

# Different Energetic Techniques for Modelling Traction Drives



UNIVERSITÀ DEGLI STUDI  
DI MODENA E REGGIO EMILIA

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# Outline

The traction system of an automatic subway is described using four different **Energetic Graphical Techniques**:

- 1) Bond-Graph (**BG**)
- 2) Energetic Macroscopic Representation (**EMR**)
- 3) Power-Oriented Graphs (**POG**)
- 4) Vectorial Bond-Graph (**VBG**).

The paper highlight the analogies and the differences between these modelling techniques in the analysis and simulation of the considered automatic subway system.

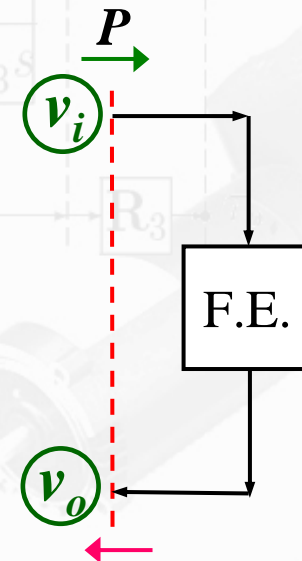
In Simulink, the four considered graphical techniques provides the same simulations.

# Dynamic Modelling: power variables

The **BG**, **EMR** and **POG** modelling techniques are based on the use of the **Power** and **Energy** variables.

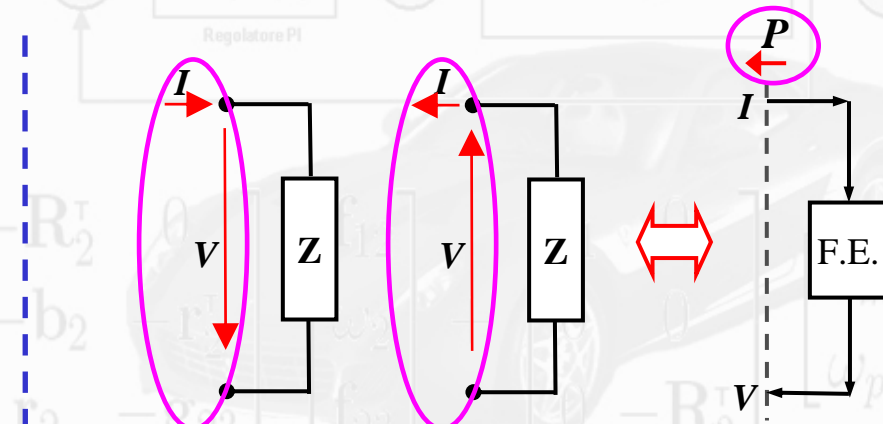
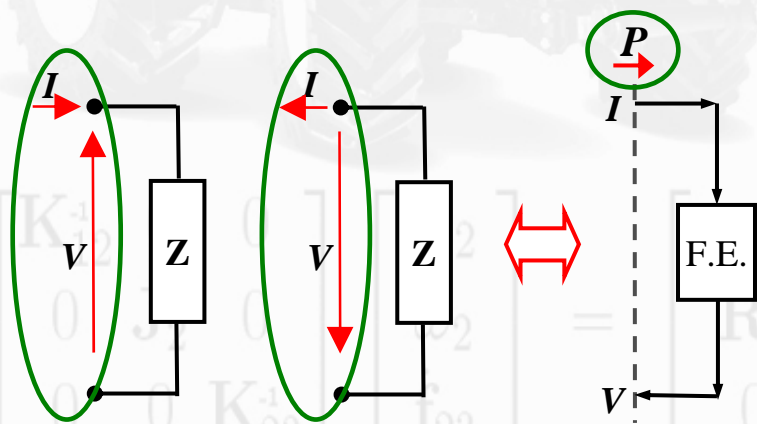
The physical elements (F.E.) interact with the external world through power sections characterized by two power variables  $v_i$  e  $v_o$ .

Each power variable has its own positive direction. According to these definitions the power  $P$  entering the physical element can be positive or negative.



The power enters into the element:

The power exits from the element:



# Dynamic Modeling of Physical Systems

## Different Energy domains:

1) Electrical; 2) Mechanical (tras./rot.); 3) Hydraulic; etc.

## The same dynamic structure:

- 2 "dynamic" elements  $D_1, D_2$  that store energy;
- 1 "static" element  $R$  that dissipates (or generates) energy;
- 2 "energy variables"  $q_1(t), q_2(t)$  used for describing the stored energy;
- 2 "power variables"  $v_1(t), v_2(t)$  used for moving the energy;

POG:

	Electrical	Mechanical	Hydraulic
$\mathcal{D}_1$	$C$ Capacitor	$M$ Mass	$C_I$ Hyd. Capacitor
$q_1$	$Q$ Charge	$p$ Momentum	$V$ Volume
<b>Across-variables</b>	$v_1$ $V$ Voltage	$v$ Velocity	$P$ Pressure
$\mathcal{D}_2$	$L$ Inductor	$E$ Spring	$L_I$ Hyd. Inductor
$q_2$	$\phi$ Flux	$x$ Displacement	$\phi_I$ Hyd. Flux
<b>Through-variables</b>	$v_2$ $I$ Current	$F$ Force	$Q$ Flow
$\mathcal{R}$	$R$ Resistor	$b$ Friction	$R_I$ Hyd. Resistor

Bond Graphs:

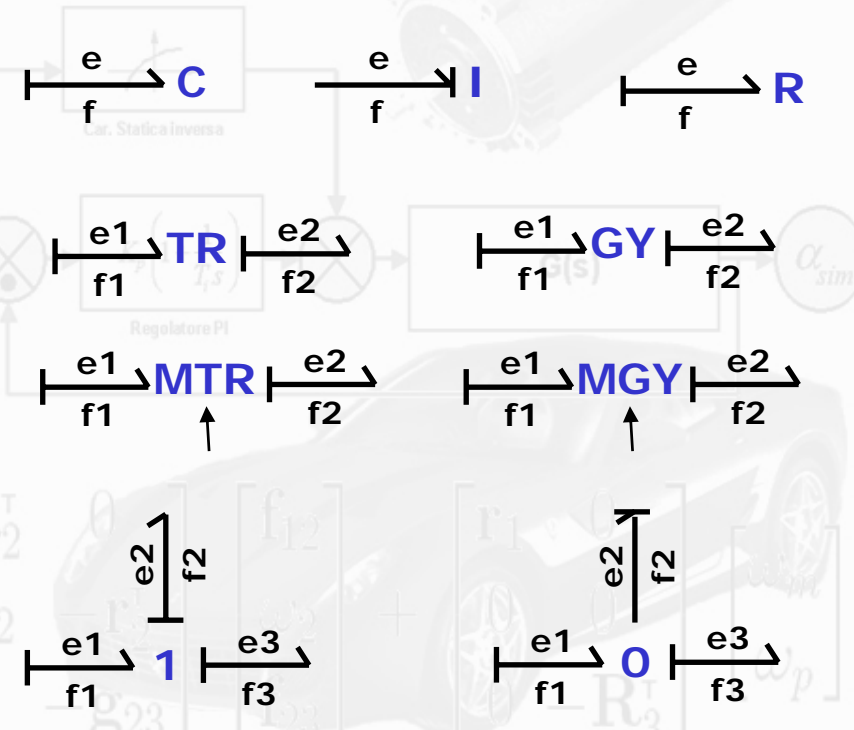
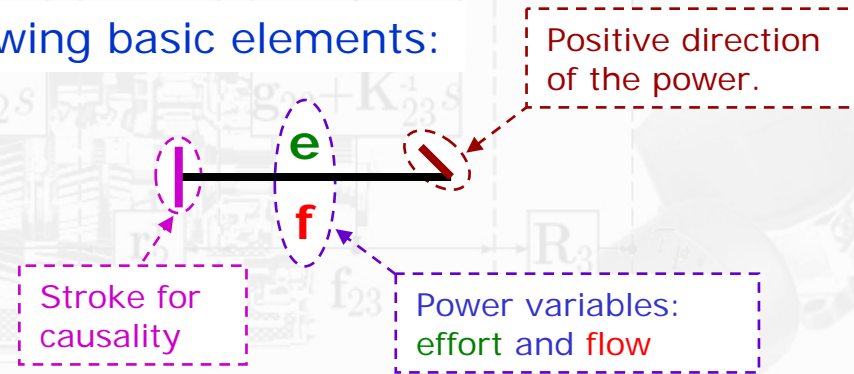
Efforts

Flows

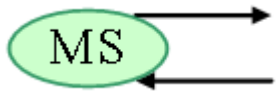
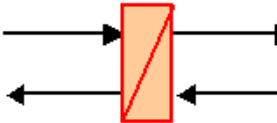
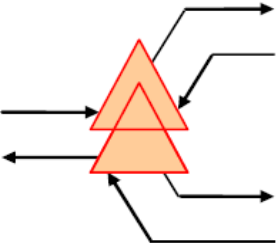
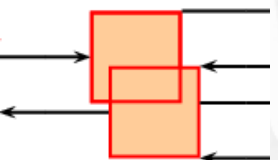
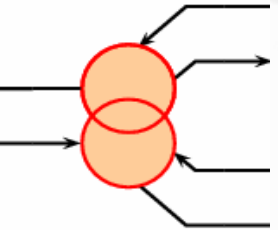
# Bond Graphs (BG)


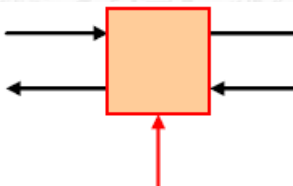
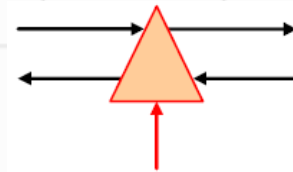
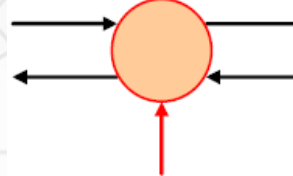
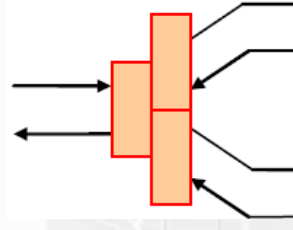
The Bond Graphs are composed by the following basic elements:

- A line representing the “bond”: **effort** above and **flow** below the line.
- An **arrow** representing the positive direction of the power.
- A **stroke** representing the integral or differential causality.
- Three basic **1-port elements** which store and/or dissipate the energy: capacitor **C**, inertia **I** and resistor **R**.
- Four **2-port elements** which simply transform the power: transformer **TR**, gyrator **GY**, modulated transformer **MTR** and modulated gyrator **MGY**.
- Two **3 port junctions** for connecting the basic elements: **0**-junction for the parallel connection and **1**-junction for the series connection.



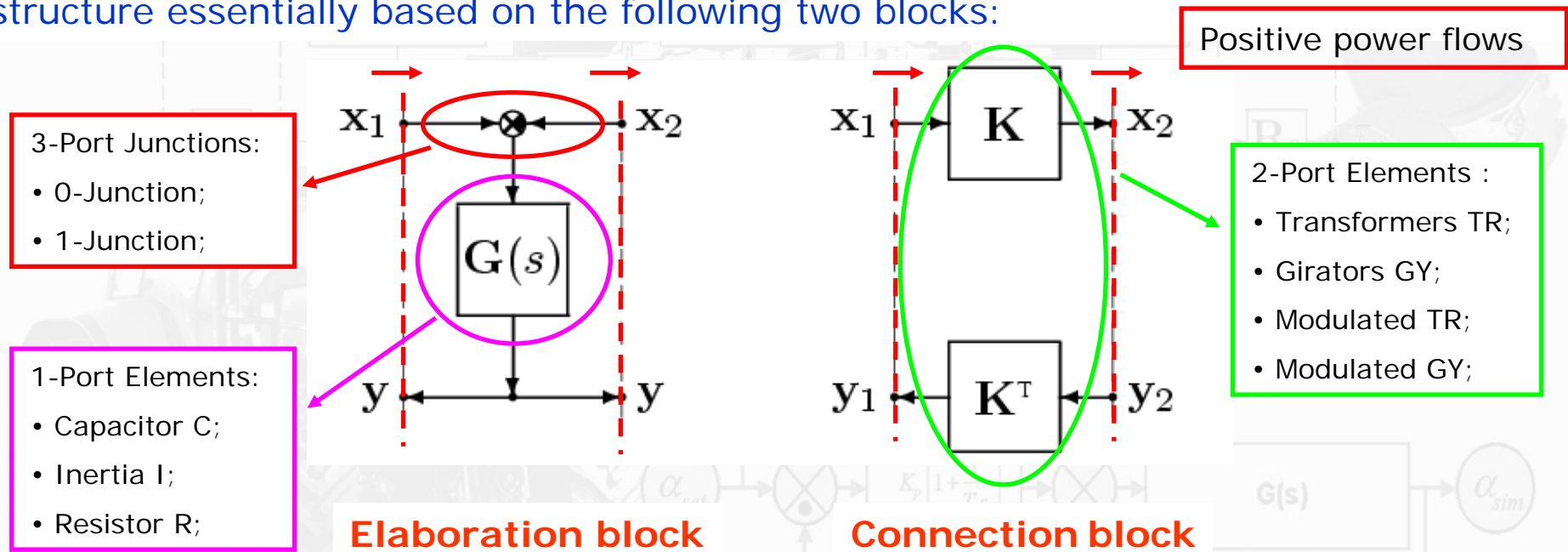
# Energetic Macroscopic Representation (EMR)

	Mechanical source of energy
	Element with energy accumulation
	Mechanical coupling (distribution of mechanical energy)
	Electrical coupling (distribution of electrical energy)
	Electromechanical coupling (distribution of electromechanical energy)

	Electrical source of energy
	Electrical converter (without energy accumulation)
	Mechanical converter (without energy accumulation)
	Electromechanical converter (without energy accumulation)
	Selector of models

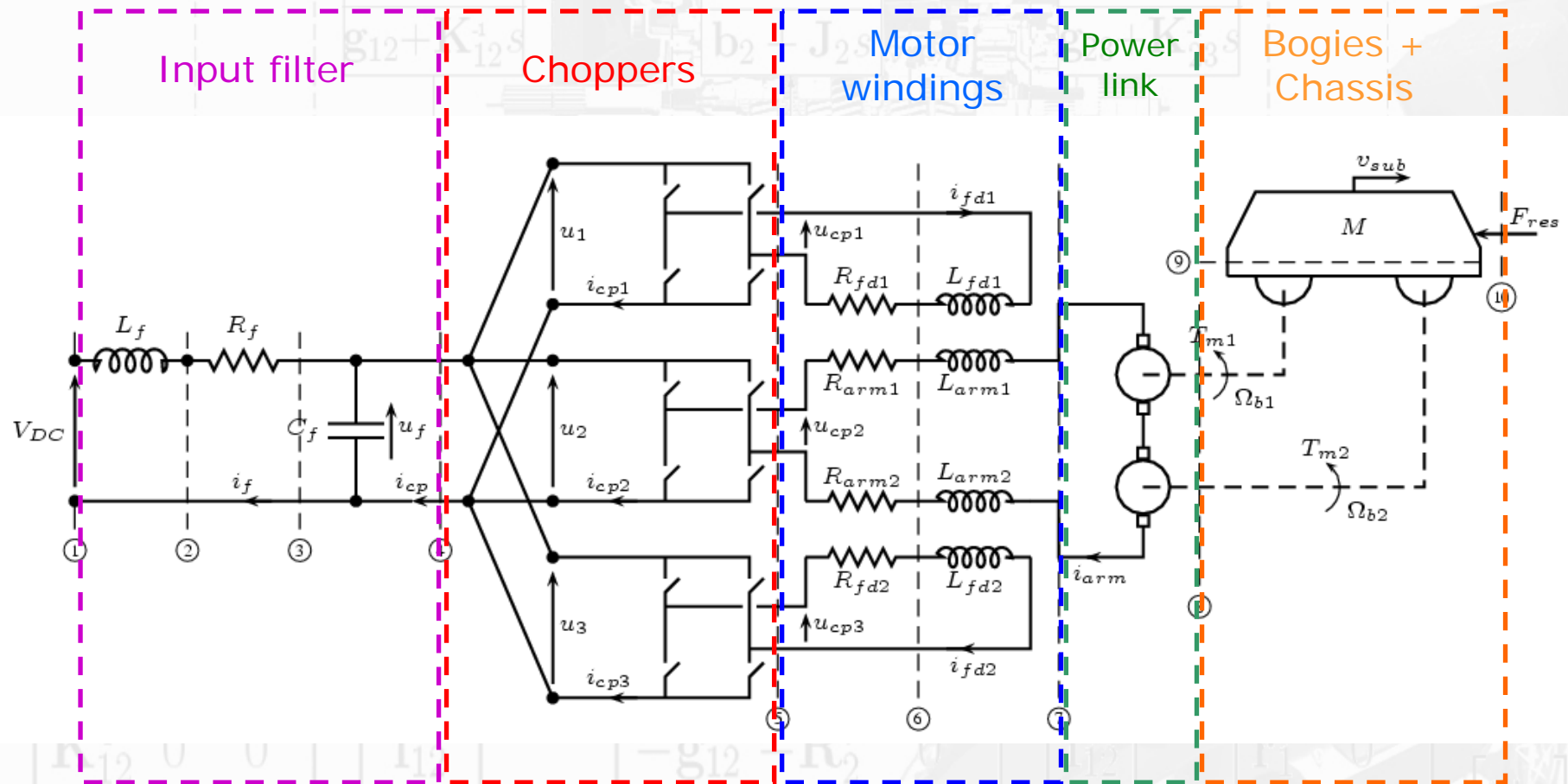
# Power-Oriented Graphs (POG)

The Power-Oriented Graphs are "block diagrams" obtained by using a "modular" structure essentially based on the following two blocks:



- POG maintains a direct correspondence between pairs of system variables and real power flows: the product of the two variables involved in each dashed line of the graph has the physical meaning of "power flowing through that section".
- The Elaboration block can store and dissipate/generate energy.
- The Connection block can only "transform" the energy.

# The Traction System of a Automatic Subway



The electrical energy is filtered and stored

The control redistribute the energy

Two DC electrical motors with the armatures in parallel

The mechanical energy is given to the subway

# Traction System: state space equations

Input filter:

$$\begin{cases} L_f \frac{d}{dt} i_f = V_{DC} - R_f i_f - u_f \\ C_f \frac{d}{dt} u_f = i_f - i_{cp} \end{cases}$$

Choppers:

$$\begin{cases} u_1 = u_2 = u_3 = u_f \\ i_{cp} = i_{cp1} + i_{cp2} + i_{cp3} \end{cases}$$

Switching functions:

$$\begin{cases} u_{cp(i)} = m_{cp(i)} u_f \\ i_{cp(i)} = m_{cp(i)} i_{load(i)} \end{cases}$$

Field and armature windings:

$$\begin{aligned} L_{fd(k)} \frac{d}{dt} i_{fd(k)} + R_{fd(k)} i_{fd(k)} &= u_{chop(k)} - e_{fd(k)} \\ L_{arm} \frac{d}{dt} i_{arm} + R_{arm} i_{arm} &= u_{cp2} - e_{arm} \end{aligned}$$

Power links: subway-bogies:

$$\begin{cases} \Omega_{b(k)} = m_{b(k)} v_{sub} \\ F_{b(k)} = m_{b(k)} T_{m(k)} \end{cases}$$

Power links between DC motor and bogies:

$$\begin{cases} T_{m(k)} = k_{dcm(k)} i_{fd(k)} i_{arm} = k_{fd(k)} i_{arm} \\ e_{arm(k)} = k_{dcm(k)} i_{fd(k)} \Omega_{b(k)} = k_{fd(k)} \Omega_{b(k)} \\ e_{arm} = e_{arm1} + e_{arm2} \end{cases}$$

Total force:

$$\begin{cases} v_{b1} = v_{b2} = v_{sub} \\ F_{tot} = F_{b1} + F_{b2} \end{cases}$$

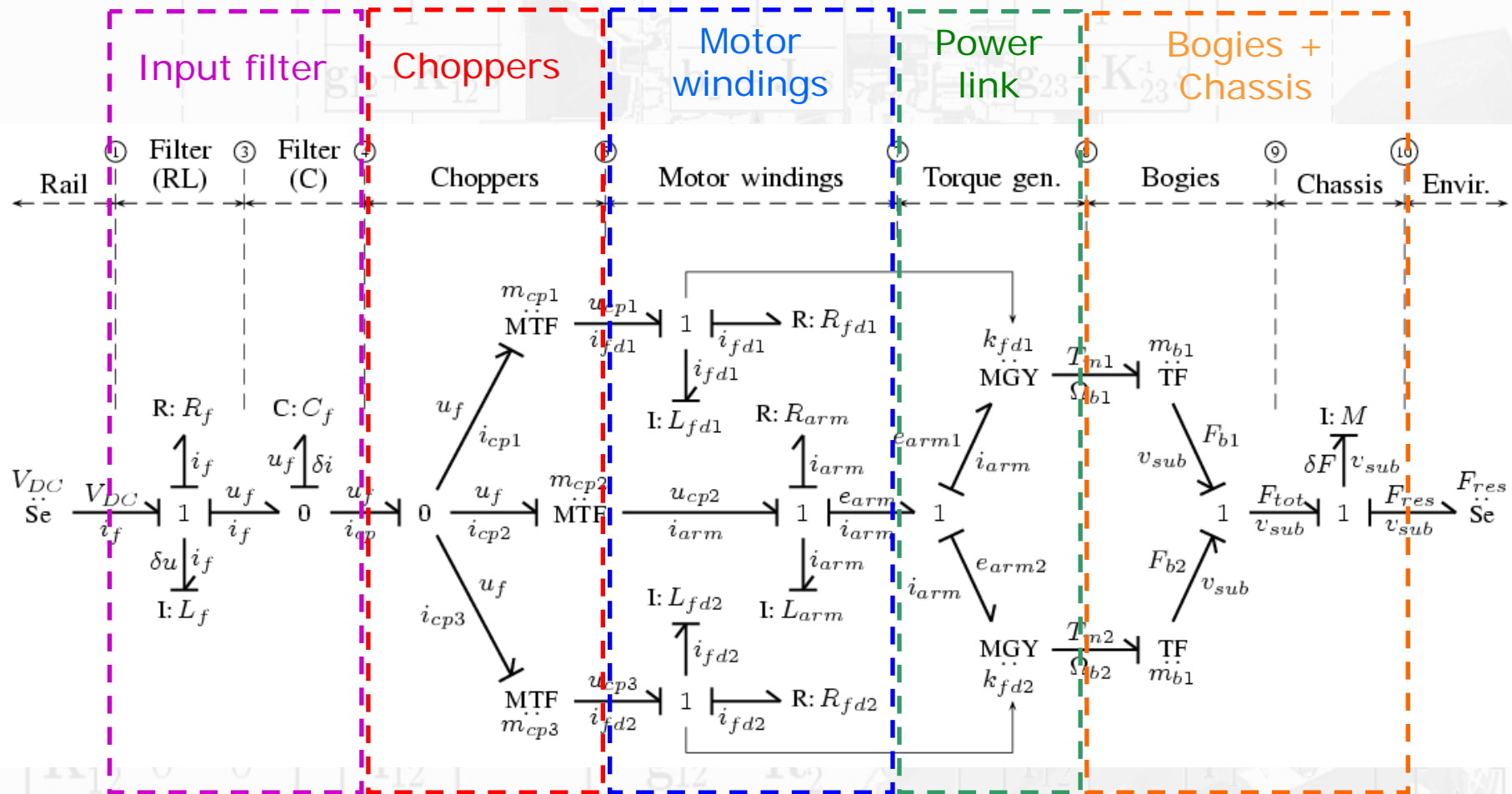
Subways dynamics:

$$M \frac{d}{dt} v_{sub} = F_{tot} - F_{res}$$

Resistive force:

$$F_{res} = F_0 + a_r v_{sub} + b_r v_{sub}^2 + M g \sin \alpha$$

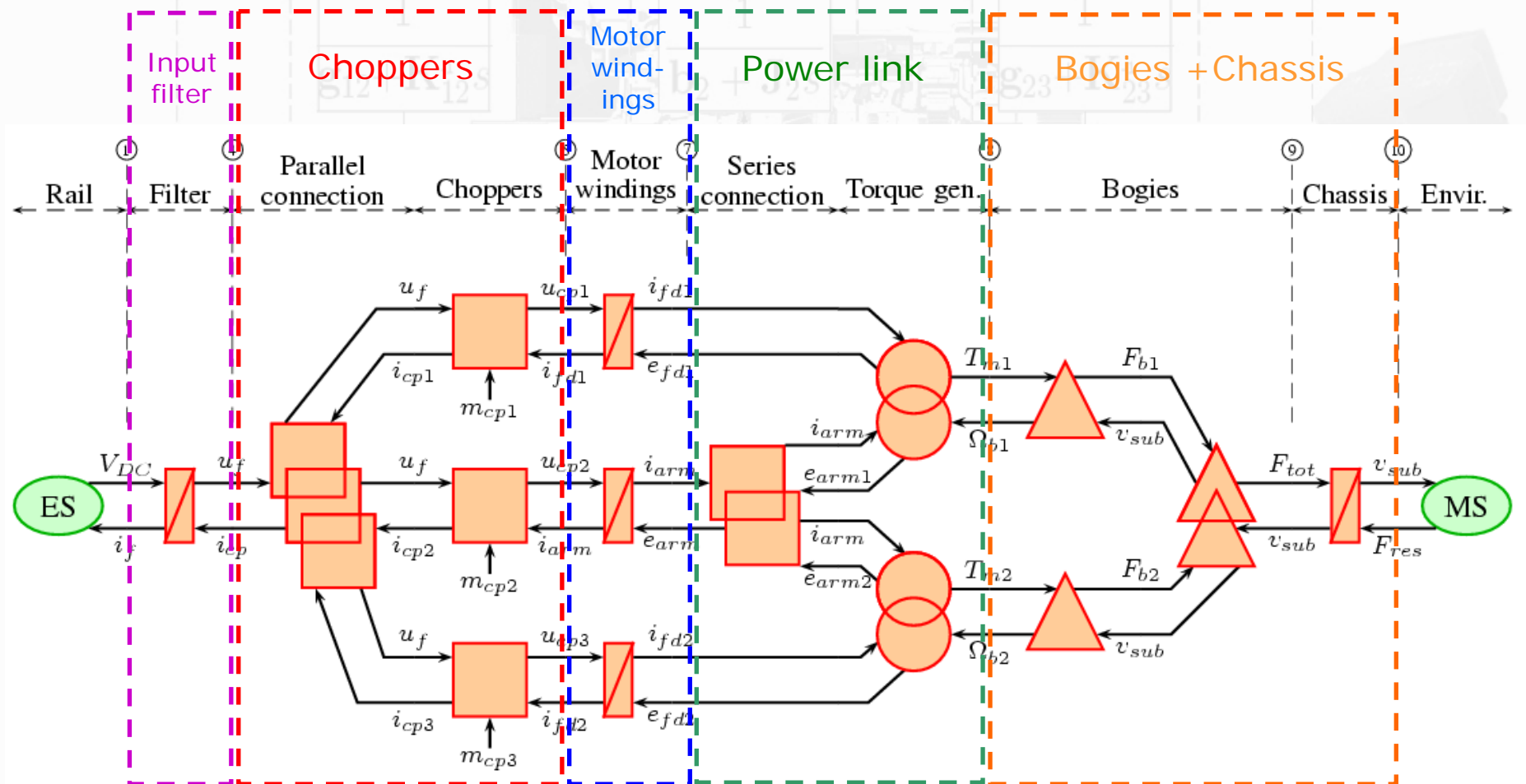
# BG description of the Subway System



- The graphical description is planar
- Each singular element is shown

- Exact mathematical description
- Not easy to read for beginners

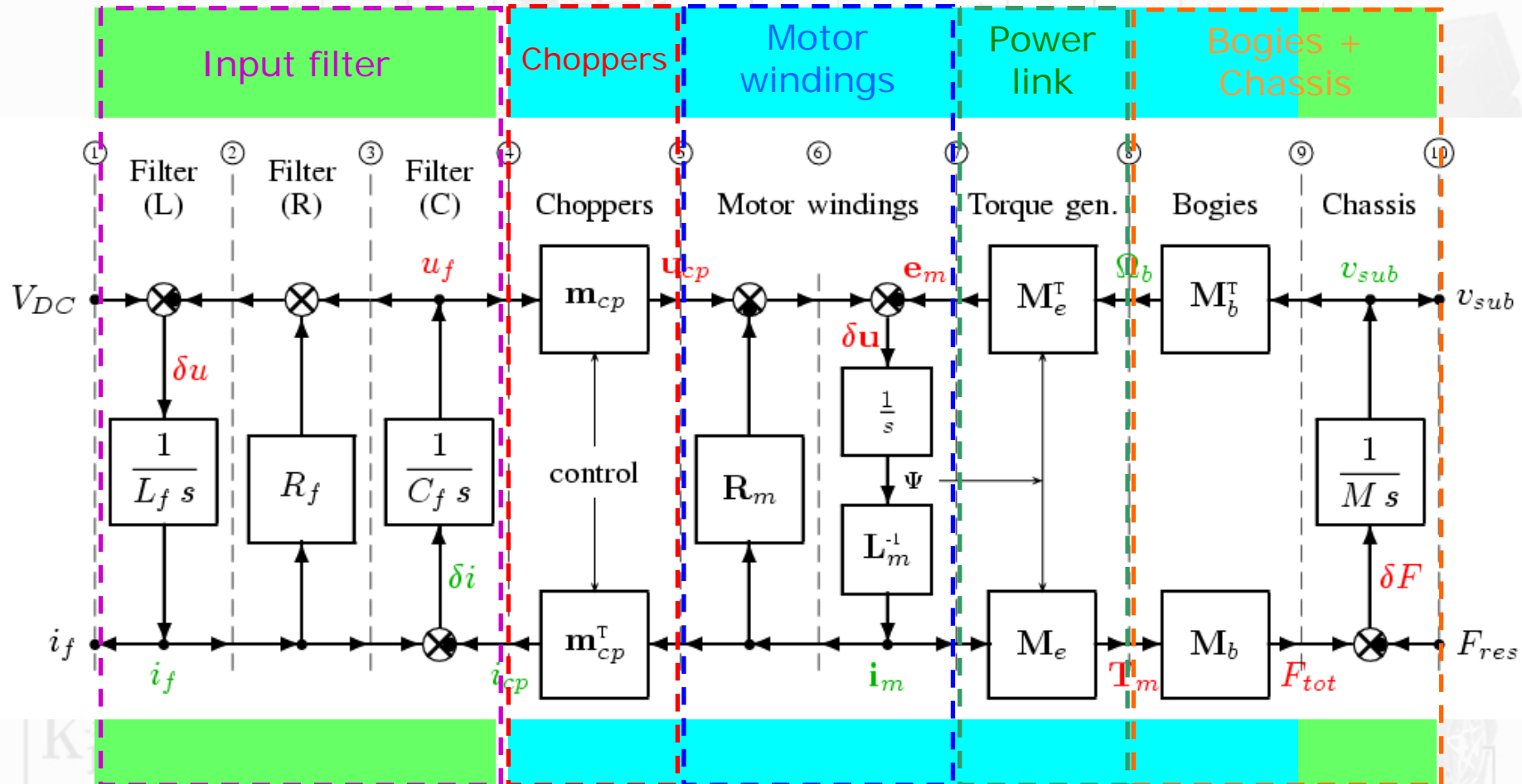
# EMR description of the Subway System



- The physical connections between the energy domains are clearly shown

- The structure is easy to understand  
- No mathematical details

# POG description of the Subway System



- Exact mathematical description
- Easy to read for beginners

- It is scalar and/or vectorial
- Linear graphical description

# POG modelling: matrices and vectors

Switching functions:

$$\mathbf{m}_{cp} = \begin{bmatrix} m_{cp1} \\ m_{cp2} \\ m_{cp3} \end{bmatrix}$$

Chopper voltages:

$$\mathbf{u}_{cp} = \begin{bmatrix} u_{cp1} \\ u_{cp2} \\ u_{cp3} \end{bmatrix}$$

Motor currents:

$$\mathbf{i}_m = \begin{bmatrix} i_{fd1} \\ i_{arm} \\ i_{fd2} \end{bmatrix}$$

DC Motor inductances:

$$\mathbf{L}_m = \begin{bmatrix} L_{fd1} & 0 & 0 \\ 0 & L_{arm} & 0 \\ 0 & 0 & L_{fd1} \end{bmatrix}$$

DC Motor resistances:

$$\mathbf{R}_m = \begin{bmatrix} R_{fd1} & 0 & 0 \\ 0 & R_{arm} & 0 \\ 0 & 0 & R_{fd1} \end{bmatrix}$$

EMF voltages:

$$\mathbf{e}_m = \begin{bmatrix} e_{fd1} \\ e_{arm} \\ e_{fd2} \end{bmatrix}$$

Motors-Bogies power link:

$$\mathbf{M}_e = \begin{bmatrix} 0 & k_{dcm1} & i_{fd1} & 0 \\ 0 & k_{dcm2} & i_{fd2} & 0 \end{bmatrix}$$

Motor torque vector:

$$\mathbf{T}_m = \begin{bmatrix} T_{m1} \\ T_{m2} \end{bmatrix}$$

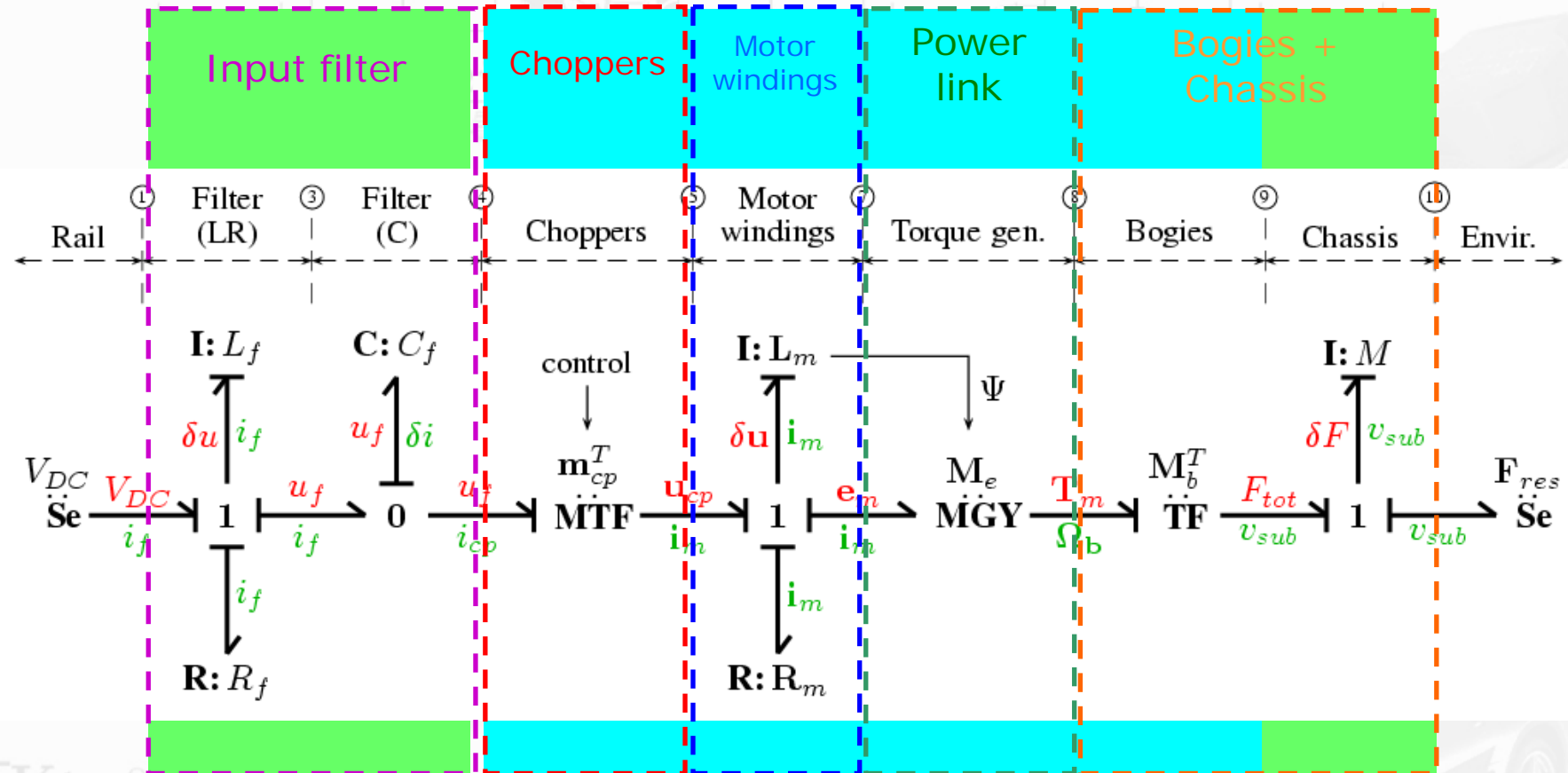
Bogies-Subway power link:

$$\mathbf{M}_b = \begin{bmatrix} m_{b1} & m_{b2} \end{bmatrix}$$

Bogies velocity vector:

$$\mathbf{\Omega}_b = \begin{bmatrix} \Omega_{b1} \\ \Omega_{b2} \end{bmatrix}$$

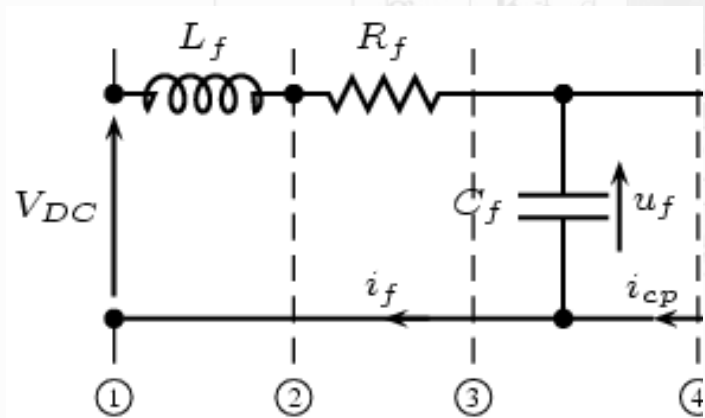
# VBG description of the Subway System



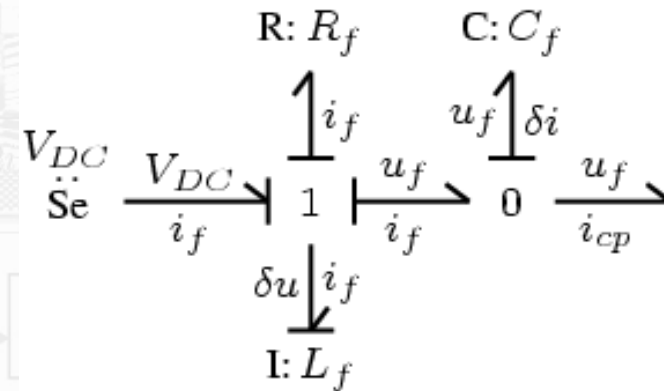
- Exact mathematical description
- The graphs are linear and/or planar
- It is scalar and/or vectorial
- No mixed effort/flow variables

# Comparison: the input filter

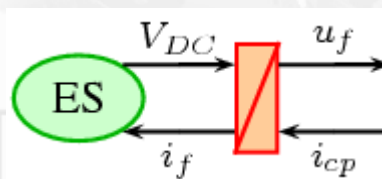
The physical system:



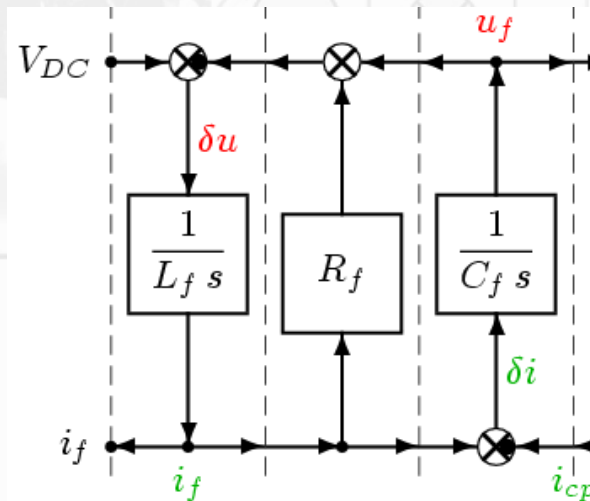
The BG:



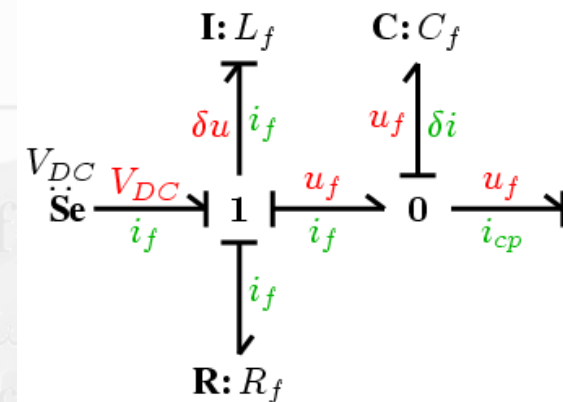
The EMR:



The POG:

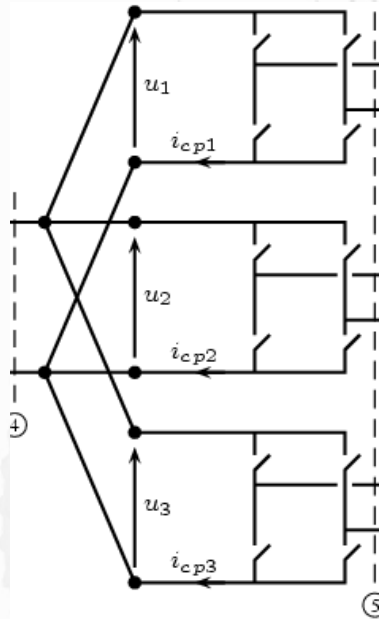


The VBG:

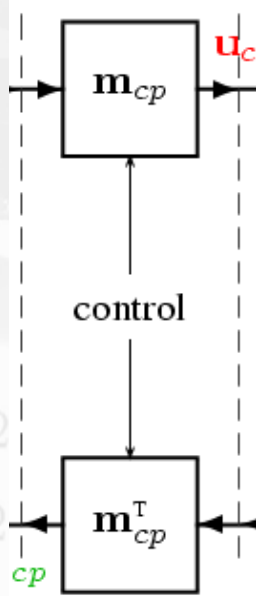


# Comparison: the choppers

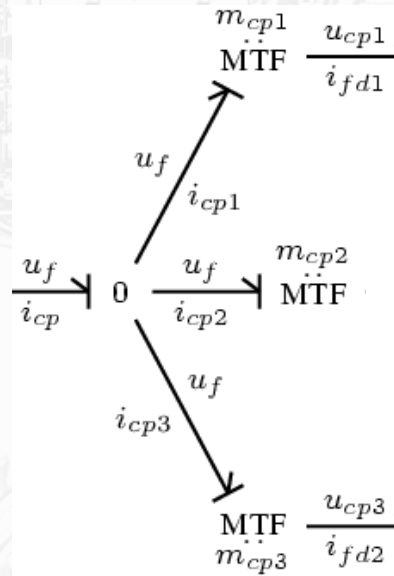
The physical system:



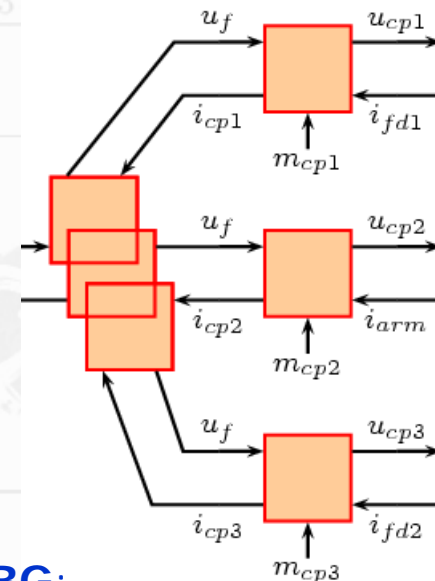
The POG:



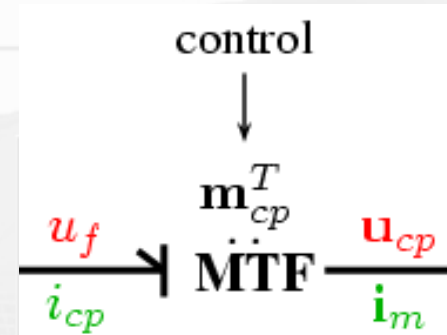
The BG:



The EMR:

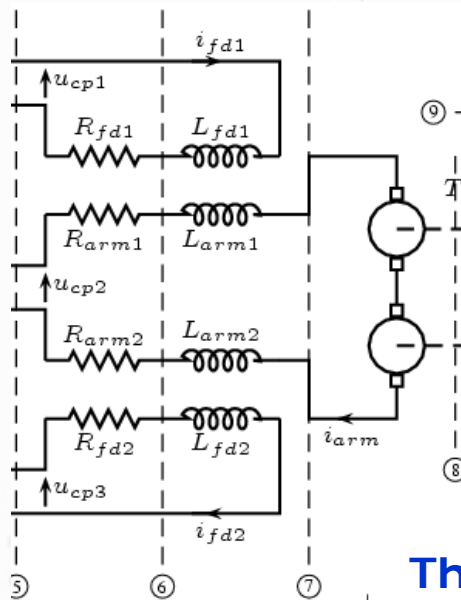


The VBG:

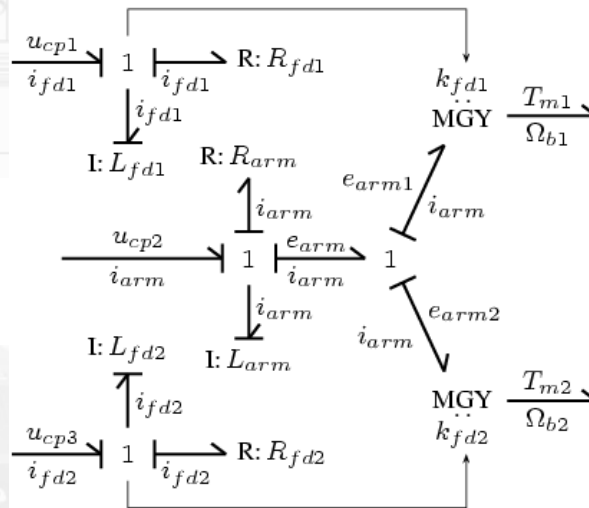


# Comparison: the electrical motors

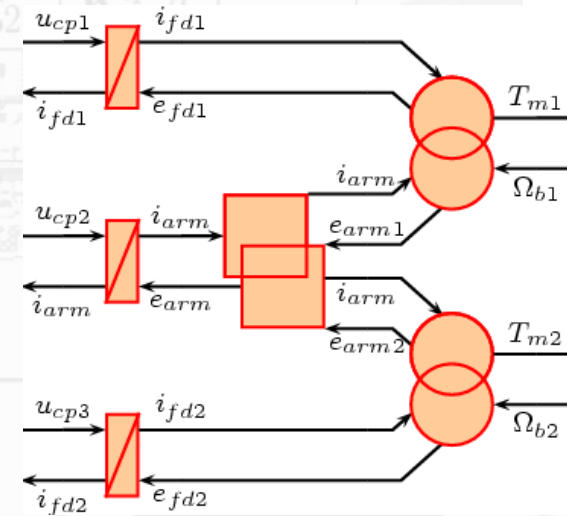
The physical system:



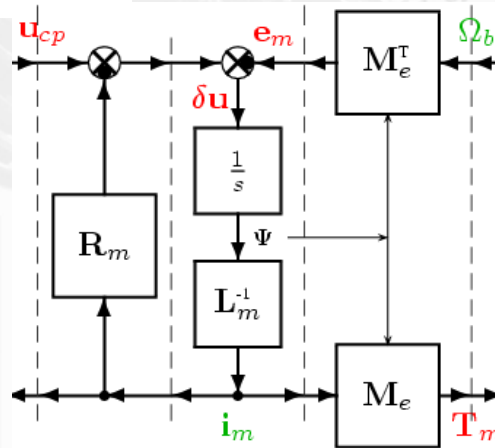
The BG:



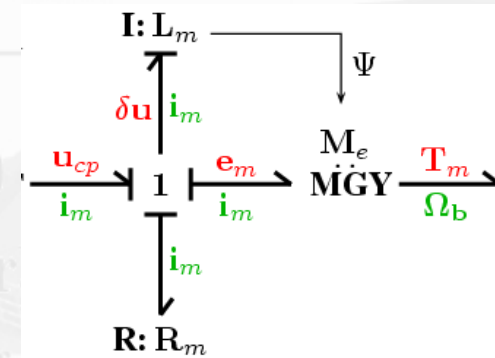
The EMR:



The POG:

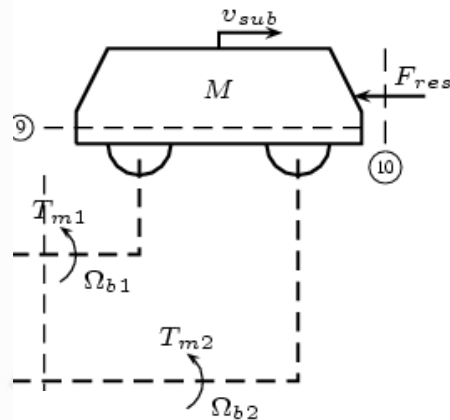


The VBG:

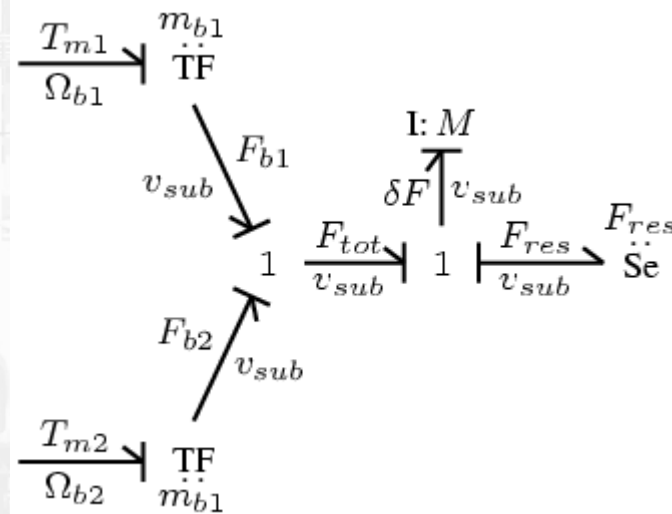


# Comparison: the bogies and the chassis

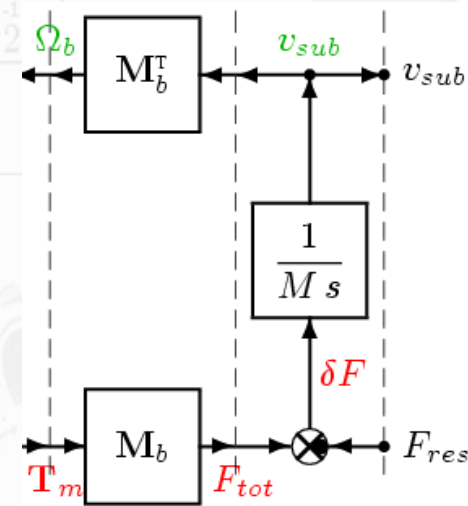
The physical system:



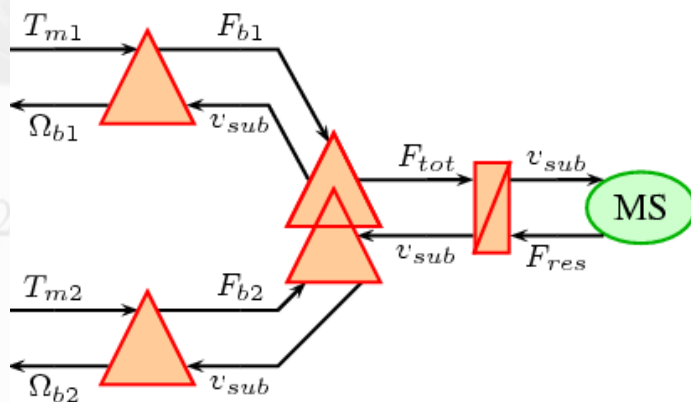
The BG:



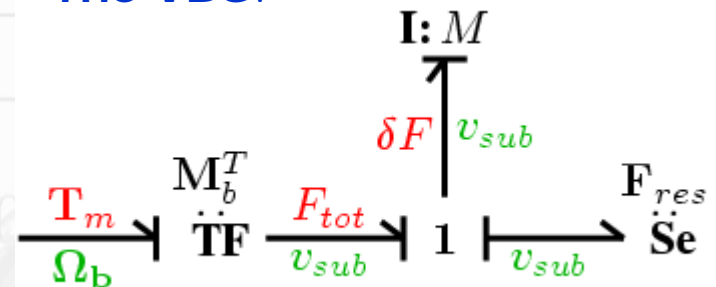
The POG:



The EMR:



The VBG:



# Simulation results

## System parameters:

Input voltage:  $V_{DC} = 750$  V;

Filter parameters:

$C_f = 6$  mF,  $R_f = 0.01$  Ohm,  $L_f = 0.9$  mH;

DC motor parameters:

$L_{fdi} = 0.5$  H,  $R_{fdi} = 2$  Ohm,

$L_{armi} = 0.54$  mH,  $R_{armi} = 0.025$  Ohm;

Torque coefficients:  $k_{dcmi} = 0.077$ ;

Bogie ratios:  $m_{b1} = m_{b2} = 18.12$ ;

Chassis mass:  $M = 15000$  kg;

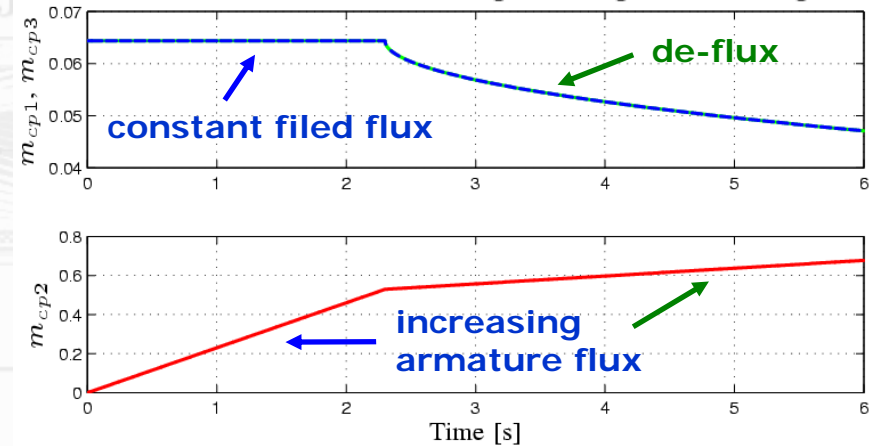
Resistance force parameters:

$F_0 = 1550$  Nm,  $a_r = 30$ ,  $b_r = 4$ ,  $\alpha = 0$ ;

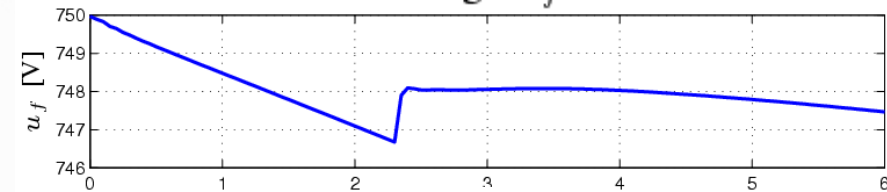
Initial conditions:

$v_{sub}(0) = 750$  V,  $i_{fdi}(0) = 24$  A.

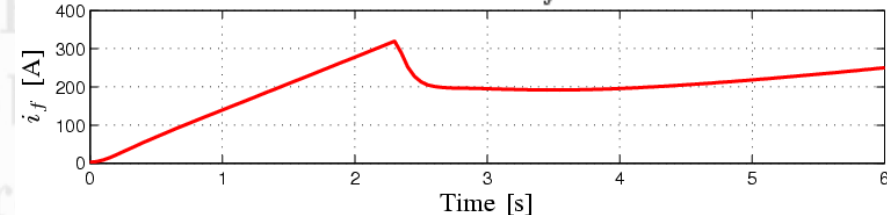
Switching functions  $m_{cp1}$ ,  $m_{cp2}$  and  $m_{cp3}$



Voltage  $u_f$



Current  $i_f$



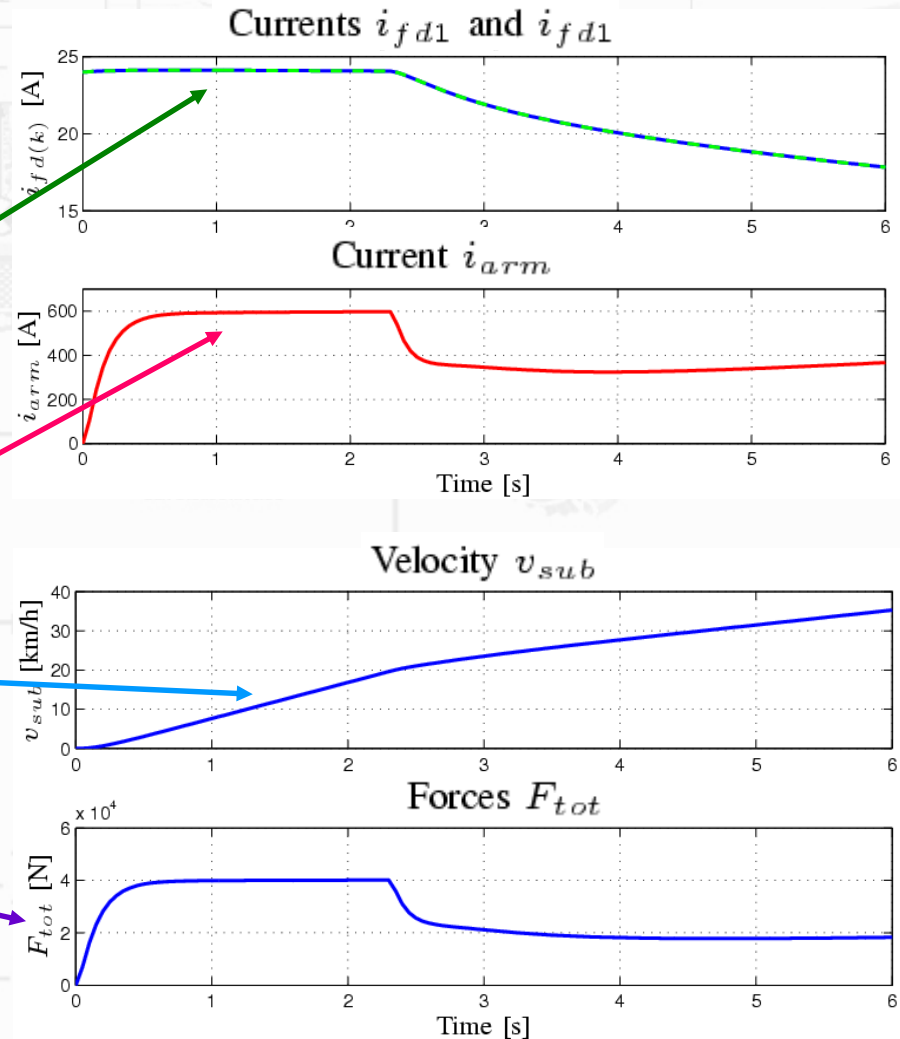
# Simulation results

All the four modelling techniques (BG, EMR, POG, VBG) provides the same simulation results.

The field currents are proportional to the field switching functions.

The armature current becomes constant because of the increasing the f.e.m. due to the rotor velocity which is proportional to the subway velocity.

The total force acting on the subway mass is proportional to its acceleration



# Comparison between **BG**, **EMR** and **POG**

Mnemonic	EMR	POG	BG/VBG
Title	<i>Energetic Macroscopic Representation</i>	<i>Power Oriented Graphs</i>	<i>(Vectorial) Bond Graphs</i>
Author	<i>A. Bouscayrol</i>	<i>R. Zanasi</i>	<i>H. M. Paynter</i>
Year	<i>2000</i>	<i>1991</i>	<i>1959</i>
Energy domains	<i>Electrical - Mechanical (extensible)</i>	<i>All known</i>	<i>All known</i>
Power variables	<i>Scalar or vectorial</i>	<i>Scalar or vectorial</i>	<i>Scalar or vectorial</i>
Causality	<i>Exclusive integral</i>	<i>Integral (preferably) or differential</i>	<i>Integral (preferably) or differential</i>
Basic elements	<i>9 Electrical - Mechanical</i>	<i>2</i>	<i>8</i>

# Comparison between **BG**, **EMR** and **POG**

Mnemonic	EMR	POG	BG/VBG
Visibility of both directions	<i>graphically visible</i>	<i>graphically visible</i>	<i>not graphically visible</i>
Assistance for the control	<i>Causal Ordering Graph (J. P. Hautier)</i>	<i>none</i>	<i>none</i>
Reference direction for power flow	<i>no</i>	<i>yes (Implicitly)</i>	<i>yes</i>
Displacement / momentum explicitly	<i>no</i>	<i>yes</i>	<i>yes</i>
Mathematical model from graphical description	<i>partially obtainable</i>	<i>directly obtainable (explicit in the graph)</i>	<i>directly obtainable (implicit in the graph)</i>
Simulink library	<i>icon library</i>	<i>none</i>	<i>add-on block library BG V.2.1</i>
Usage hints	<i>user defined subsystems always</i>	<i>standard blocks</i>	<i>blocks and editor as usual</i>
Main objective	<i>Simulation and control</i>	<i>Simulation and analysis</i>	<i>Simulation and design</i>

# Conclusions

1) All the four Energetic Graphical Techniques:

- Bond-Graph (**BG**)
- Energetic Macroscopic Representation (**EMR**)
- Power-Oriented Graphs (**POG**)
- Vectorial Bond-Graph (**VBG**)

are very good for modelling physical systems

2) Each technique has its own advantages and limitations

*A final personal hope:* the development of a joined program that automatically converts the graphical schemes from one energetic technique to the others.