Multi-phase Synchronous Motors

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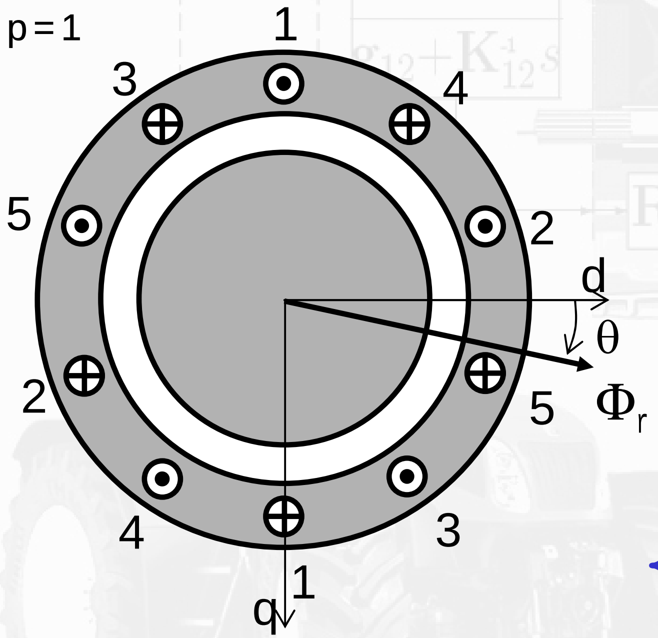
Multi-phase Syncronous motor (m=5)



Electrical part

$$\frac{d\tilde{\mathbf{\Phi}}_c(\mathbf{I},\theta)}{dt} = \mathbf{V} - \mathbf{R}\,\mathbf{I}$$

system of differential equations describing the electrical part

where:

$\tilde{\mathbf{\Phi}}_c(\mathbf{I},\theta) = \mathbf{L}\,\mathbf{I} + \mathbf{\Phi}_c(\theta)$ chained total magnetic fluxes

$\mathbf{V}$ voltage input vector

$\mathbf{R}$ dissipating matrix (diagonal)

$$\mathbf{L} = p\begin{bmatrix} L_1 & M_{12} & M_{13} & \cdots & M_{1m} \\ M_{12} & L_2 & M_{23} & \cdots & M_{2m} \\ M_{13} & M_{23} & L_3 & \cdots & M_{3m} \\ \vdots & \vdots & & \ddots & \vdots \\ M_{1m} & M_{2m} & M_{3m} & \cdots & L_m \end{bmatrix}$$

positive-definite symmetric matrix

$$\frac{d(\mathbf{L}\,\mathbf{I})}{dt} = \mathbf{V} - \mathbf{R}\mathbf{I} - p\,\frac{\partial \mathbf{\Phi}_c(\theta)}{\partial\theta}\,\omega_r$$

$\mathbf{K}_\tau(\theta)$ TORQUE VECTOR

Multi-phase Syncronous motor

Mechanical part

$$\frac{d(J_r\omega_r)}{dt} = \tau_r - \tau_e - b_r\,\omega_r$$

differential equation describing the mechanical part

Energy stored in the electro-mechanical system:

$$E(\mathbf{I},\omega_r,\theta_r) = \frac{1}{2}\mathbf{I}^{\mathrm{T}}\mathbf{L}\mathbf{I} + \mathbf{I}^{\mathrm{T}}\,\boldsymbol{\Phi}_c(p\theta_r) + \frac{1}{2}J_r\omega_r^2$$

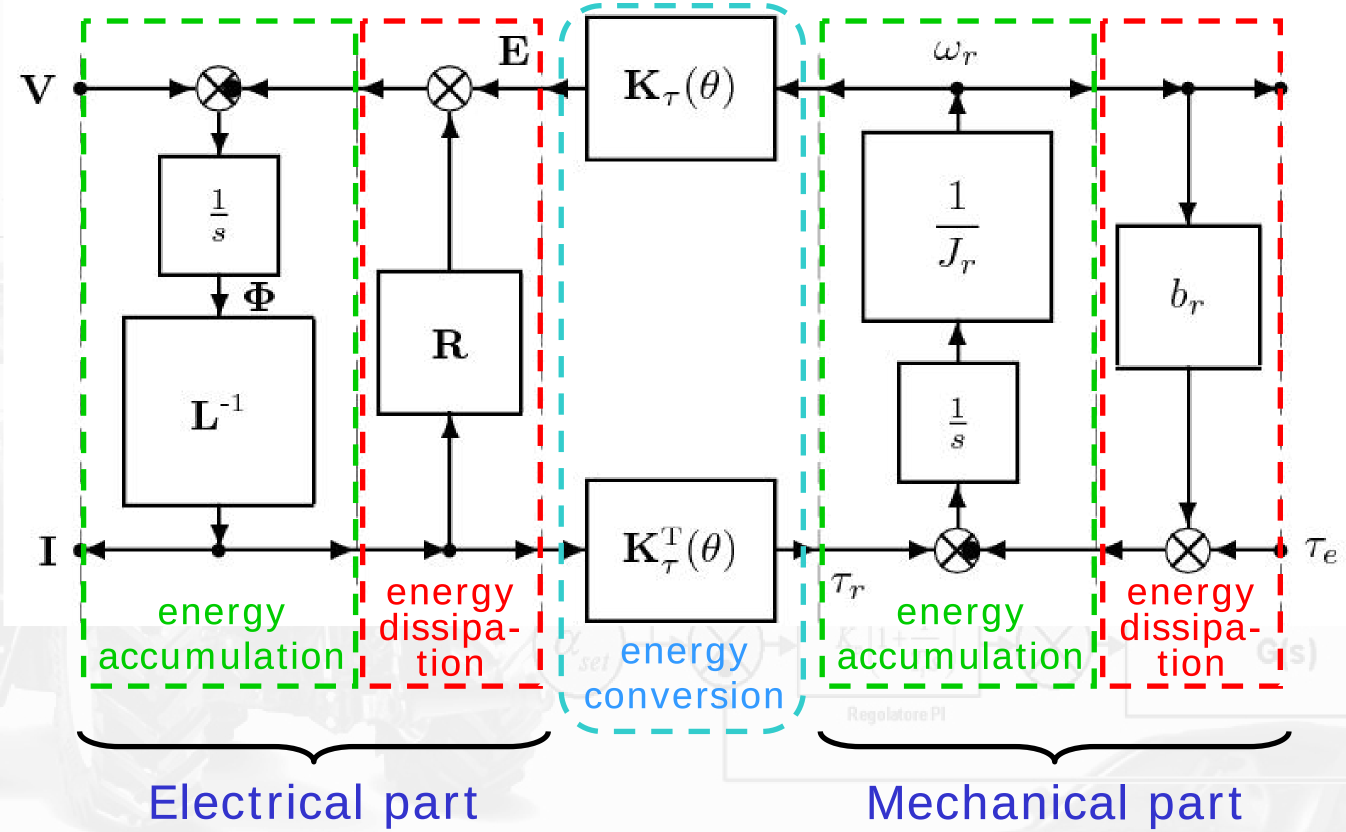
$\mathbf{K}_\tau^{\mathrm{T}}(\theta)\,\mathbf{I}$ = electromotive torque

$\mathbf{K}_\tau(\theta)\omega$ = counter-electromotive forces

Function $\phi_c(\theta)$ of chained rotor flux can be developed in Fourier series of cosines with only odd harmonics, so vector $K_\tau(\theta)$ can be written as:

$$\mathbf{K}_\tau(\theta) = p\,\varphi_c \left\| \left[-\sum_{n=1:2}^{\infty} n\,a_n \sin[n(\theta - h\,\gamma)] \right] \right\|_{0:m-1}^{h}$$

POG scheme of the multi-phase motor



State-Space description of the system:

$$\left[\begin{array}{c|c} \mathbf{L} & 0 \\ \hline 0 & J_r\end{array}\right]\left[\begin{array}{c}\dot{\mathbf{I}}\\ \hline \dot{\omega}_r\end{array}\right] = -\left[\begin{array}{c|c} \mathbf{R} & \mathbf{K}_\tau(\theta)\\ \hline -\mathbf{K}_\tau^{\mathrm{T}}(\theta) & b_r\end{array}\right]\left[\begin{array}{c} \mathbf{I}\\ \hline \omega_r\end{array}\right] + \left[\begin{array}{c} \mathbf{V} \\ \hline -\tau_e\end{array}\right]$$

Orthonormal transformations

Let's consider the following orthonormal transformation and then apply it to the system:

$$
{}^t\mathbf{T}^{\mathrm{T}}_{\omega} = {}^{\omega}\mathbf{T}_t = \sqrt{\frac{2}{m}}
\begin{bmatrix}
\left\|\left[\begin{matrix}\cos(k\,(\theta - h\,\gamma)) \\ \sin(k\,(\theta - h\,\gamma))\end{matrix}\right]\right\|^{k\quad h}_{1:2:m-2\quad 0:m-1} \\
\left\|\left[\ \frac{1}{\sqrt{2}}\ \right]\right\|^{h}_{0:m-1}
\end{bmatrix}
$$

$\longrightarrow$ from Σ_t to Σ_ω

The system dynamic equations become:

$$
\left[\begin{array}{c|c} {}^{\omega}\mathbf{L} & 0 \\ \hline 0 & J_r \end{array}\right]
\left[\begin{array}{c} {}^{\omega}\dot{\mathbf{I}} \\ \hline \dot{\omega}_r \end{array}\right]
= -\left[\begin{array}{c|c} {}^{\omega}\mathbf{R} + \mathbf{J}_{\omega}\,{}^{\omega}\mathbf{L} & {}^{\omega}\mathbf{K}_{\tau} \\ \hline -\,{}^{\omega}\mathbf{K}^{\mathrm{T}}_{\tau} & b_r \end{array}\right]
\left[\begin{array}{c} {}^{\omega}\mathbf{I} \\ \hline \omega_r \end{array}\right]
+ \left[\begin{array}{c} {}^{\omega}\mathbf{V} \\ \hline -\tau_e \end{array}\right]
$$

where the transformed matrices are:

$$
{}^{\omega}\mathbf{L} = p
\begin{bmatrix}
\Delta_0 + \frac{m\,M_0}{2} & 0 & 0 & 0 & \cdots & 0 \\
0 & \Delta_0 + \frac{m\,M_0}{2} & 0 & 0 & \cdots & 0 \\
0 & 0 & \Delta_0 & 0 & \cdots & 0 \\
0 & 0 & 0 & \Delta_0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & 0 & \cdots & \Delta_0
\end{bmatrix}
$$

and

$$
\mathbf{J}_{\omega} =
\begin{bmatrix}
\left\|\left[\begin{matrix} 0 & -k\,\omega \\ k\,\omega & 0 \end{matrix}\right]\right\|^{k}_{1:2:m-2} & 0 \\
0 & 0
\end{bmatrix}
$$

${}^{\omega}\mathbf{R} = \mathbf{R}$ $\longrightarrow$ diagonal matrices

The torque vector

In the transformed space the structure of the torque vector is quite simple:

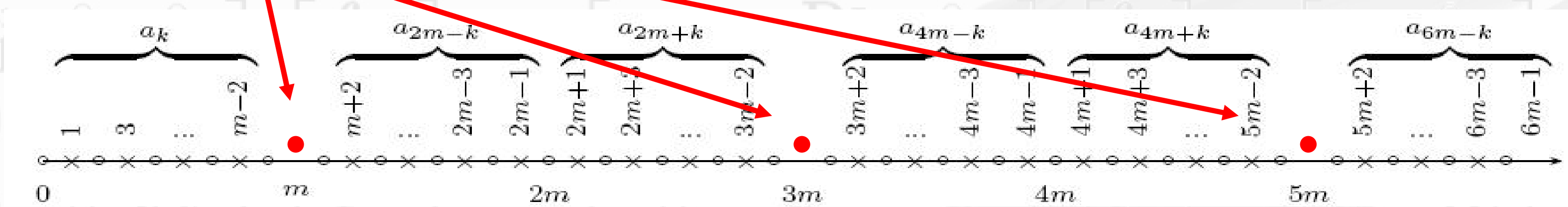
$$
{}^{\omega}\mathbf{K}_{\tau}(\theta) = -p\,\varphi_c\sqrt{\frac{m}{2}}
\begin{bmatrix}
\left[\begin{bmatrix}
\sum\limits_{n=0:2m}^{\infty}[(n+k)\,a_{n+k}+(n-k)\,a_{n-k}]\sin(n\theta) \\
\sum\limits_{n=0:2m}^{\infty}[(n+k)\,a_{n+k}-(n-k)\,a_{n-k}]\cos(n\theta)
\end{bmatrix}\right]^{k}_{1:2:m-2} \\
-\sqrt{2}\sum\limits_{n=m:2m}^{\infty} n a_n \sin(n\theta)
\end{bmatrix}
=
\begin{bmatrix}
\left[\begin{bmatrix}
{}^{\omega}K_{kd}(\theta) \\
{}^{\omega}K_{kq}(\theta)
\end{bmatrix}\right]^{k}_{1:2:m-2} \\
{}^{\omega}K_{m}(\theta)
\end{bmatrix}
$$

"direct" terms

"quadrature" terms

This term is not present if the phases are star-connected:

- The "**d**" and "**q**" terms have only harmonics with frequency multiple of 2m
- The harmonics of the last term don't influence the "**d**" and "**q**" terms



The torque vector

Proposition 1. The torque vector can be constant only if the flux vector can be expressed in Fourier series as follows:

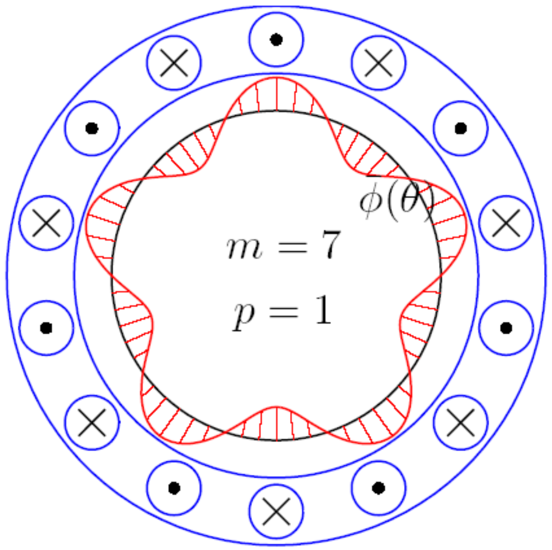
$$\bar{\phi}(\theta) = \sum_{i=1:2}^{m-2} a_i \cos(i\,\theta)$$

All the constant components of the torque vector are obtained for n=0:

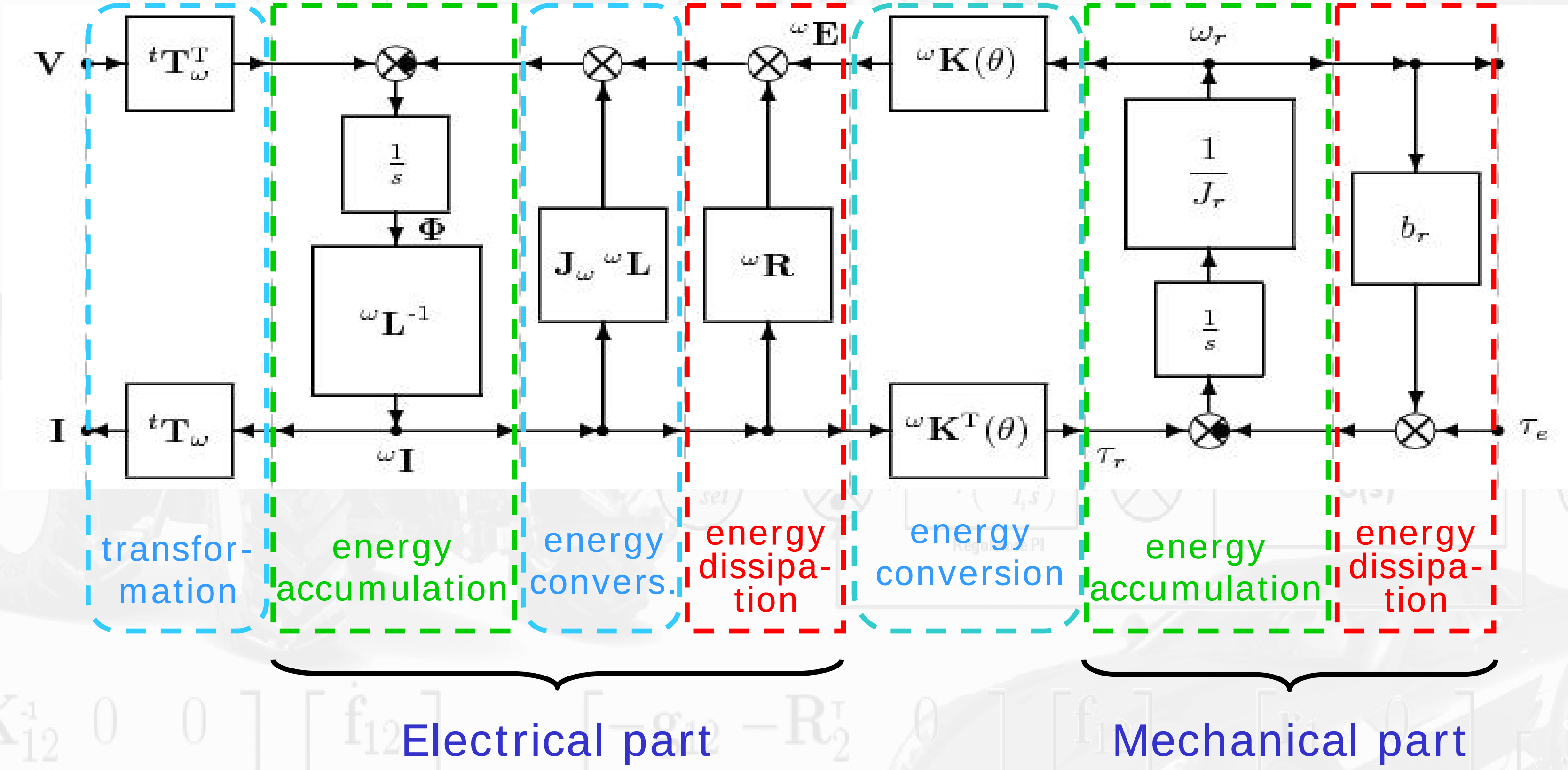
$${}^{\omega}\mathbf{K}_{\tau}^{T}(\theta)|_{n=0} = -\varphi_c\, p \sqrt{\frac{m}{2}} \left[\, \big\|\big[\; 0 \quad k\, a_k \;\big]\big\|_{1:2:m-2}^{k} \;\; 0 \, \right]$$

Main result. Among all the fluxes providing a constant vector, the one that minimizes the module of the current vector (and therefore the dissipated power) is given by:

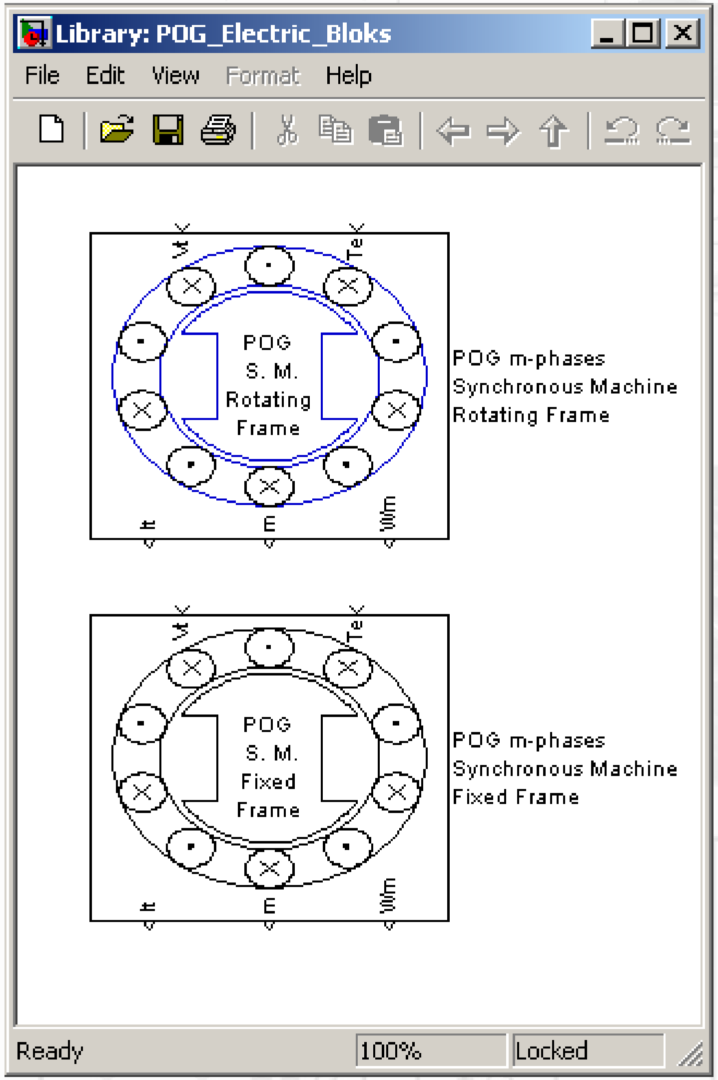
$$\bar{\phi}(\theta) = \cos((m-2)\,\theta)$$



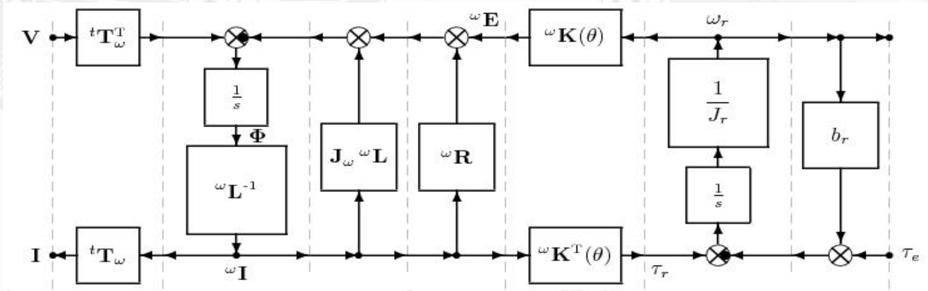
POG scheme in the transformed space

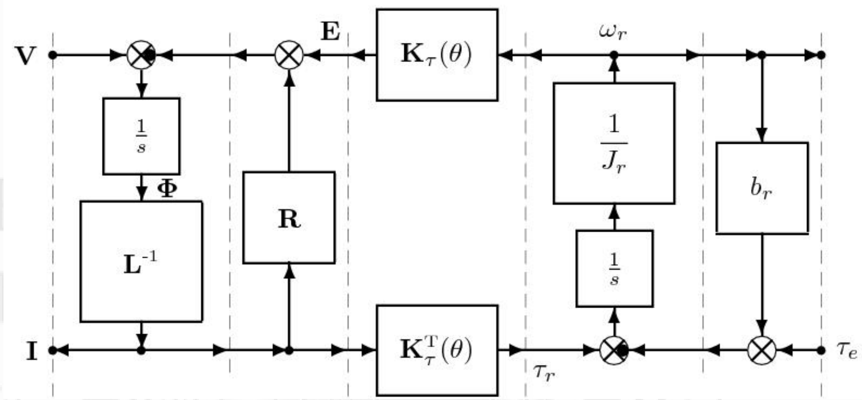


POG Blocks in Simulink



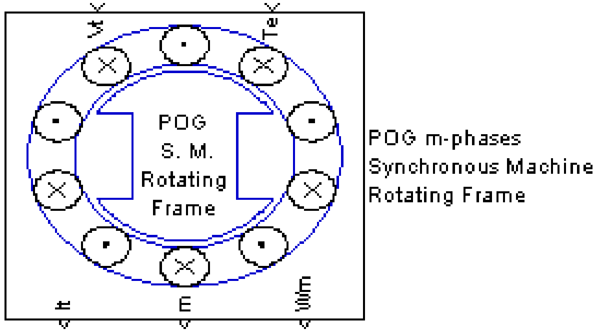
POG m-phases Syncronous Motors

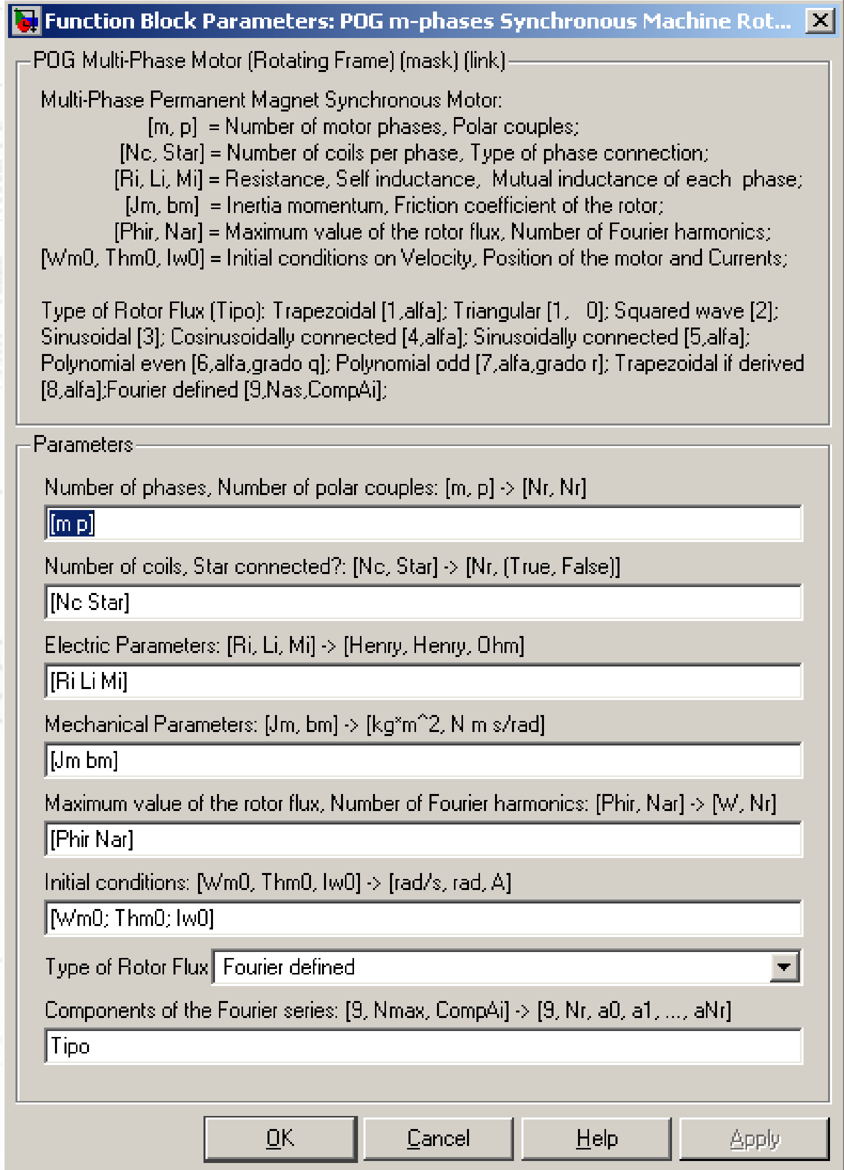




POG Blocks in Simulink: the musk

POG m-phases Motors:





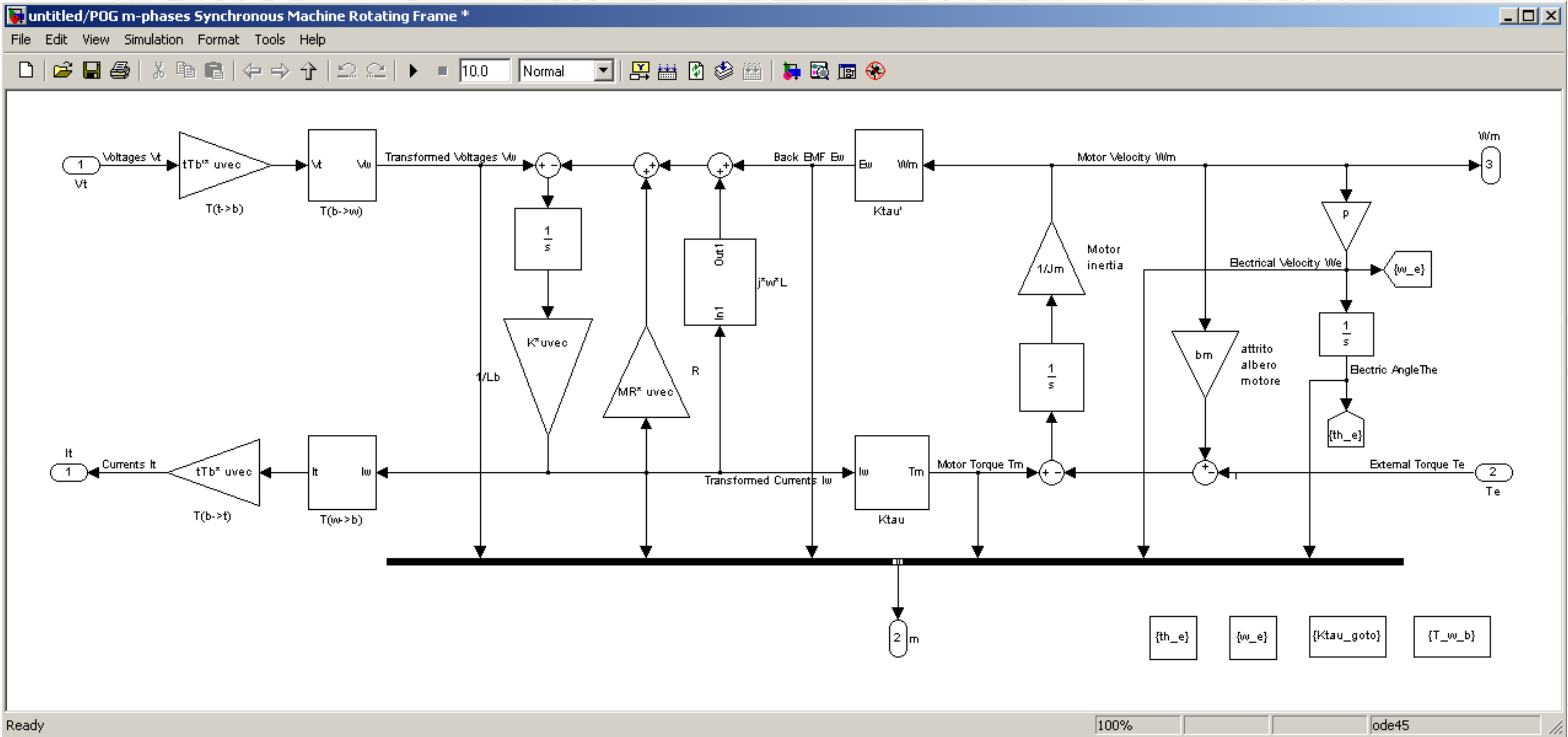
Types of rotor flux:

1. Trapezoidal
2. Triangular
3. Squared wave
4. Sinusoidal
5. Cosinusoidally connected
6. Sinusoidally connected
7. Polynomial even
8. Polynomial odd
9. Trapezoidal if derived
10. Fourier defined

The stator can also be “star connected”

POG Blocks in Simulink

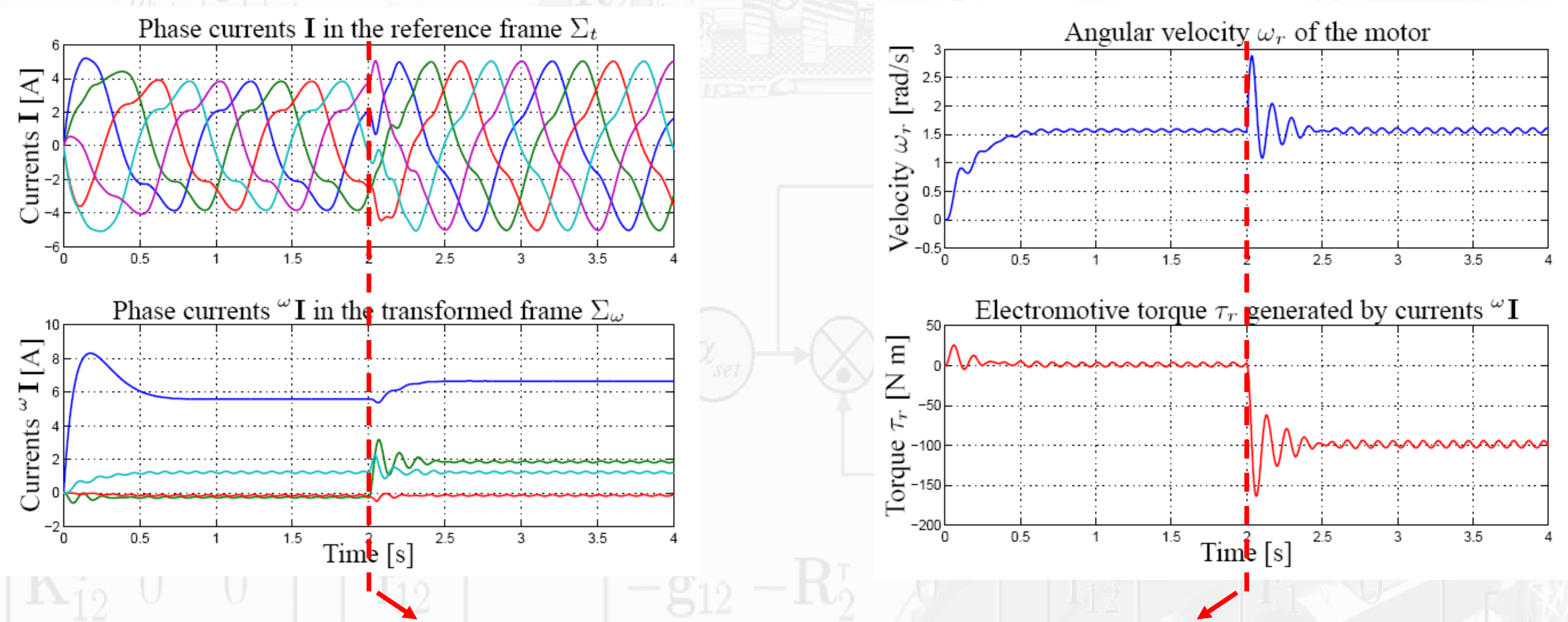
The POG block diagrams can be directly inserted in Simulink !!



Simulations (1)

The POG scheme has been implemented in Simulink.

Simulation implemented with cosinusoidal interpolated rotor flux with 200 harmonics.

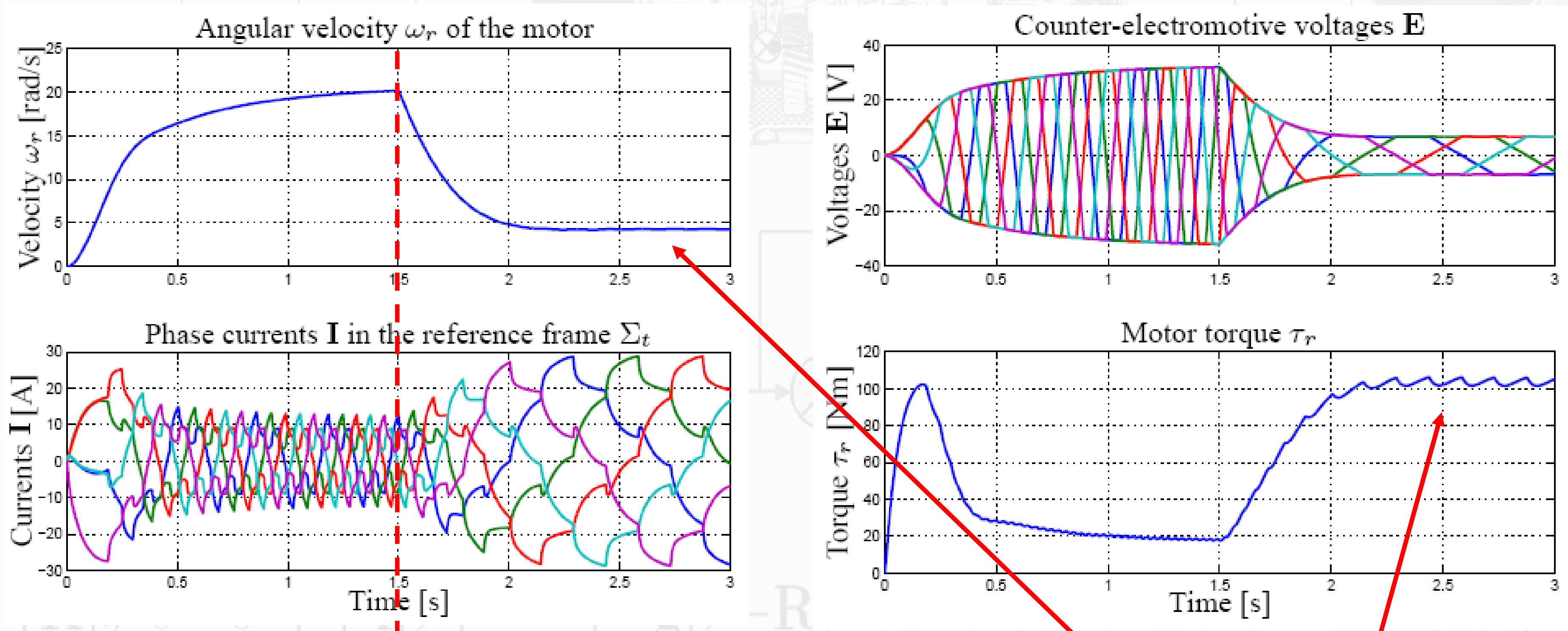


At time t=2s the load torque τ_e switches from 0 to 100 Nm

Simulations (2)

Simulation implemented with even polynomial interpolation ($q=2, \alpha=\pi/5$)

Counter-electromotive voltages are trapezoidal



At time t=1.5s the load torque τ_e switches from 0 to 100 Nm

This control generates high torques when velocity is small

Conclusions

- Power-Oriented Graphs (POG) are a simple and powerful graphical technique that can be used for modelling all types of physical systems involving power flows.

- POG are easily understandable, simple to use and suitable both for teaching and for research.