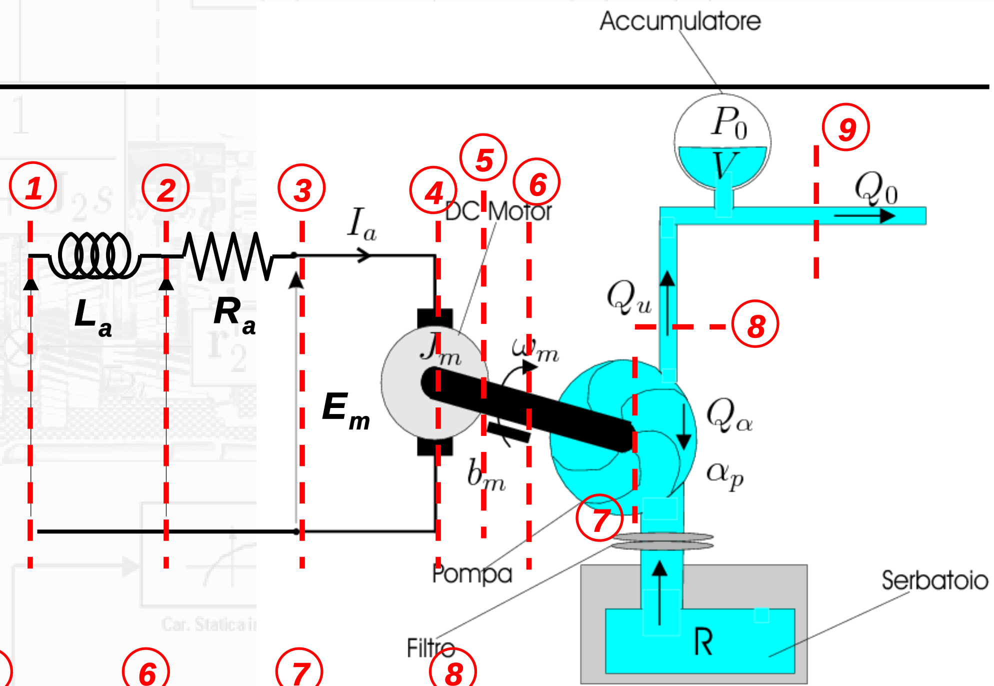
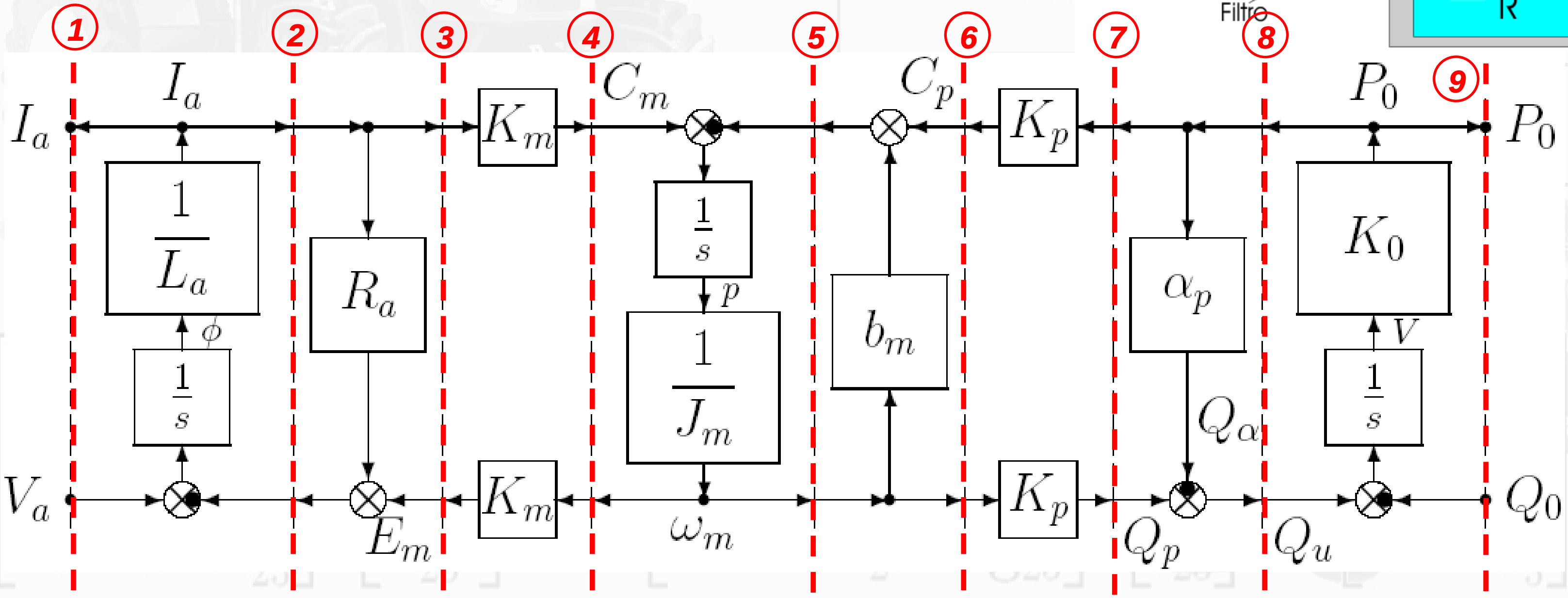
Example of POG modeling and Model Reduction

A DC motor connected to an hydraulic pump:

There is a direct correspondence between the POG blocks and the physical elements …



The POG model:



POG models can be directly inserted in Simulink.

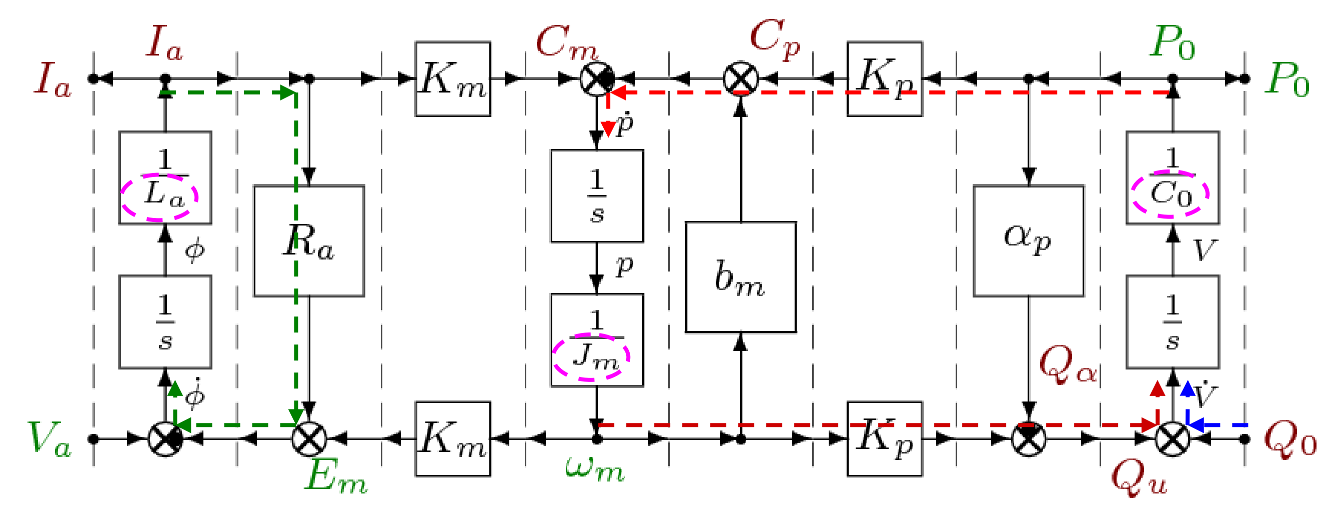
State space model of the POG schemes

The POG state space description of the DC motor with hydraulic pump:

$$\underbrace{\begin{bmatrix} L_a & 0 & 0 \\ 0 & J_m & 0 \\ 0 & 0 & C_0 \end{bmatrix}}_{\mathbf{L}} \underbrace{\begin{bmatrix} \dot{I}_a \\ \dot{\omega}_m \\ \dot{P}_0 \end{bmatrix}}_{\dot{\mathbf{x}}} = \underbrace{\begin{bmatrix} -R_a & -K_m & 0 \\ K_m & -b_m & -K_p \\ 0 & K_p & -\alpha_p \end{bmatrix}}_{-\mathbf{A}} \underbrace{\begin{bmatrix} I_a \\ \omega_m \\ P_0 \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}}_{\mathbf{B}} \underbrace{\begin{bmatrix} V_a \\ Q_0 \end{bmatrix}}_{\mathbf{u}}$$

$$\mathbf{y} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{C}} \mathbf{x}$$

The coefficients of matrices $-\mathbf{A}$ and $\mathbf{B}$ are the gains of all the paths that link the state and input variables $\mathbf{x}$ and $\mathbf{u}$ to the inputs of the integrators:



The elements of matrix **L** are the coefficients of the constitutive relations.

Matrices **A**, **B** and **L** can be obtained by direct inspection of the POG scheme.

State space models of the POG schemes

The POG model of the DC motor with hydraulic pump:

$$\underbrace{\begin{bmatrix} L_a & 0 & 0 \\ 0 & J_m & 0 \\ 0 & 0 & C_0 \end{bmatrix}}_{\mathbf{L}} \underbrace{\begin{bmatrix} \dot{I}_a \\ \dot{\omega}_m \\ \dot{P}_0 \end{bmatrix}}_{\dot{\mathbf{x}}} = \underbrace{\begin{bmatrix} -R_a & -K_m & 0 \\ K_m & -b_m & -K_p \\ 0 & K_p & -\alpha_p \end{bmatrix}}_{-\mathbf{A}} \underbrace{\begin{bmatrix} I_a \\ \omega_m \\ P_0 \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}}_{\mathbf{B}} \underbrace{\begin{bmatrix} V_a \\ Q_0 \end{bmatrix}}_{\mathbf{u}}$$

$$\mathbf{y} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{C}} \mathbf{x}$$

The POG models can be represented by the compact vectorial scheme:

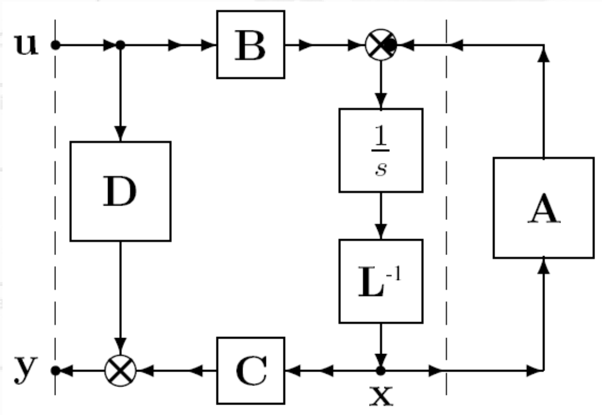
Energy matrix

Power matrix

$$\begin{cases} \mathbf{L}\,\dot{\mathbf{x}} = -\mathbf{A}\,\mathbf{x} + \mathbf{B}\,\mathbf{u} \\ \mathbf{y} = \mathbf{C}\,\mathbf{x} + \mathbf{D}\,\mathbf{u} \end{cases}$$

Stored Energy: $E_s = \frac{1}{2}\mathbf{x}^{\mathrm{T}}\mathbf{L}\,\mathbf{x}$

Dissipating Power: $P_d = \mathbf{x}^{\mathrm{T}}\mathbf{A}\,\mathbf{x}$



As contains only the dissipating elements:

$$\mathbf{A}_s = \frac{\mathbf{A} + \mathbf{A}^{\mathrm{T}}}{2} = \begin{bmatrix} R_a & 0 & 0 \\ 0 & b_m & 0 \\ 0 & 0 & \alpha_p \end{bmatrix}$$

Aw contains only the connection elements:

$$\mathbf{A}_w = \frac{\mathbf{A} - \mathbf{A}^{\mathrm{T}}}{2} = \begin{bmatrix} 0 & K_m & 0 \\ -K_m & 0 & K_p \\ 0 & -K_p & 0 \end{bmatrix}$$

POG modeling reduction: using a "congruent" transformation

When an eigenvalue of matrix ***L*** goes to zero (or to infinity), the system degenerates towards a lower dynamic dimension system. The "reduced system" can be obtained by using a congruent transformation $x = Tz$ where ***T*** is a rectangular matrix:

congruent transformation

$$\begin{cases} \mathbf{T}^T\mathbf{L}\mathbf{T}\dot{\mathbf{z}} = \mathbf{T}^T\mathbf{A}\mathbf{T}\mathbf{z} + \mathbf{T}^T\mathbf{B}\mathbf{u} \\ \mathbf{y} = \mathbf{B}^T\mathbf{T}\mathbf{z} \end{cases} \Leftrightarrow \begin{cases} \overline{\mathbf{L}}\dot{\mathbf{z}} = \overline{\mathbf{A}}\mathbf{z} + \overline{\mathbf{B}}\mathbf{u} \\ \mathbf{y} = \overline{\mathbf{B}}^T\mathbf{z} \end{cases}$$

Transformed system

When a parameter goes to zero a static relation between state variable arises:

$$J_m = 0 \Rightarrow K_m I_a - b_m \omega_m - K_p P_0 = 0 \Rightarrow \omega_m = \frac{K_m}{b_m} I_a - \frac{K_p}{b_m} P_0$$

A <u>rectangular</u> state space transformation can be easily obtained:

$$\underbrace{\begin{bmatrix} I_a \\ \omega_m \\ P_0 \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} 1 & 0 \\ \frac{K_m}{b_m} & -\frac{K_p}{b_m} \\ 0 & 1 \end{bmatrix}}_{\mathbf{T}} \underbrace{\begin{bmatrix} I_a \\ P_0 \end{bmatrix}}_{\mathbf{z}}$$

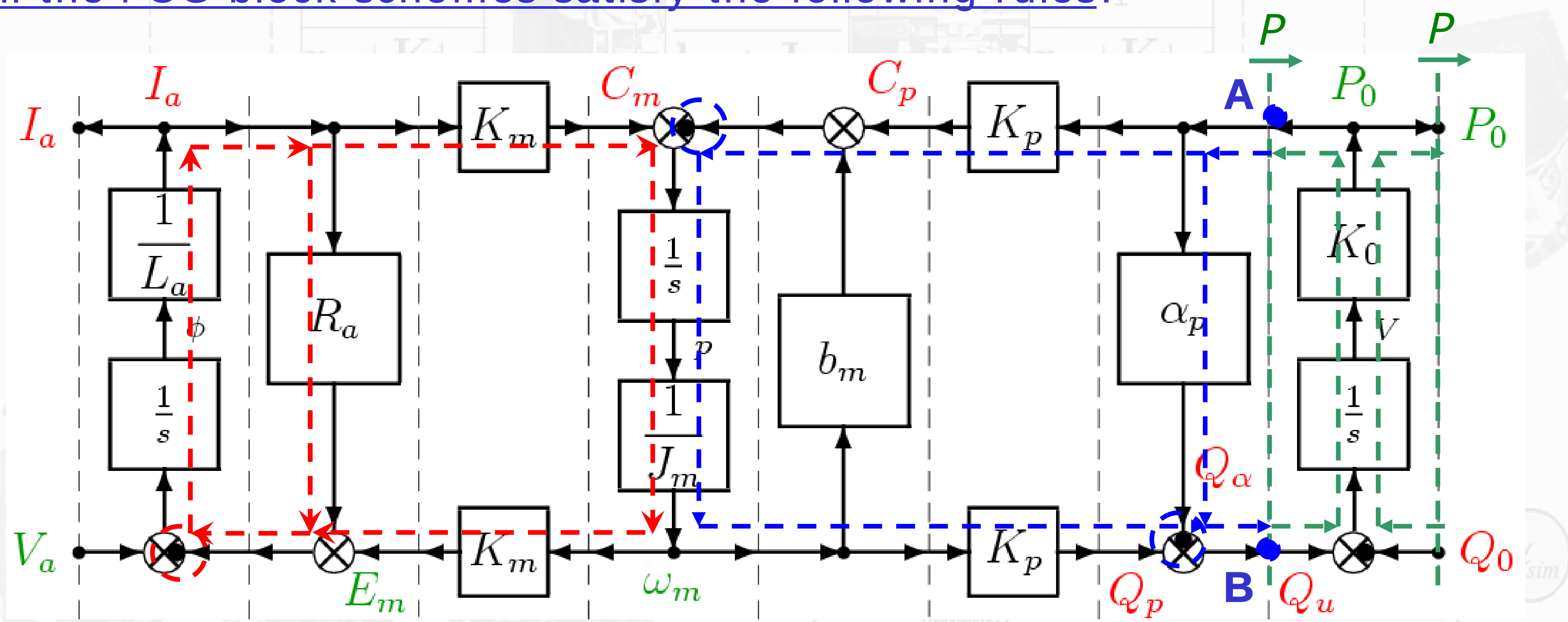
Old state vector (x) — **Rectangular matrix** (T) — **New state vector** (z)

Applying the congruent transformation <u>one directly obtains the reduced system</u>:

$$\begin{bmatrix} L_a & 0 \\ 0 & \frac{1}{K_0} \end{bmatrix} \begin{bmatrix} \dot{I}_a \\ \dot{P}_0 \end{bmatrix} = \begin{bmatrix} -R_a - \frac{K_m^2}{b_m} & \frac{K_m K_p}{b_m} \\ \frac{K_m K_p}{b_m} & -\alpha_p - \frac{K_p^2}{b_m} \end{bmatrix} \begin{bmatrix} I_a \\ P_0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} V_a \\ Q_0 \end{bmatrix}$$

POG schemes: graphical rules

All the POG block schemes satisfy the following rules:



1) *the loops of a POG scheme always contains an "odd" number of minus signs* (i.e. of black spots in the summation elements).

2) chosen two generic points A and B of a POG scheme, *all the paths that go from A to B contain either an "odd" number or an "even" number of minus signs*.

3) the direction of the power flowing through a section is positive if an "even" number of minus signs is present along one of the paths which goes from the input to the output of the section.