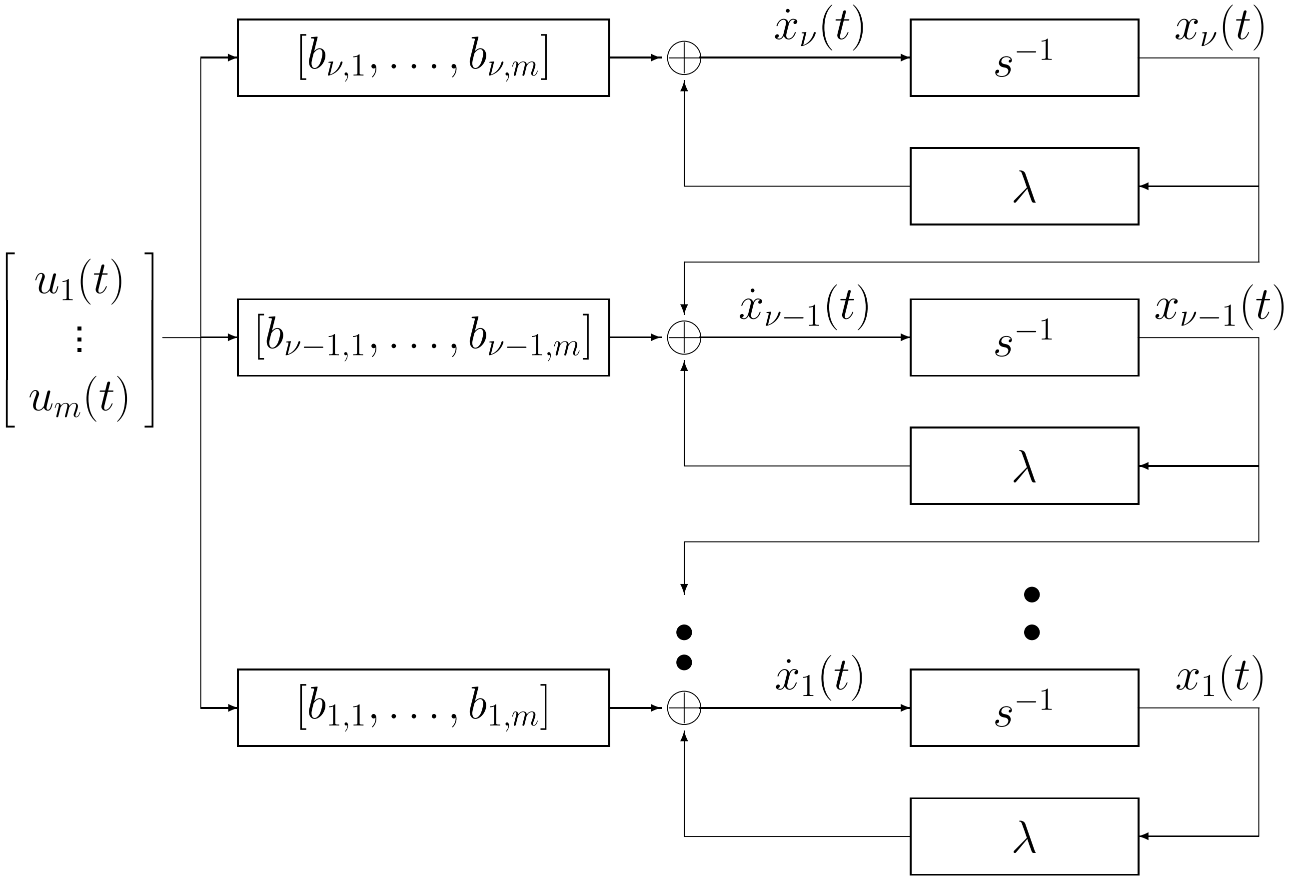


Rappresentazione grafica - caso continuo

L'interazione tra i modi corrispondenti ad un miniblocco di Jordan di dimensione ν, può essere evidenziata mediante uno schema a blocchi corrispondente alla relazione matriciale:

$$\begin{bmatrix} \dot{x}_1(t) \\ \vdots \\ \dot{x}_{\nu-1}(t) \\ \dot{x}_\nu(t) \end{bmatrix} = \begin{bmatrix} \lambda & 1 & \dots & 0 \\ \vdots & & \vdots & \vdots \\ 0 & 0 & \dots & 1 \\ 0 & 0 & \dots & \lambda \end{bmatrix} \begin{bmatrix} x_1(t) \\ \vdots \\ x_{\nu-1}(t) \\ x_\nu(t) \end{bmatrix} + \begin{bmatrix} b_{1,1} & \dots & b_{1,m} \\ \vdots & & \vdots \\ b_{\nu-1,1} & \dots & b_{\nu-1,m} \\ b_{\nu,1} & \dots & b_{\nu,m} \end{bmatrix} \begin{bmatrix} u_1(t) \\ \vdots \\ u_m(t) \end{bmatrix}$$

descrive ν sistemi lineari del primo ordine interagenti tra di loro:



Autovalori reali multipli - Esempio

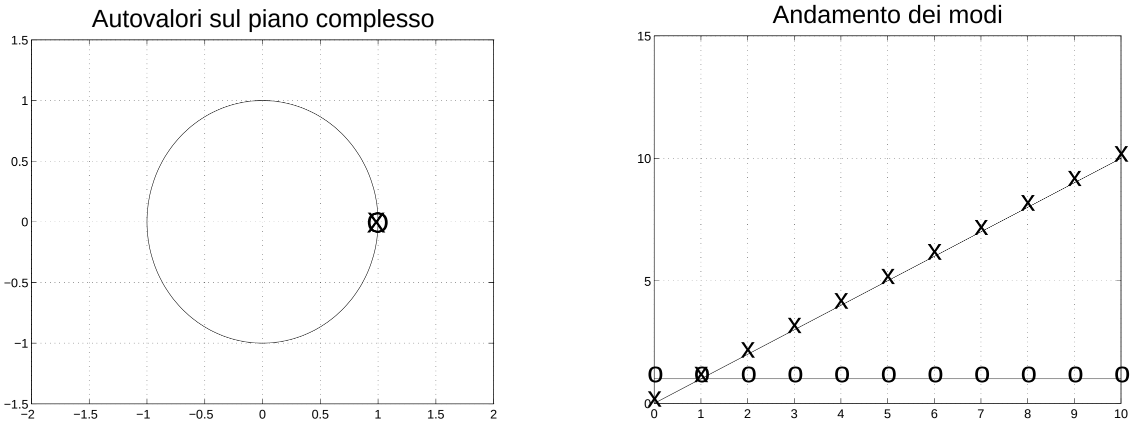
Sia il sistema in forma di Jordan con autovalore λ di molteplicità due:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

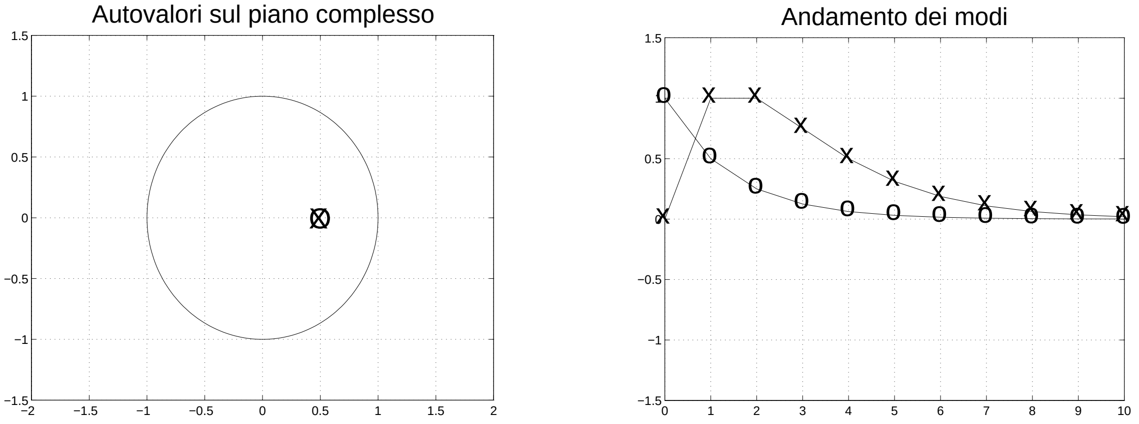
La traiettoria dello stato è quindi definita dalle relazioni:

$$\begin{cases} x_1(k) = \lambda^k x_1(0) + k\lambda^{k-1} x_2(0), & x_1(0) = 1 \\ x_2(k) = \lambda^k x_2(0) & x_2(0) = 1 \end{cases}$$

Analizziamo i due modi $m_1 = \lambda^k$ e $m_2 = k\lambda^{k-1}$:



Autovalore doppio $\lambda = 1$, modi m_1 e m_2



Autovalore doppio $\lambda = 0.5$, modi m_1 e m_2

Autovalori reali multipli - Esempio

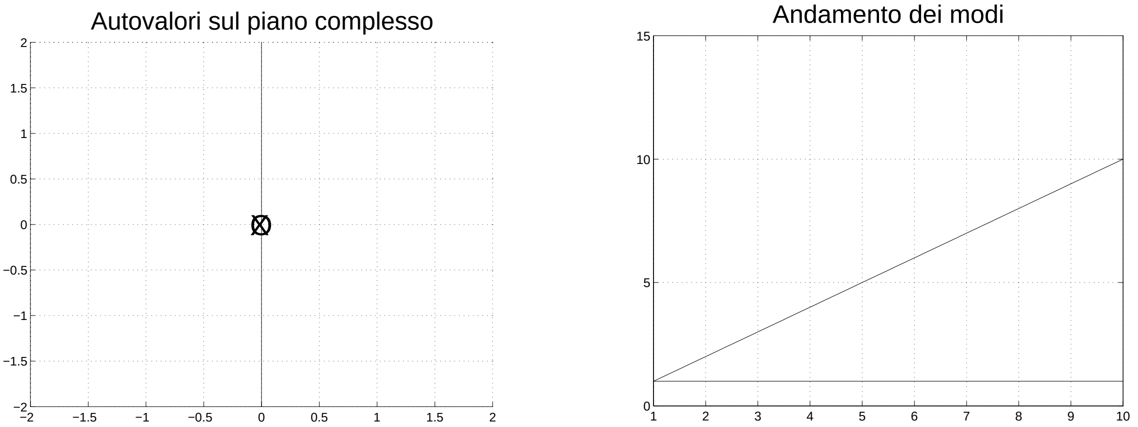
Sia il sistema in forma di Jordan con autovalore λ di molteplicità due:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

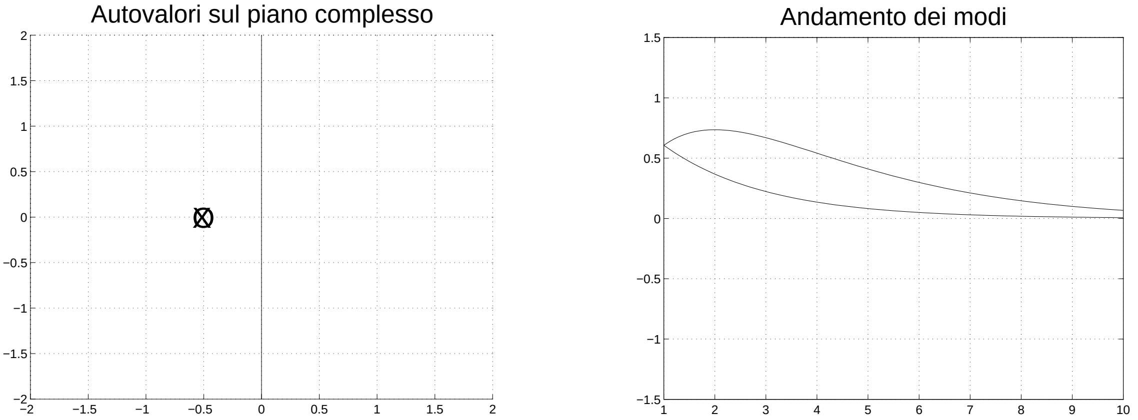
La traiettoria dello stato è quindi definita dalle relazioni:

$$\begin{cases} x_1(t) = e^{\lambda t} x_1(0) + t e^{\lambda t} x_2(0), & x_1(0) = 1 \\ x_2(t) = e^{\lambda t} x_2(0) & x_2(0) = 1 \end{cases}$$

Analizziamo i due modi $m_1 = e^{\lambda t}$ e $m_2 = t e^{\lambda t}$:



Autovalore doppio $\lambda = 0$, modi m_1 e m_2



Autovalore doppio $\lambda = -0.5$, modi m_1 e m_2