# The POG Modeling Technique Applied to Electrical Systems


UNIVERSITÀ DEGLI STUDI
DI MODENA E REGGIO EMILIA

Roberto ZANASI

Computer Science Engineering Department (DII)
University of Modena and Reggio Emilia
Italy

E-mail: roberto.zanasi@unimo.it

# Outline

- Main characteristics of the Power-Oriented Graphs (POG) modelling technique

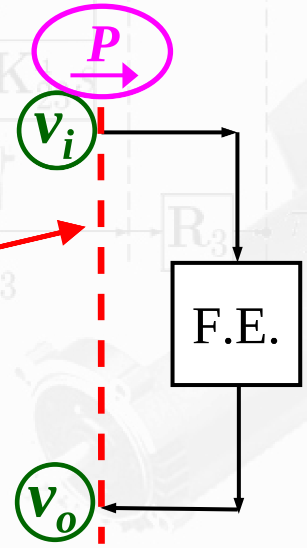
- POG modelling examples:
  1. DC motor connected to an hydraulic pump
  2. Three-phase brushless motor
  3. Three-phase asynchronous motor

# POG Dynamic Modeling: Physical sections

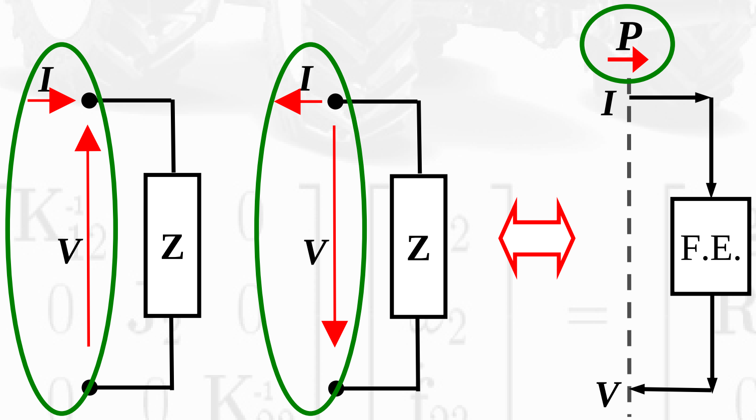
The physical elements (F.E.) interact with the external world through sections. Each section is characterized by two power variables $v_i$ e $v_o$.

In POG a section is denoted by using a dashed line.

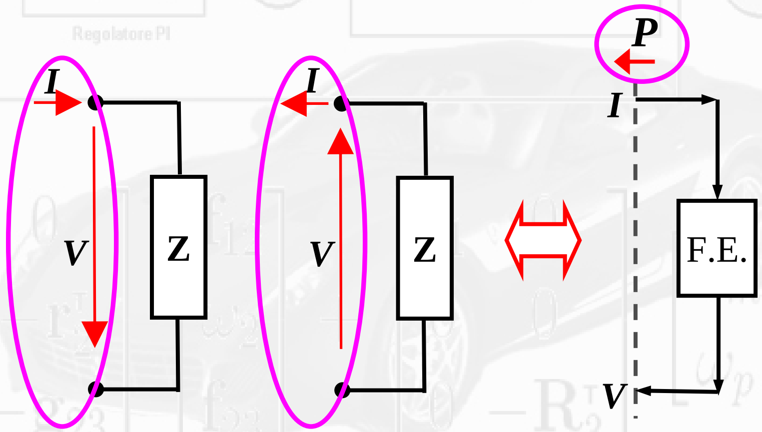
Each power variable has its own positive direction. The power flowing through a section can be positive or negative. An arrow over the dashed line is used for denoting the positive direction of the power $P$.



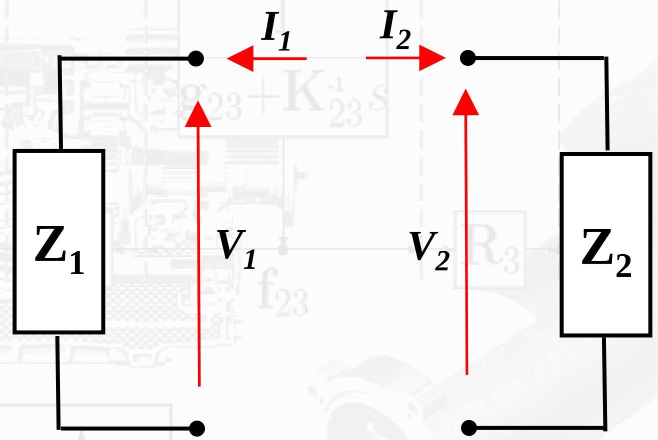

The power enters into the element:




The power exits from the element:



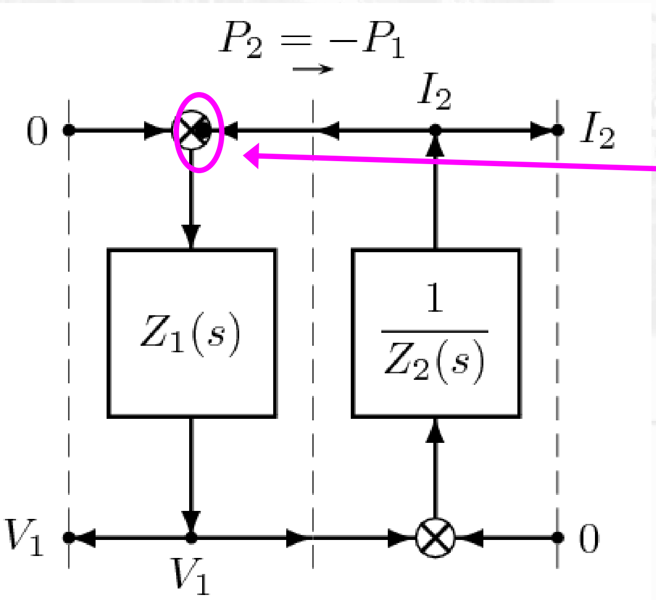
# POG Dynamic Modeling: Connections



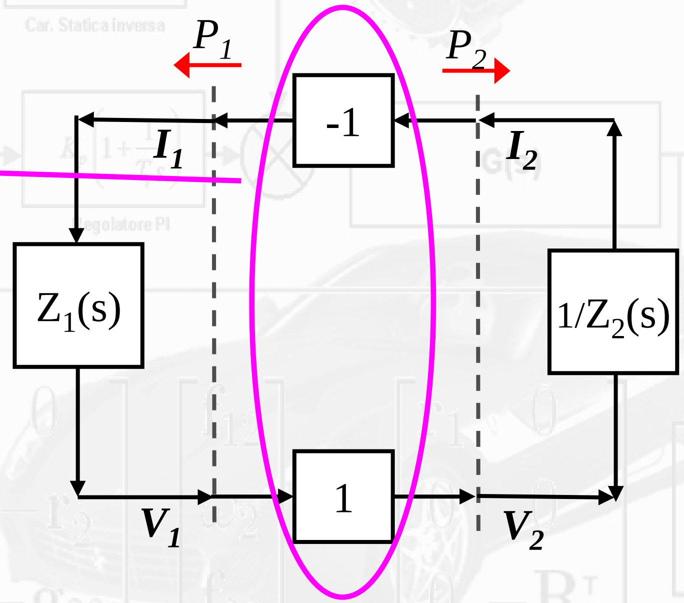

Example: connection of two electrical elements $Z_1$, $Z_2$.

If the powers $P_1$, $P_2$ enter into the two electrical elements, the variables $I_1$, $V_1$, $I_2$, $V_2$ cannot have all the same positive direction.

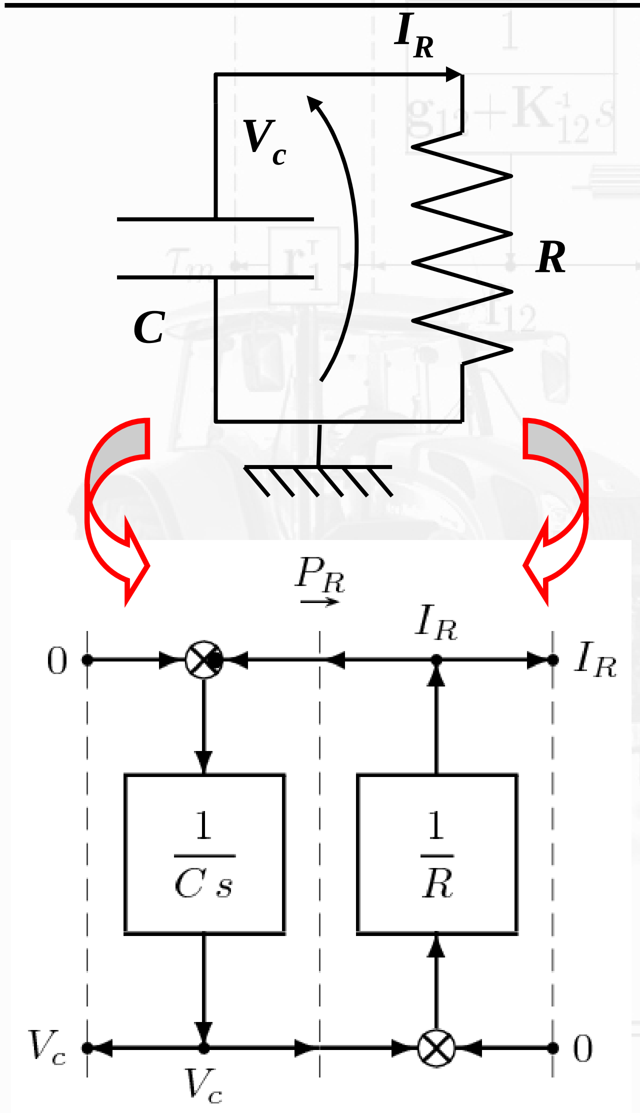
In this case a "connection block" is used for converting the power variables.

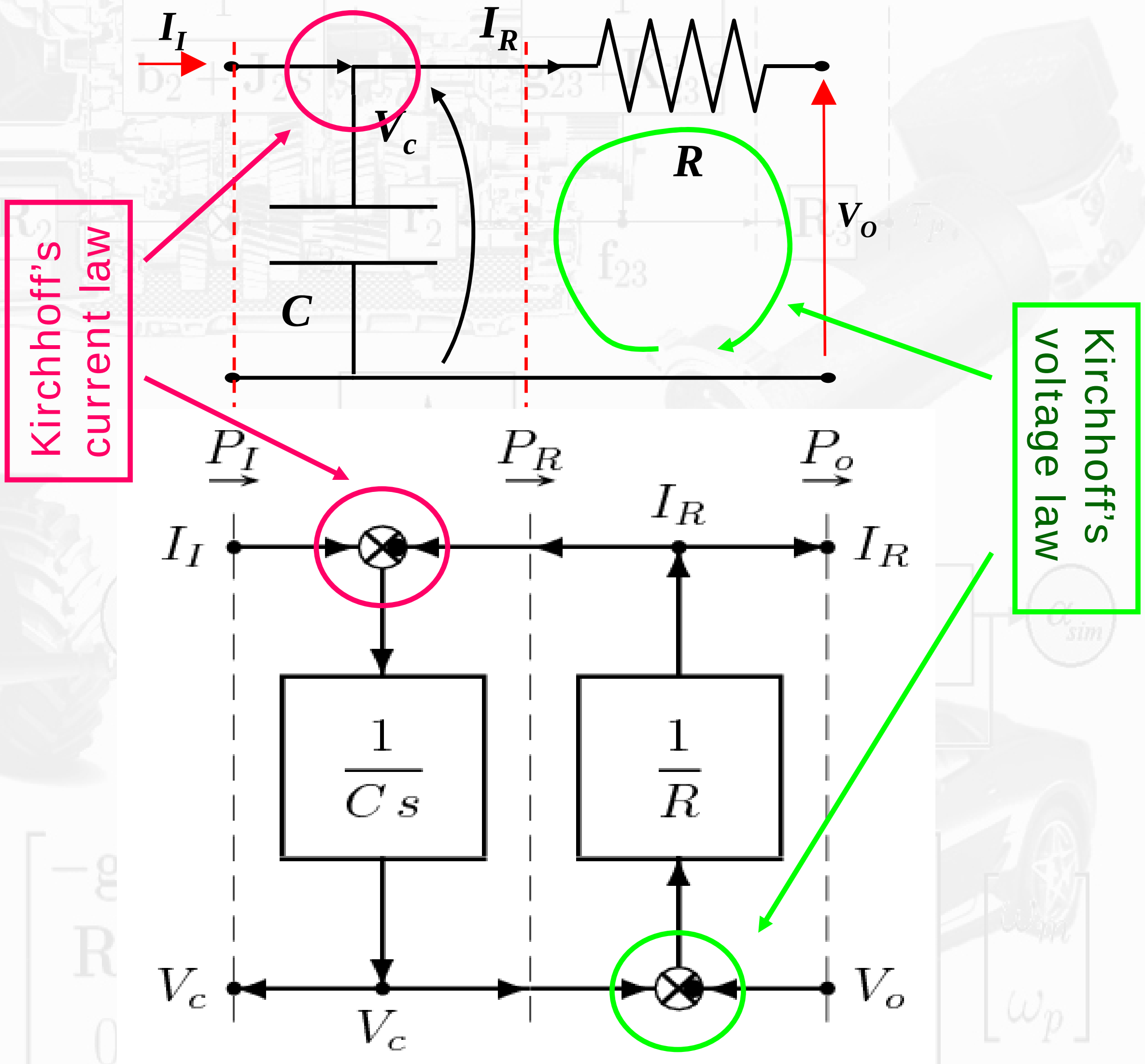



Equivalent description (scalar case)



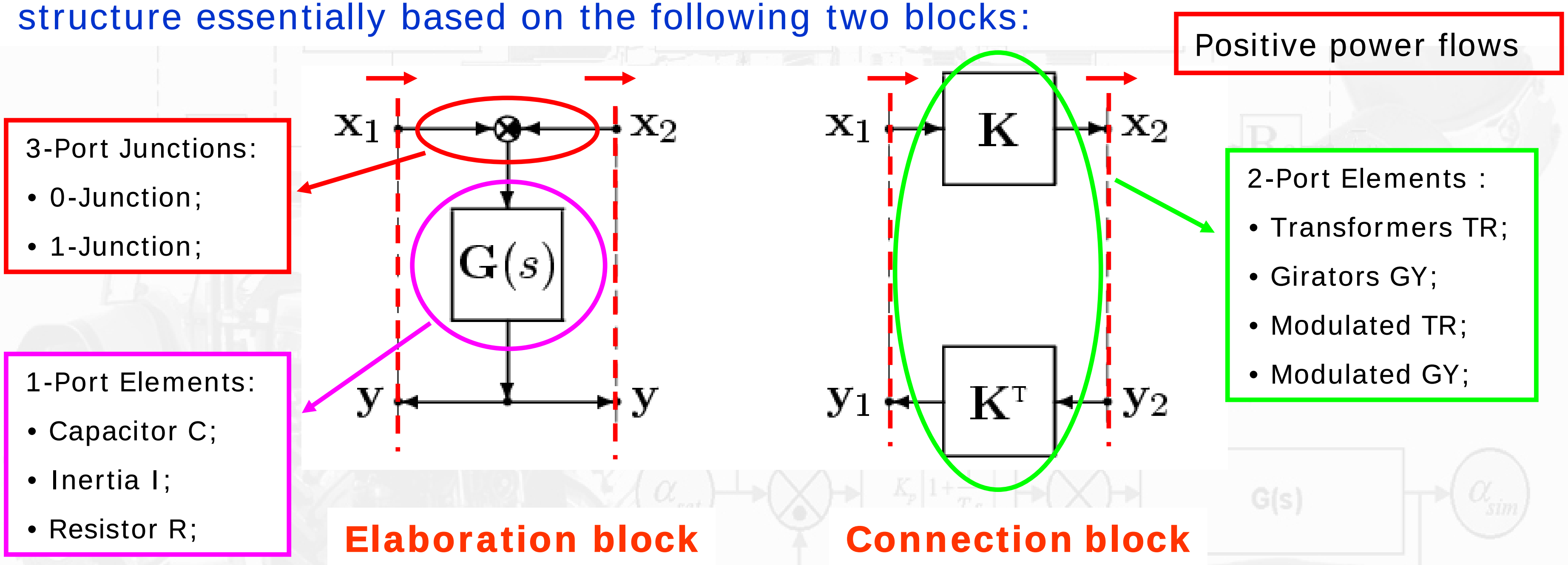
# Dynamic Modeling: Electrical examples



# Introduction
# Power-Oriented Graphs (POG)

The Power-Oriented Graphs are "block diagrams" obtained by using a "modular" structure essentially based on the following two blocks:




- POG maintains a direct correspondence between pairs of system variables and real power flows: the product of the two variables involved in each dashed line of the graph has the physical meaning of ``power flowing through that section''.
- The Elaboration block can store and dissipate/generate energy.
- The Connection block can only "transform" the energy.

# Dynamic Modeling of Physical Systems

Different Energy domains:
1) Electrical; 2) Mechanical (tras./rot.); 3) Hydraulic; etc.

The same dynamic structure:
- 2 "dynamic" elements $D_1$, $D_2$ that store energy;
- 1 "static" element $R$ that dissipates (or generates) energy;
- 2 "energy variables" $q_1(t)$, $q_2(t)$ used for describing the stored energy;
- 2 "power variables" $v_1(t)$, $v_2(t)$ used for moving the energy;

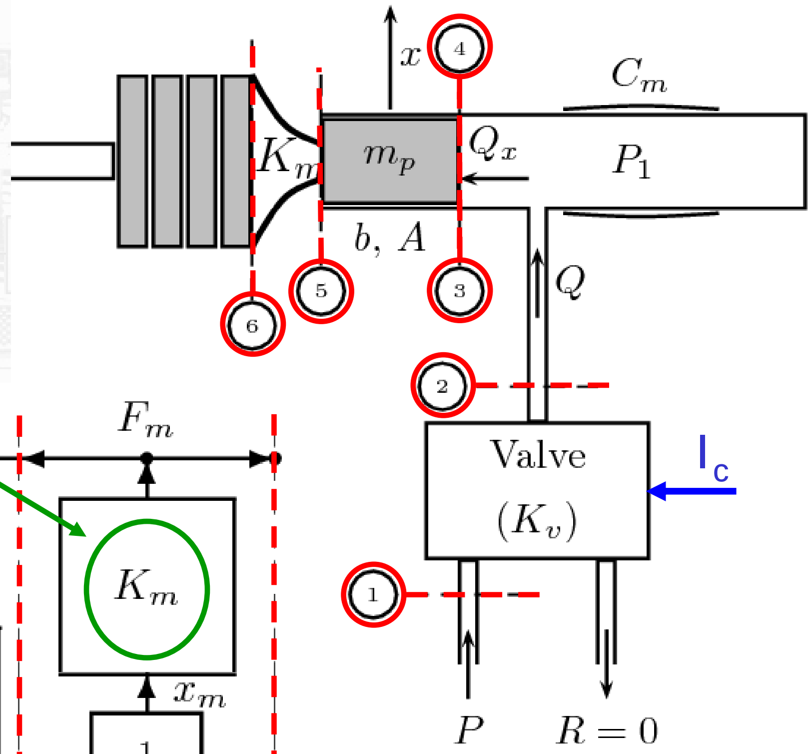
| | | Electrical | | Mechanical | | Hydraulic | |
|---|---|---|---|---|---|---|---|
| | $\mathcal{D}_1$ | $C$ | Capacitor | $M$ | Mass | $C_I$ | Hyd. Capacitor |
| | $q_1$ | $Q$ | Charge | $p$ | Momentum | $V$ | Volume |
| Across-variables | $v_1$ | $V$ | Voltage | $v$ | Velocity | $P$ | Pressure |
| | $\mathcal{D}_2$ | $L$ | Inductor | $E$ | Spring | $L_I$ | Hyd. Inductor |
| | $q_2$ | $\phi$ | Flux | $x$ | Displacement | $\phi_I$ | Hyd. Flux |
| Through-variables | $v_2$ | $I$ | Current | $F$ | Force | $Q$ | Flow |
| | $\mathcal{R}$ | $R$ | Resistor | $b$ | Friction | $R_I$ | Hyd. Resistor |

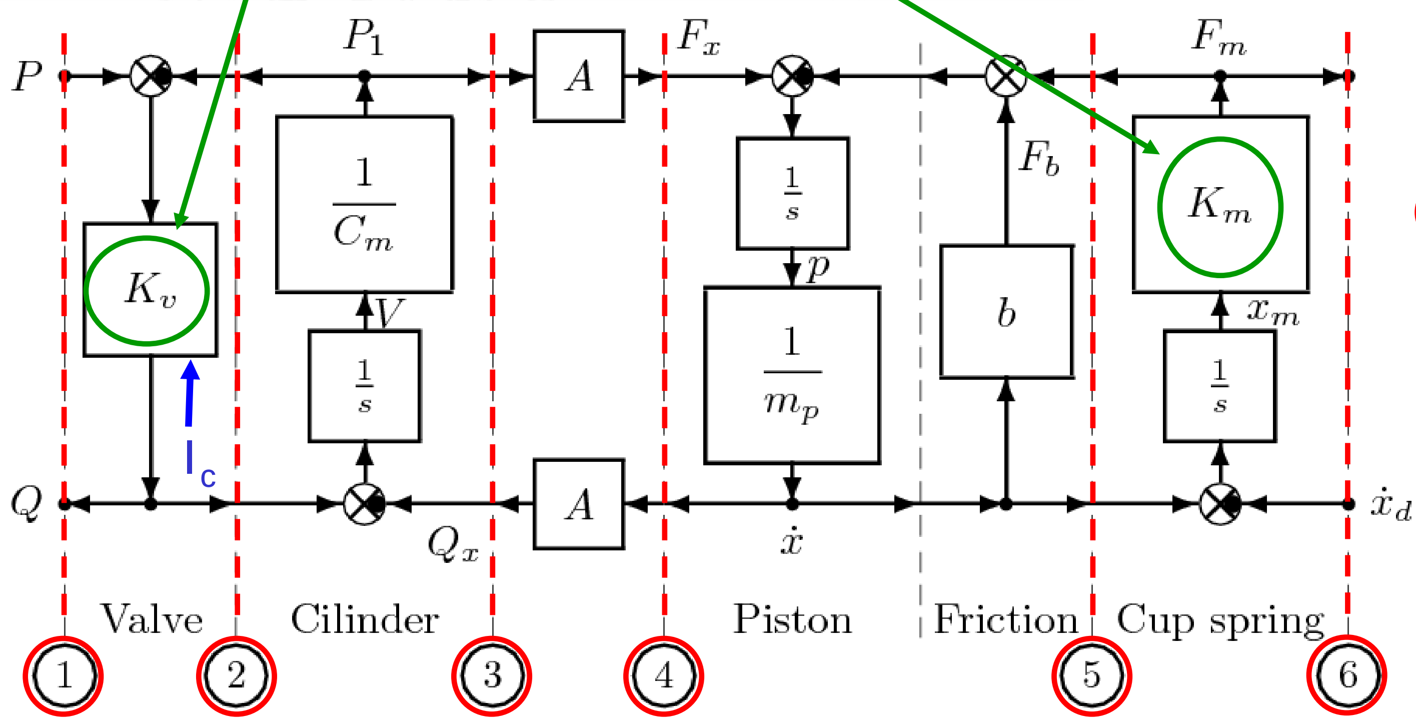
In POG the new symbols and the new definitions are "minimal"

# Example of POG modeling: an electro-hydraulic clutch

The electro-hydraulic clutch:

Strongly nonlinear elements

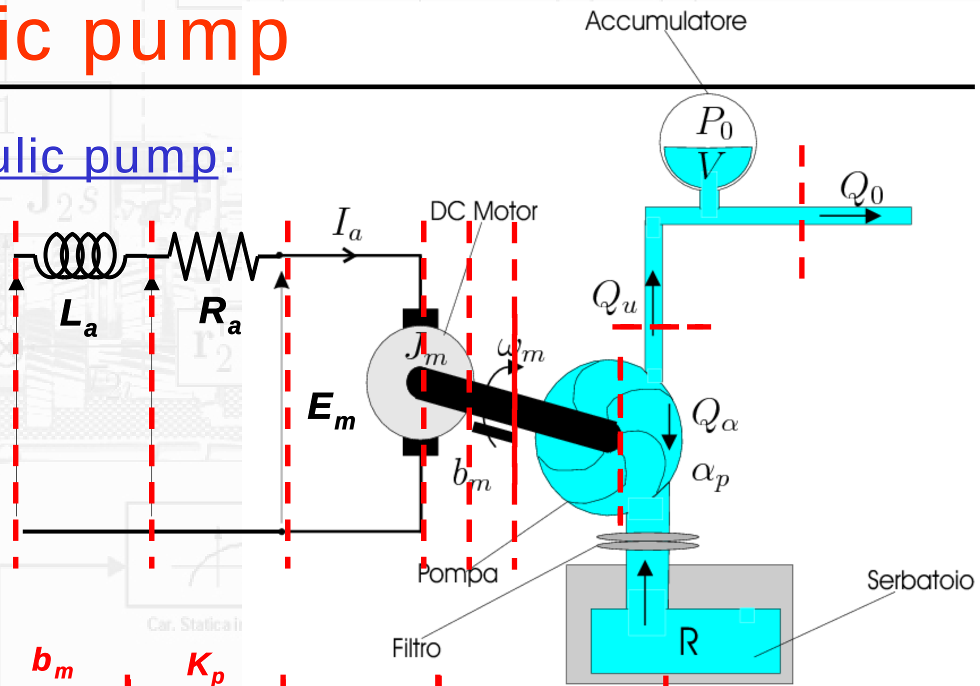



Control input

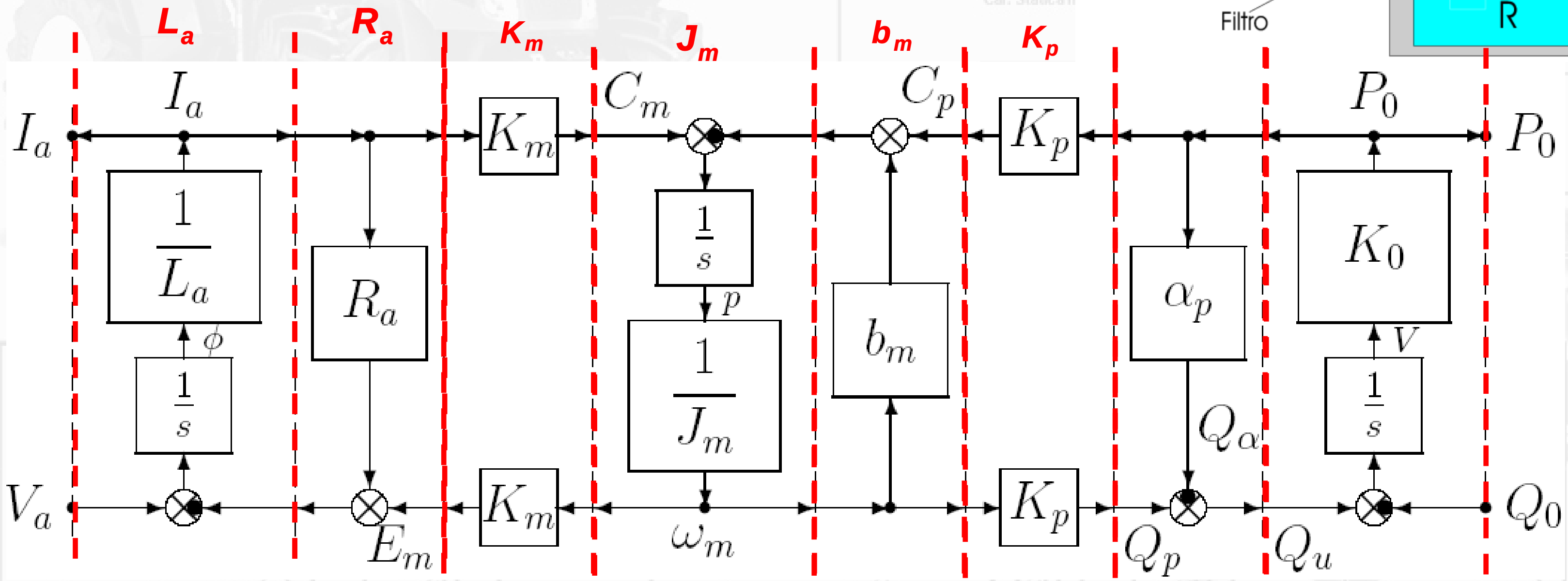
# Example of POG modeling: DC electric motor with an hydraulic pump

A DC motor connected to an hydraulic pump:




There is a direct correspondence between the POG blocks and the physical elements …

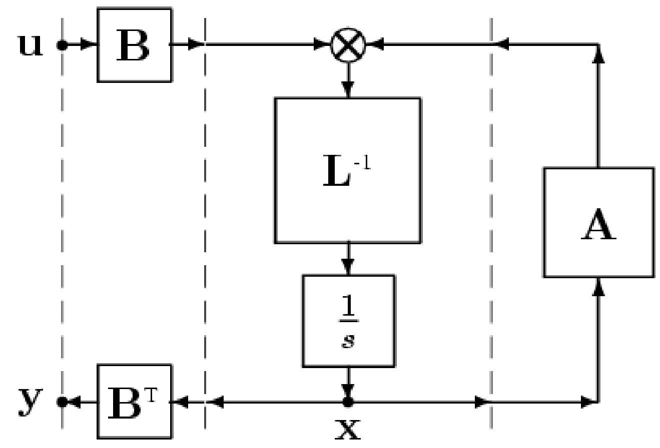
The POG model:



# Introduction
# Power-Oriented Graphs - LTI Systems

- Direct correspondence between POG and state space descriptions:

$$\begin{cases} \mathbf{L}\,\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \\ \mathbf{y} = \mathbf{B}^{\mathrm{T}}\mathbf{x} \end{cases}$$




Stored Energy: $E_s = \frac{1}{2}\mathbf{x}^{\mathrm{T}}\mathbf{L}\,\mathbf{x}$

Dissipating Power: $P_d = \mathbf{x}^{\mathrm{T}}\mathbf{A}\,\mathbf{x}$

A “power” state space description of the DC motor with hydraulic pump:

$$\underbrace{\begin{bmatrix} L_a & 0 & 0 \\ 0 & J_m & 0 \\ 0 & 0 & \frac{1}{K_0} \end{bmatrix}}_{\mathbf{L}} \underbrace{\begin{bmatrix} \dot{I}_a \\ \dot{\omega}_m \\ \dot{P}_0 \end{bmatrix}}_{\dot{\mathbf{x}}} = \underbrace{\begin{bmatrix} -R_a & -K_m & 0 \\ K_m & -b_m & -K_p \\ 0 & K_p & -\alpha_p \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} I_a \\ \omega_m \\ P_0 \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix}}_{\mathbf{B}} \underbrace{\begin{bmatrix} V_a \\ Q_0 \end{bmatrix}}_{\mathbf{u}}$$

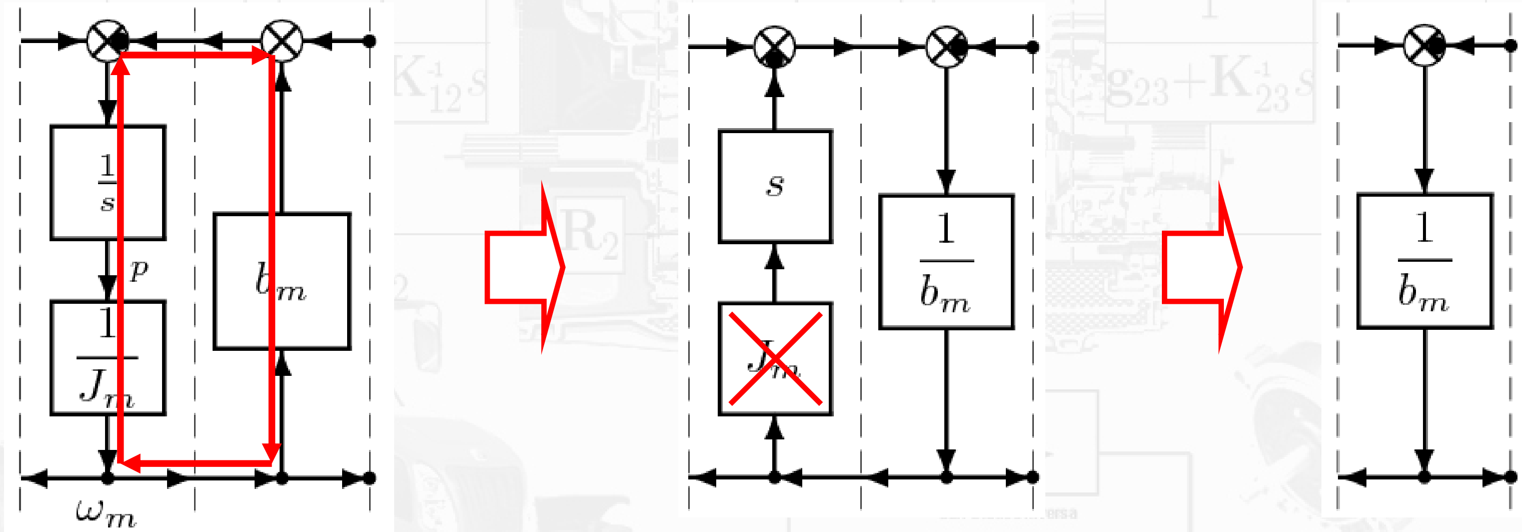
$$\mathbf{y} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{C}} \mathbf{x}$$

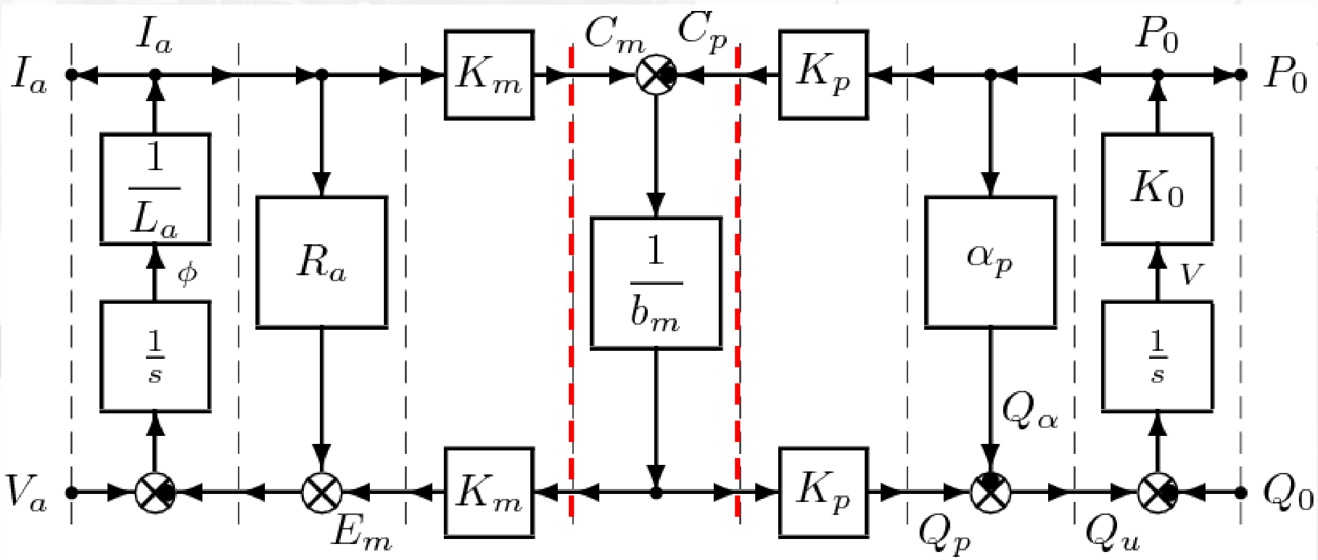
Which is the “reduced model” when $J_m$->0 ?

Two possible solutions:

1) graphically inverting a path …;
2) using a congruent transformation

# POG modeling reduction: graphically inverting a path



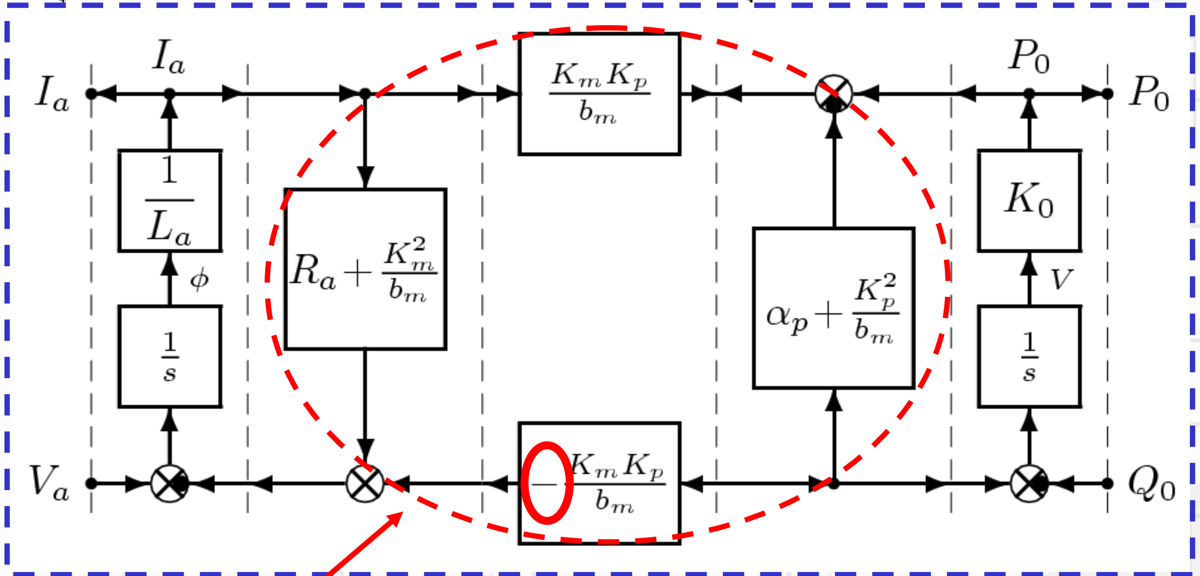




POG reduced model !

# POG modeling reduction: using a “congruent” transformation

When an eigenvalue of matrix ***L*** goes to zero (or to infinity), the system degenerates towards a lower dynamic dimension system. The “reduced system ” can be obtained by using a “congruent” transformation $x = \mathbf{T}z$ where ***T*** is a rectangular matrix:

$$\begin{cases} \mathbf{T}^T\mathbf{L}\mathbf{T}\dot{\mathbf{z}} = \mathbf{T}^T\mathbf{A}\mathbf{T}\mathbf{z} + \mathbf{T}^T\mathbf{B}\mathbf{u} \\ \mathbf{y} = \mathbf{B}^T\mathbf{T}\mathbf{z} \end{cases} \Leftrightarrow \begin{cases} \overline{\mathbf{L}}\dot{\mathbf{z}} = \overline{\mathbf{A}}\mathbf{z} + \overline{\mathbf{B}}\mathbf{u} \\ \mathbf{y} = \overline{\mathbf{B}}^T\mathbf{z} \end{cases}$$




$$J_m = 0$$

$$\Downarrow$$

$$K_m I_a - b_m \omega_m - K_p P_0 = 0$$

$$\Downarrow$$

$$\omega_m = \frac{K_m}{b_m} I_a - \frac{K_p}{b_m} P_0$$

$$\Downarrow$$

$$\underbrace{\begin{bmatrix} I_a \\ \omega_m \\ P_0 \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} 1 & 0 \\ \frac{K_m}{b_m} & -\frac{K_p}{b_m} \\ 0 & 1 \end{bmatrix}}_{\mathbf{T}} \underbrace{\begin{bmatrix} I_a \\ P_0 \end{bmatrix}}_{\mathbf{z}}$$

$$\Downarrow$$

Dissipation 2D element

$$\begin{bmatrix} L_a & 0 \\ 0 & \frac{1}{K_0} \end{bmatrix} \begin{bmatrix} \dot{I}_a \\ \dot{P}_0 \end{bmatrix} = \begin{bmatrix} -R_a - \frac{K_m^2}{b_m} & \frac{K_m K_p}{b_m} \\ \frac{K_m K_p}{b_m} & -\alpha_p - \frac{K_p^2}{b_m} \end{bmatrix} \begin{bmatrix} I_a \\ P_0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} V_a \\ Q_0 \end{bmatrix}$$

# POG modeling of Electrical Motors

Let us consider Electric Motors "energetically" characterized by:

1) the magnetic flux "***LI***" generated by the stator and/or rotor currents $I_s$ and $I_r$;

2) the magnetic flux "$\varphi$ ($\theta_r$)" of the permanent magnets (if present);

3) the momentum "$J_r\,\omega_r$" generated by rotor velocity $\omega_r$;

The Energy *K* stored in the system can be expressed as follows:

$$K = \frac{1}{2}\dot{\mathbf{q}}^{\mathrm{T}}\,\mathbf{L}(\theta_r)\,\dot{\mathbf{q}} + \dot{\mathbf{q}}^{\mathrm{T}}\,\varphi(\theta_r)$$

$$\mathbf{L}(\theta_r) = \mathbf{L}(\theta_r)^{\mathrm{T}} > 0$$

where $\dot{\mathbf{q}} = \left[\; \mathbf{I}_s \quad \mathbf{I}_s \quad \omega_r \;\right]$ and $\omega_r$ is the rotor angular position.

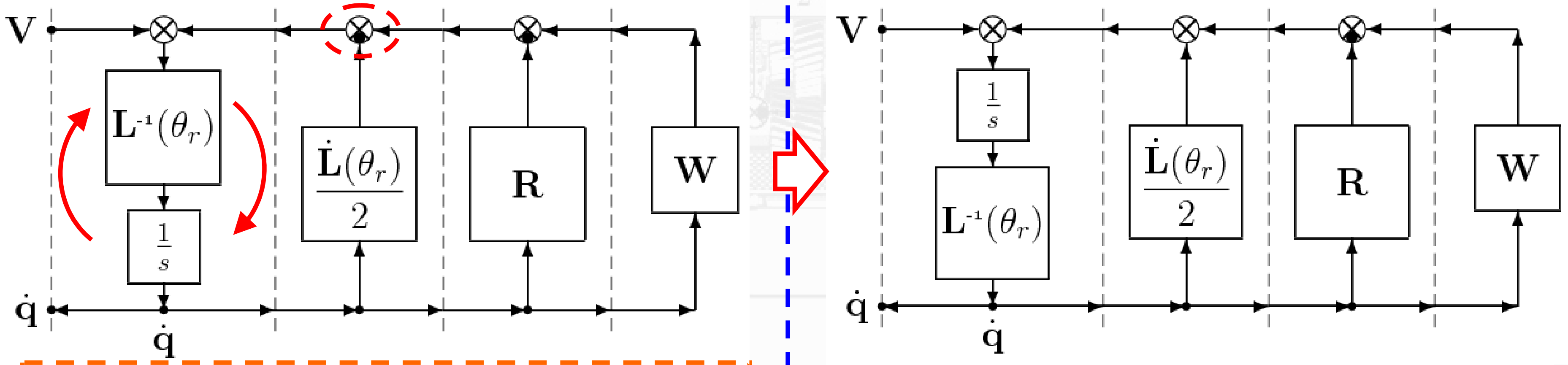
The dynamic equations of the system are:

$$\mathbf{L}\,\ddot{\mathbf{q}} + \frac{1}{2}\dot{\mathbf{L}}\,\dot{\mathbf{q}} = \mathbf{V} - \mathbf{R}\,\dot{\mathbf{q}} + \underbrace{\left[\frac{\partial(\frac{1}{2}\dot{\mathbf{q}}^{\mathrm{T}}\,\mathbf{L} + \varphi^{\mathrm{T}})}{\partial \mathbf{q}^{\mathrm{T}}} - \frac{\partial(\frac{1}{2}\mathbf{L}\,\dot{\mathbf{q}} + \varphi)}{\partial \mathbf{q}}\right]}_{\mathbf{W}}\dot{\mathbf{q}}$$

Where **R** is a symmetric matrix (energy "dissipation/generation") and **W** is a skew-symmetric matrix (energy "redistribution"): $\mathbf{R} = \mathbf{R}^{\mathrm{T}}$ , $\mathbf{W} = -\mathbf{W}^{\mathrm{T}}$

# POG modeling of Electrical Motors

Two different but equivalent POG graphical representations:




$$\mathbf{L}(\theta_r)\,\ddot{\mathbf{q}} + \frac{1}{2}\dot{\mathbf{L}}(\theta_r)\,\dot{\mathbf{q}} = \mathbf{V} - \mathbf{R}\,\dot{\mathbf{q}} + \mathbf{W}\,\dot{\mathbf{q}}$$

$$\frac{d}{dt}\left[\mathbf{L}(\theta_r)\,\dot{\mathbf{q}}\right] = \mathbf{V} + \frac{1}{2}\dot{\mathbf{L}}(\theta_r)\,\dot{\mathbf{q}} - \mathbf{R}\,\dot{\mathbf{q}} + \mathbf{W}\dot{\mathbf{q}}$$

The dynamic equations can be easily interpreted from a "power" point of view.

Multiplying $\dot{\mathbf{q}}^{\mathrm{T}}$ on the left of the first equation one obtains:

$$\underbrace{\dot{\mathbf{q}}^{\mathrm{T}}\,\mathbf{L}\,\ddot{\mathbf{q}} + \frac{1}{2}\dot{\mathbf{q}}^{\mathrm{T}}\,\dot{\mathbf{L}}\,\dot{\mathbf{q}}}_{\dot{K}} = \underbrace{\dot{\mathbf{q}}^{\mathrm{T}}\,\mathbf{V}}_{P_e} - \underbrace{\dot{\mathbf{q}}^{\mathrm{T}}\,\mathbf{R}\,\dot{\mathbf{q}}}_{P_d} + \underbrace{\dot{\mathbf{q}}^{\mathrm{T}}\,\mathbf{W}\,\dot{\mathbf{q}}}_{0}$$

Stored energy variation

Entering power

Dissipated power

Redistributed power

# Brushless motor: the three-phase stator circuit

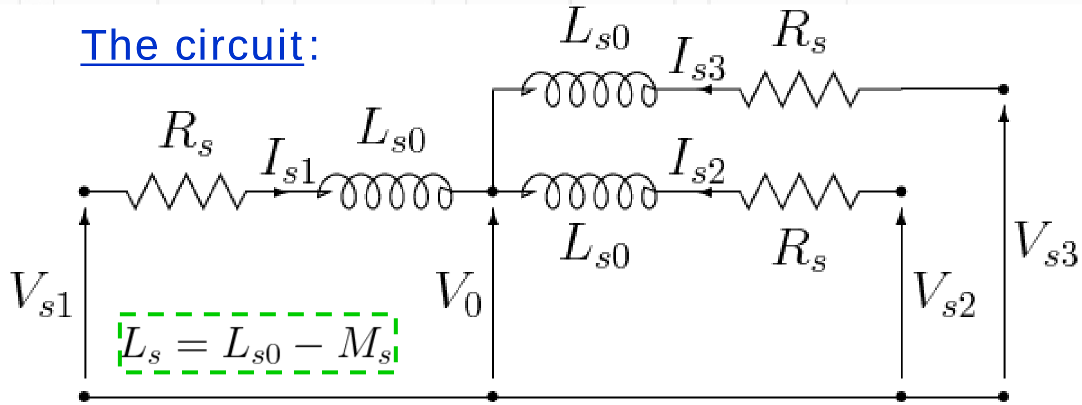
The constraint:

$$I_{s1} + I_{s2} + I_{s3} = 0$$

The dynamic equations:

$${}^t\mathbf{L}_s\, {}^t\dot{\mathbf{I}}_s = -\,{}^t\mathbf{R}_s\, {}^t\mathbf{I}_s + {}^t\mathbf{V}_s - \mathbf{V}_0$$

The circuit:




that is, expanded:

$$\underbrace{\begin{bmatrix} L_{s0} & M_s & M_s \\ M_s & L_{s0} & M_s \\ M_s & M_s & L_{s0} \end{bmatrix}}_{{}^t\mathbf{L}_s} \underbrace{\begin{bmatrix} \dot{I}_{s1} \\ \dot{I}_{s2} \\ \dot{I}_{s3} \end{bmatrix}}_{{}^t\dot{\mathbf{I}}_s} = -\underbrace{\begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix}}_{{}^t\mathbf{R}_s} \underbrace{\begin{bmatrix} I_{s1} \\ I_{s2} \\ I_{s3} \end{bmatrix}}_{{}^t\mathbf{I}_s} + \underbrace{\begin{bmatrix} V_{s1} - V_0 \\ V_{s2} - V_0 \\ V_{s3} - V_0 \end{bmatrix}}_{{}^t\mathbf{V}_s - \mathbf{V}_0}$$

Static "dq"

By using a congruent transformation ${}^t\mathbf{I}_s = {}^t\mathbf{T}_b\, {}^b\mathbf{I}_s$ one obtains the "reduced system":

$${}^t\mathbf{T}_b = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

$$\Rightarrow \quad \underbrace{\begin{bmatrix} L_s & 0 \\ 0 & L_s \end{bmatrix}}_{{}^b\mathbf{L}_s} \underbrace{\begin{bmatrix} \dot{I}_{sd} \\ \dot{I}_{sq} \end{bmatrix}}_{{}^b\dot{\mathbf{I}}_s} = -\underbrace{\begin{bmatrix} R_s & 0 \\ 0 & R_s \end{bmatrix}}_{{}^b\mathbf{R}_s} \underbrace{\begin{bmatrix} I_{sd} \\ I_{sq} \end{bmatrix}}_{{}^b\mathbf{I}_s} + \underbrace{\begin{bmatrix} V_{sd} \\ V_{sq} \end{bmatrix}}_{{}^b\mathbf{V}_s}$$

# Brushless motor: the rotating frame

By using a orthonormal transformation ${}^b\mathbf{I}_s = {}^b\mathbf{T}_\omega\, {}^\omega\mathbf{I}_s$ ...

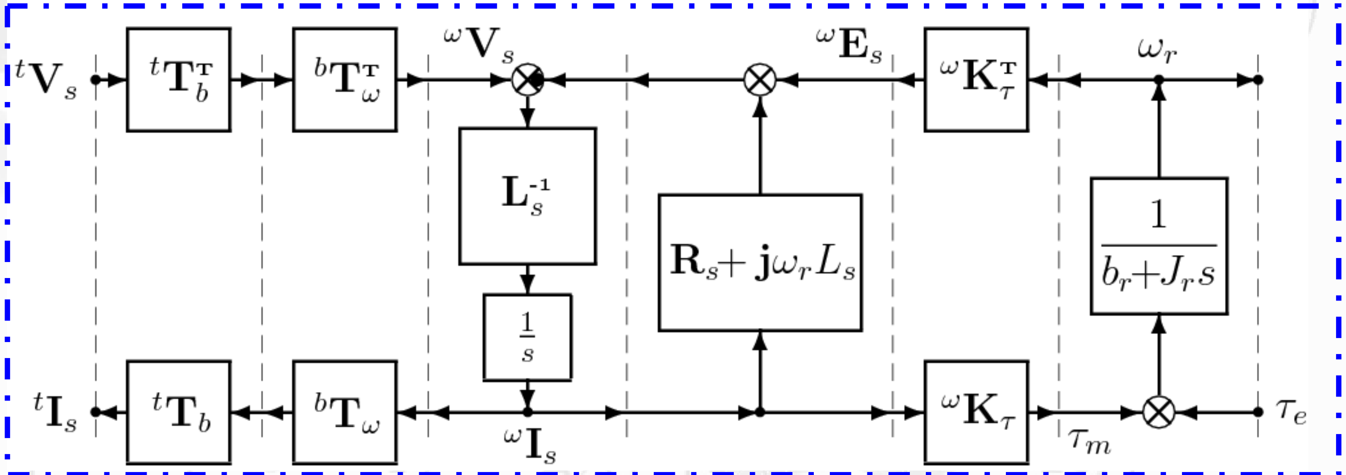
$$
{}^b\mathbf{T}_\omega = \begin{bmatrix} \cos\theta_r & -\sin\theta_r \\ \sin\theta_r & \cos\theta_r \end{bmatrix}
\Rightarrow
\left[\begin{array}{c|c} \mathbf{L}_s & \mathbf{0} \\ \hline \mathbf{0} & J_r \end{array}\right]
\left[\begin{array}{c} {}^\omega\dot{\mathbf{I}}_s \\ \hline \dot{\omega}_r \end{array}\right]
= -\left[\begin{array}{c|c} {}^\omega\mathbf{R}_s + \mathbf{j}\omega_r L_s & {}^\omega\mathbf{K}_\tau^{\mathrm{T}} \\ \hline -{}^\omega\mathbf{K}_\tau & b_r \end{array}\right]
\left[\begin{array}{c} {}^\omega\mathbf{I}_s \\ \hline \omega_r \end{array}\right]
+ \left[\begin{array}{c} {}^\omega\mathbf{V}_s \\ \hline -\tau_e \end{array}\right]
$$

... one obtains the "two-phase rotating" dynamic model of the system.

$$
\left[\begin{array}{cc|c} L_s & 0 & 0 \\ 0 & L_s & 0 \\ \hline 0 & 0 & J_r \end{array}\right]
\left[\begin{array}{c} {}^\omega\dot{I}_{sd} \\ {}^\omega\dot{I}_{sq} \\ \hline \dot{\omega}_r \end{array}\right]
= -\left[\begin{array}{cc|c} R_s & -\omega_r L_s & \frac{\partial\, {}^\omega\varphi_d}{\partial\theta_r} \\ \omega_r L_s & R_s & \frac{\partial\, {}^\omega\varphi_q}{\partial\theta_r} \\ \hline -\frac{\partial\, {}^\omega\varphi_d}{\partial\theta_r} & -\frac{\partial\, {}^\omega\varphi_q}{\partial\theta_r} & b_r \end{array}\right]
\left[\begin{array}{c} {}^\omega I_{sd} \\ {}^\omega I_{sq} \\ \hline \omega_r \end{array}\right]
+ \left[\begin{array}{c} {}^\omega V_{sd} \\ {}^\omega V_{sq} \\ \hline -\tau_e \end{array}\right]
$$

Expanded form where $\vec{\varphi}(\theta_r)$ is the magnetic flux generated by the permanent magnets

POG dynamic model of the brushless motor



# Brushless motor: sinusoidal magnetic flux

If the magnetic flux of the permanent magnets is sinusoidal …

$$^{t}\vec{\varphi}(\theta_r) = \varphi_0 \begin{bmatrix} \cos\theta_r \\ \cos(\theta_r - \frac{2\pi}{3}) \\ \cos(\theta_r + \frac{2\pi}{3}) \end{bmatrix}$$

Three-phase

$$^{b}\vec{\varphi}(\theta_r) = \sqrt{\frac{3}{2}}\varphi_0 \begin{bmatrix} \cos\theta_r \\ \sin\theta_r \end{bmatrix}$$

Two-phase static

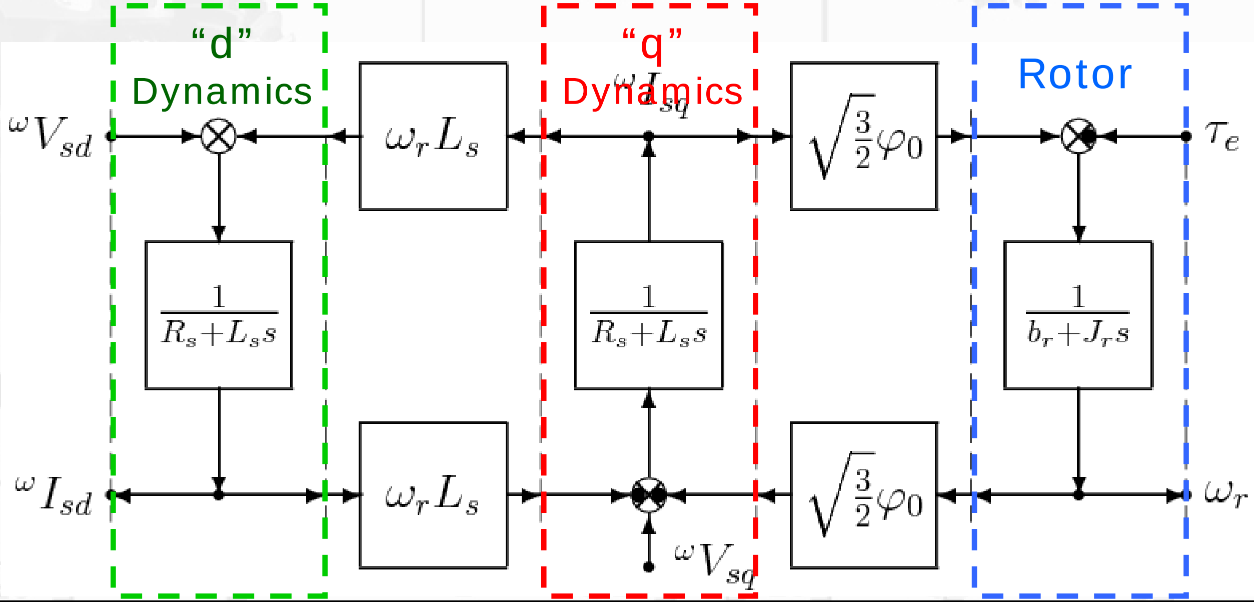
$$^{\omega}\vec{\varphi}(\theta_r) = \begin{bmatrix} \sqrt{\frac{3}{2}}\varphi_0 \\ 0 \end{bmatrix}$$

Two-phase rotating

… the dynamic equations of brushless motor strongly simplify …

$$\left[\begin{array}{cc|c} L_s & 0 & 0 \\ 0 & L_s & 0 \\ \hline 0 & 0 & J_r \end{array}\right] \left[\begin{array}{c} {}^{\omega}\dot{I}_{sd} \\ {}^{\omega}\dot{I}_{sq} \\ \hline \dot{\omega}_r \end{array}\right] = - \left[\begin{array}{cc|c} R_s & -\omega_r L_s & 0 \\ \omega_r L_s & R_s & \sqrt{\frac{3}{2}}\varphi_0 \\ \hline 0 & -\sqrt{\frac{3}{2}}\varphi_0 & b_r \end{array}\right] \left[\begin{array}{c} {}^{\omega}I_{sd} \\ {}^{\omega}I_{sq} \\ \hline \omega_r \end{array}\right] + \left[\begin{array}{c} {}^{\omega}V_{sd} \\ {}^{\omega}V_{sq} \\ \hline -\tau_e \end{array}\right]$$

… as well as the POG graphical representation.

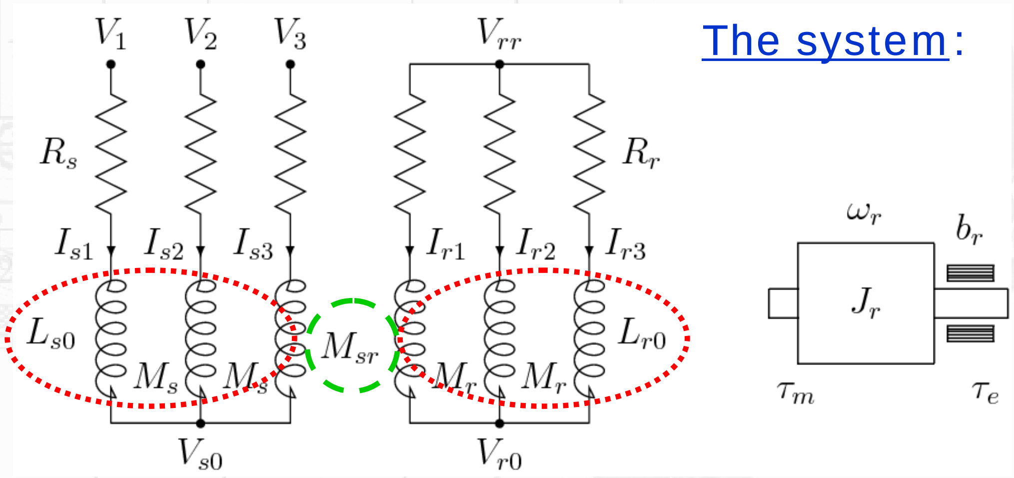


# Asyncronous three-phase motor: the stator and rotor circuits

The variables. Stator and rotor currents and voltages:

$${}^t\mathbf{I}_s,\ {}^t\mathbf{I}_r,\ {}^t\mathbf{V}_s,\ {}^t\mathbf{V}_r$$

The system:




The constraints:

$$I_{s1} + I_{s2} + I_{s3} = 0$$
$$I_{r1} + I_{r2} + I_{r3} = 0$$

The dynamic equations:

$$\frac{d}{dt}\left(\begin{bmatrix} {}^t\mathbf{L}_s & {}^t\mathbf{M}_{sr}^{\mathrm{T}} \\ {}^t\mathbf{M}_{sr} & {}^t\mathbf{L}_r \end{bmatrix}\begin{bmatrix} {}^t\mathbf{I}_s \\ {}^t\mathbf{I}_r \end{bmatrix}\right) = -\begin{bmatrix} {}^t\mathbf{R}_s & \mathbf{0} \\ \mathbf{0} & {}^t\mathbf{R}_r \end{bmatrix}\begin{bmatrix} {}^t\mathbf{I}_s \\ {}^t\mathbf{I}_r \end{bmatrix} + \begin{bmatrix} {}^t\mathbf{V}_s \\ {}^t\mathbf{V}_r \end{bmatrix}$$

where ${}^t\mathbf{R}_s = R_s\mathbf{I}_3,\quad {}^t\mathbf{R}_r = R_r\mathbf{I}_3$ and

$${}^t\mathbf{L}_s = \begin{bmatrix} L_{s0} & M_s & M_s \\ M_s & L_{s0} & M_s \\ M_s & M_s & L_{s0} \end{bmatrix} \qquad {}^t\mathbf{L}_r = \begin{bmatrix} L_{r0} & M_r & M_r \\ M_r & L_{r0} & M_r \\ M_r & M_r & L_{r0} \end{bmatrix}$$

**Stator and Rotor Self-Inductances**

$${}^t\mathbf{M}_{sr} = M_{sr}\begin{bmatrix} \cos(\theta_r) & \cos(\theta_r - \frac{2\pi}{3}) & \cos(\theta_r + \frac{2\pi}{3}) \\ \cos(\theta_r + \frac{2\pi}{3}) & \cos(\theta_r) & \cos(\theta_r - \frac{2\pi}{3}) \\ \cos(\theta_r - \frac{2\pi}{3}) & \cos(\theta_r + \frac{2\pi}{3}) & \cos(\theta_r) \end{bmatrix}$$

**Stator / Rotor Mutual-Inductances**

# Asyncronous motor: dynamic model

Applying the two-phase "static" transformation (6->4):

$$\begin{bmatrix} {}^t\mathbf{I}_s \\ {}^t\mathbf{I}_r \end{bmatrix} = \begin{bmatrix} {}^t\tilde{\mathbf{T}}_b & \mathbf{0} \\ \mathbf{0} & {}^t\tilde{\mathbf{T}}_b \end{bmatrix} \begin{bmatrix} {}^b\mathbf{I}_s \\ {}^b\mathbf{I}_r \end{bmatrix} \qquad \text{where} \qquad {}^t\tilde{\mathbf{T}}_b = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

... and then the two-phase "rotating" transformation ($\theta_d = \theta_s - \theta_r$ ):

$$\begin{bmatrix} {}^b\mathbf{I}_s \\ {}^b\mathbf{I}_r \end{bmatrix} = \begin{bmatrix} \mathbf{e}^{\mathbf{j}\theta_s} & \mathbf{0} \\ \mathbf{0} & \mathbf{e}^{\mathbf{j}\theta_d} \end{bmatrix} \begin{bmatrix} {}^\omega\mathbf{I}_s \\ {}^\omega\mathbf{I}_r \end{bmatrix} \qquad \text{where} \qquad \mathbf{j} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \qquad \mathbf{e}^{\mathbf{j}\theta} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

... one obtains the following "full" dynamic model:

$$\left[\begin{array}{cc|c} \mathbf{L}_s & \mathbf{L}_{sr} & \mathbf{0} \\ \mathbf{L}_{sr} & \mathbf{L}_r & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{0} & J_r \end{array}\right] \left[\begin{array}{c} {}^\omega\dot{\mathbf{I}}_s \\ {}^\omega\dot{\mathbf{I}}_r \\ \hline \dot{\omega}_r \end{array}\right] = - \left[\begin{array}{cc|c} \mathbf{R}_s + \mathbf{j}\omega_s L_s & \mathbf{j}(\omega_s - \frac{\omega_r}{2}) L_{sr} & \mathbf{j}\,{}^\omega\mathbf{I}_r \frac{1}{2} L_{sr} \\ \mathbf{j}(\omega_s - \frac{\omega_r}{2}) L_{sr} & \mathbf{R}_r + \mathbf{j}\omega_d L_r & -\mathbf{j}\,{}^\omega\mathbf{I}_s \frac{1}{2} L_{sr} \\ \hline {}^\omega\mathbf{I}_r^{\mathrm{T}}\, \mathbf{j} \frac{1}{2} L_{sr} & -\,{}^\omega\mathbf{I}_s^{\mathrm{T}}\, \mathbf{j} \frac{1}{2} L_{sr} & b_r \end{array}\right] \left[\begin{array}{c} {}^\omega\mathbf{I}_s \\ {}^\omega\mathbf{I}_r \\ \hline \omega_r \end{array}\right] + \left[\begin{array}{c} {}^\omega\mathbf{V}_s \\ \mathbf{0} \\ \hline -\tau_e \end{array}\right]$$

where $\mathbf{R}_s = R_s\,\mathbf{I}_2$ , $\mathbf{R}_r = R_r\,\mathbf{I}_2$, $\mathbf{L}_s = L_s\,\mathbf{I}_2$ , $\mathbf{L}_r = L_r\,\mathbf{I}_2$ , $\mathbf{L}_{sr} = L_{sr}\,\mathbf{I}_2$ ,
$L_s = L_{s0} - M_s$ , $L_r = L_{r0} - M_r$ , $L_{sr} = \frac{3}{2} M_{sr}$ .

# Asyncronous motor: dynamic model

Dynamic model in a compact form:

$$\underbrace{\left[\begin{array}{c|c} {}^{\omega}\mathbf{L}_e & \mathbf{0} \\ \hline \mathbf{0} & J_r \end{array}\right]}_{{}^{\omega}\mathbf{L}} \underbrace{\left[\begin{array}{c} {}^{\omega}\dot{\mathbf{I}}_e \\ \hline \dot{\omega}_r \end{array}\right]}_{{}^{\omega}\dot{\mathbf{I}}} = - \underbrace{\left[\begin{array}{c|c} {}^{\omega}\mathbf{R}_e - \Omega_{sd} & {}^{\omega}\mathbf{K}_\tau^{\mathrm{T}} \\ \hline -{}^{\omega}\mathbf{K}_\tau & b_r \end{array}\right]}_{({}^{\omega}\mathbf{R} + {}^{\omega}\mathbf{W})} \underbrace{\left[\begin{array}{c} {}^{\omega}\mathbf{I}_e \\ \hline \omega_r \end{array}\right]}_{{}^{\omega}\mathbf{I}} + \underbrace{\left[\begin{array}{c} {}^{\omega}\mathbf{V}_e \\ \hline -\tau_e \end{array}\right]}_{{}^{\omega}\mathbf{V}}$$

The energy matrix represents the stored energy:

$$E_s = \frac{1}{2}\, {}^{\omega}\mathbf{I}^{\mathrm{T}}\, {}^{\omega}\mathbf{L}\, {}^{\omega}\mathbf{I}$$

The symmetric part of the system matrix represents the energy dissipations:

$${}^{\omega}\mathbf{R} = \left[\begin{array}{c|c} {}^{\omega}\mathbf{R}_e & \mathbf{0} \\ \hline \mathbf{0} & b_r \end{array}\right]$$

The skew symmetric part of the system matrix represents the energy redistribution:

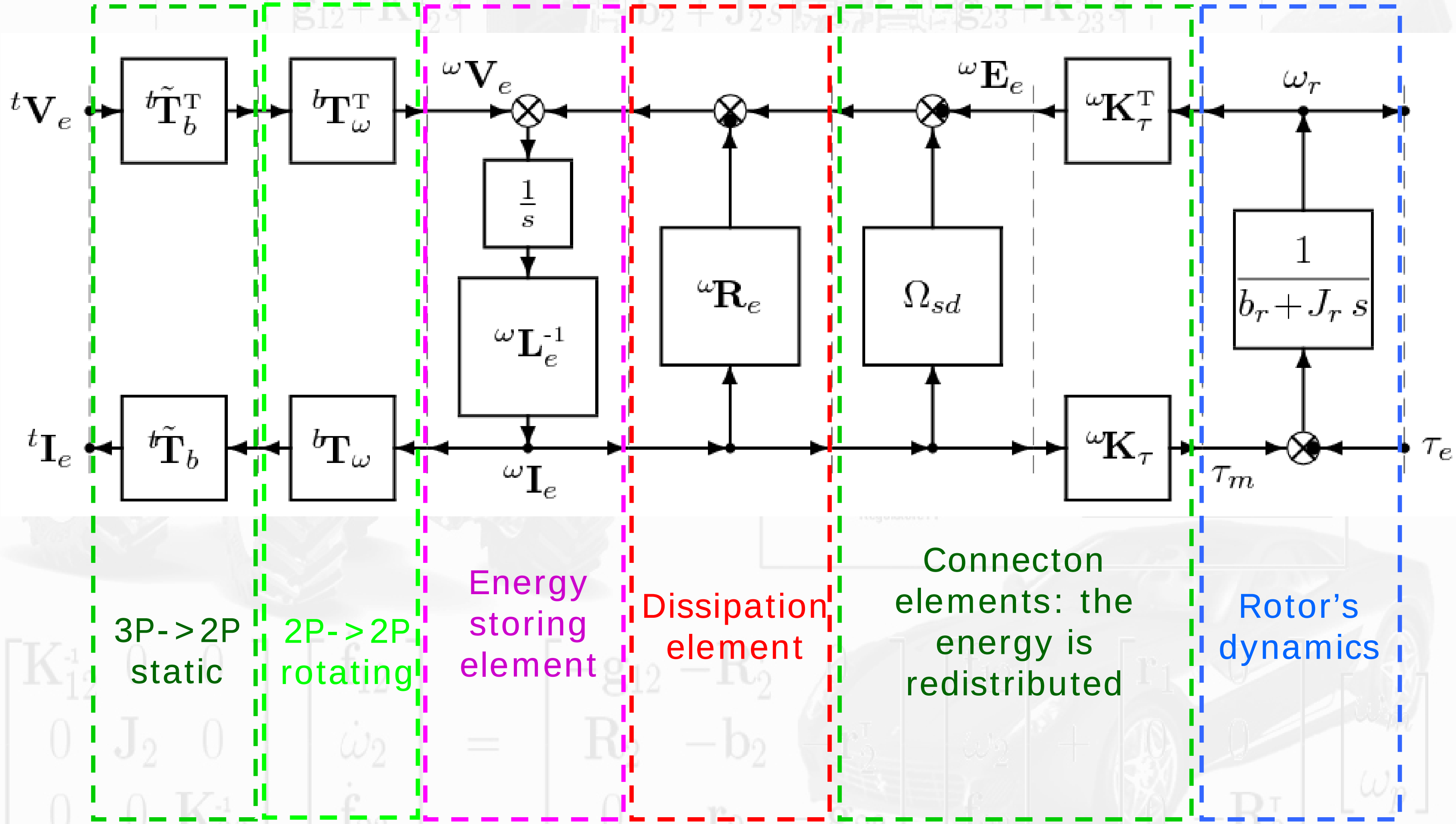
$${}^{\omega}\mathbf{W} = \left[\begin{array}{c|c} -\Omega_{sd} & {}^{\omega}\mathbf{K}_\tau^{\mathrm{T}} \\ \hline -{}^{\omega}\mathbf{K}_\tau & 0 \end{array}\right]$$

The active torque applied to the rotor:

$$\tau_m = \frac{\partial E_a}{\partial \theta_r} = \left[\; -{}^{\omega}\mathbf{I}_r^{\mathrm{T}}\, \mathbf{j}\tfrac{1}{2}L_{sr} \quad {}^{\omega}\mathbf{I}_s^{\mathrm{T}}\, \mathbf{j}\tfrac{1}{2}L_{sr} \;\right] {}^{\omega}\mathbf{I}_e = {}^{\omega}\mathbf{K}_\tau\, {}^{\omega}\mathbf{I}_e$$

# Asyncronous motor: POG model

The POG graphical representation:



# Conclusions

- Power-Oriented Graphs (POG) are a simple and powerful graphical technique that can be used for modeling all types of physical systems involving power flows.

- POG are easily understandable, simple to use and suitable both for teaching and for research.