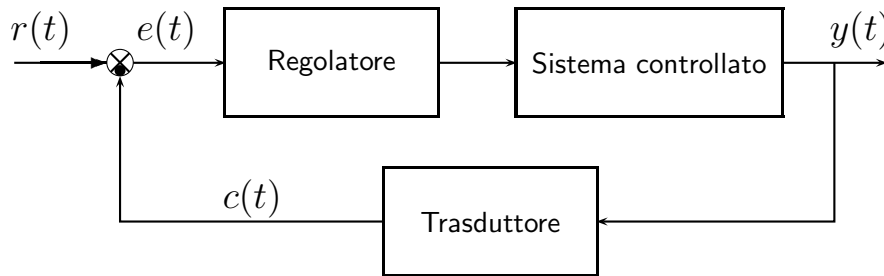


## Standard PID regulators

- Block diagram of a feedback control:



- In the control of many industrial processes the dynamic characteristics of the controlled systems can vary within wide limits: it is economically convenient to unify the control devices.
- Typically, standard control apparatuses are used, but provided with correction devices with adjustable parameters within wide limits, so that they can be adapted to the particular regulation system in which they are inserted.
- Place  $G_c(s) = M(s)/E(s)$ , we distinguish the following standard types:

1. *Proportional regulator (P)* :

$$G_c(s) = K_p ;$$

2. *Integral regulator (I)* :

$$G_c(s) = \frac{K_p}{T_i s} ;$$

3. *Proportional-integral regulator (PI)* :

$$G_c(s) = K_p \left( 1 + \frac{1}{T_i s} \right) ;$$

4. *Proportional-derivative regulator (PD)* :

$$G_c(s) = K_p (1 + T_d s) ;$$

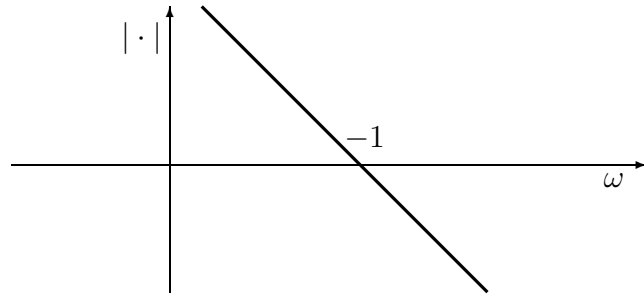
5. *Proportional-integral-derivative regulator (PID)* :

$$G_c(s) = K_p \left( 1 + T_d s + \frac{1}{T_i s} \right) .$$

The constant  $K_p$  is called *sensitivity to proportional* /,  $T_d$  *derivative time constant* /,  $T_i$  *time constant of the integral action* /.

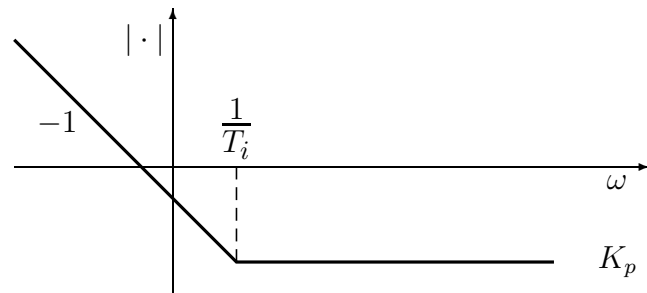
- Diagramma di Bode dei moduli di un regolatore standard I:

$$G(s) = \frac{K_p}{T_i s}$$



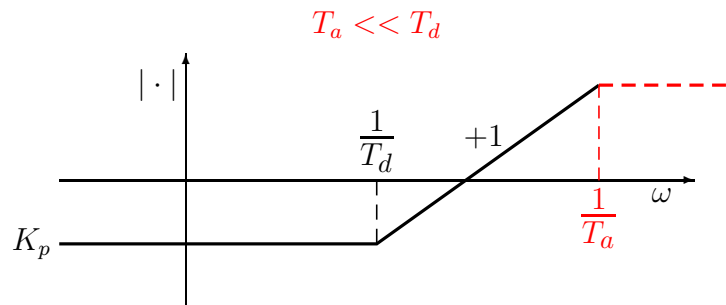
- Diagramma di Bode dei moduli di un regolatore standard PI:

$$\begin{aligned} G(s) &= K_p \left( 1 + \frac{1}{T_i s} \right) \\ &= \frac{K_p (T_i s + 1)}{T_i s} \end{aligned}$$



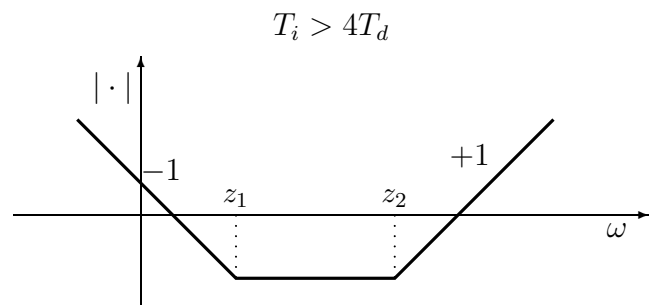
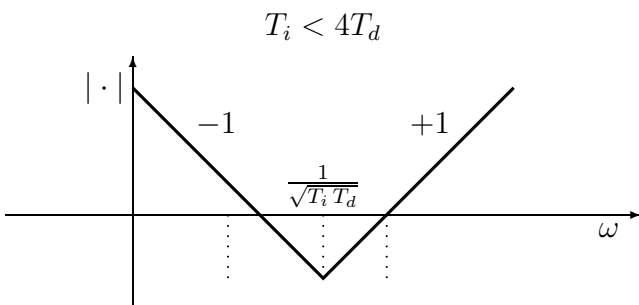
- Diagramma di Bode dei moduli di un regolatore standard PD:

$$\begin{aligned} G(s) &\simeq K_p (1 + T_d s) \\ &= \frac{K_p (1 + T_d s)}{(1 + T_a s)} \end{aligned}$$



- Diagramma di Bode dei moduli di un regolatore standard PID:

$$G(s) = K_p \left( 1 + \frac{1}{T_i s} + T_d s \right) = \frac{K_p (T_i T_d s^2 + T_i s + 1)}{T_i s} = \frac{K_p T_d (s - z_1)(s - z_2)}{s}$$

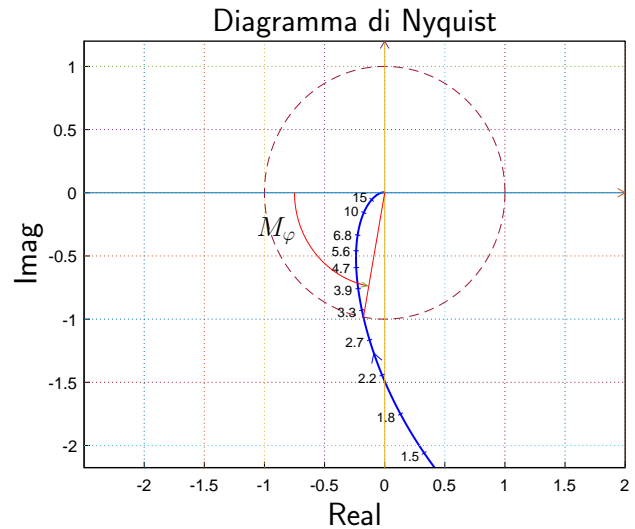
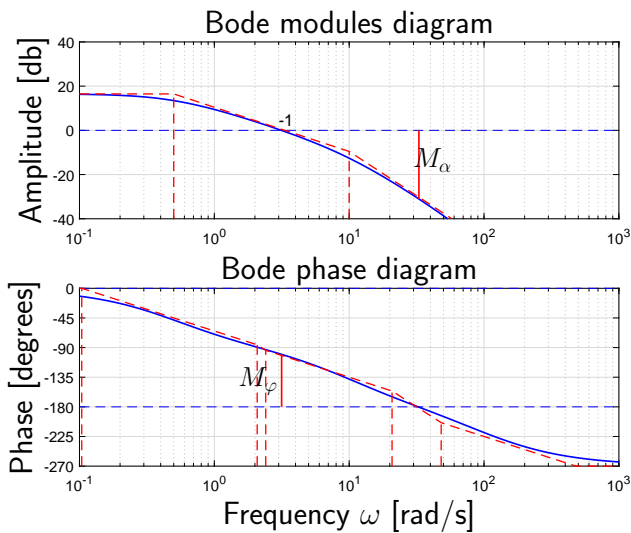


- Per sistemi a fase minima vale questa regola qualitativa: la pendenza “ $p$ ” con cui il diagramma di Bode dei moduli interseca la retta a guadagno unitario fornisce una “stima” del margine di fase  $M_\varphi$  del sistema stesso:

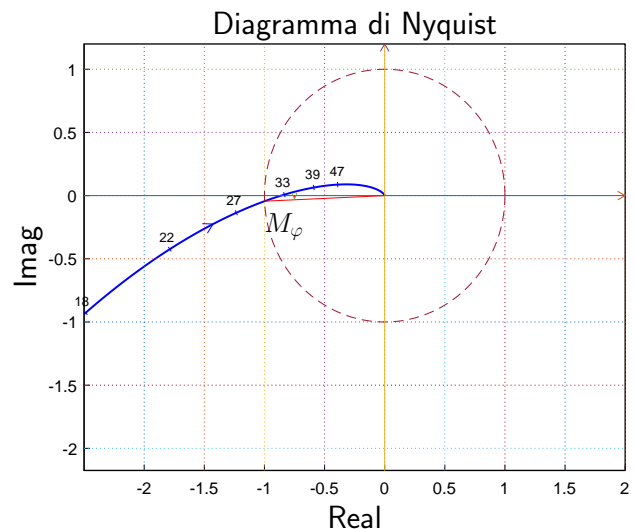
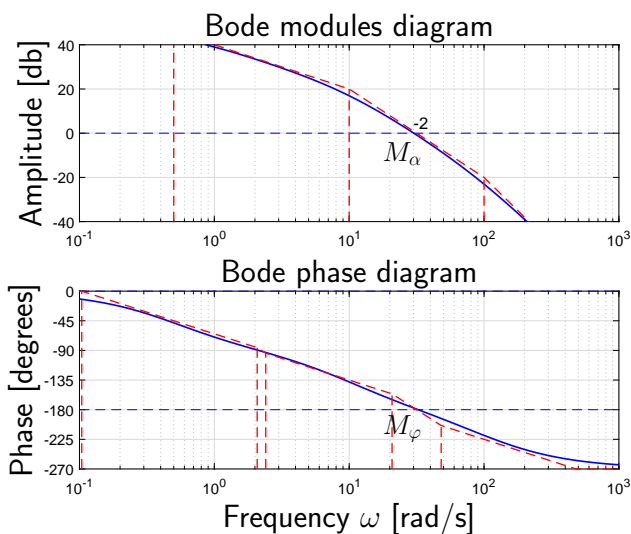
$$M_\varphi = (2 + p) \frac{\pi}{2}$$

Questa regola “qualitativa” si basa sull’utilizzo della formula di Bode.

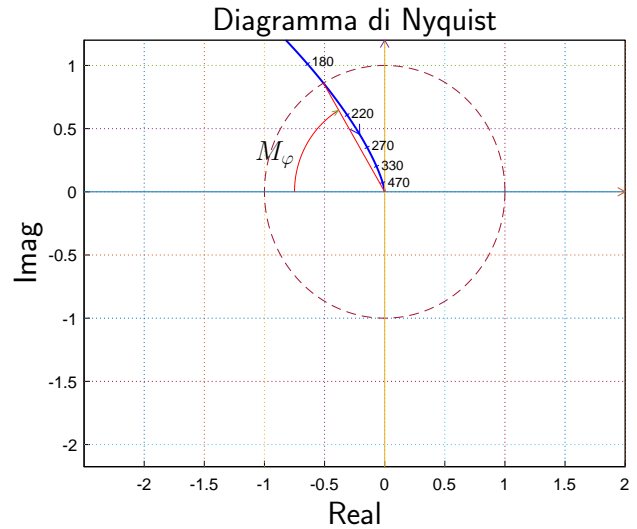
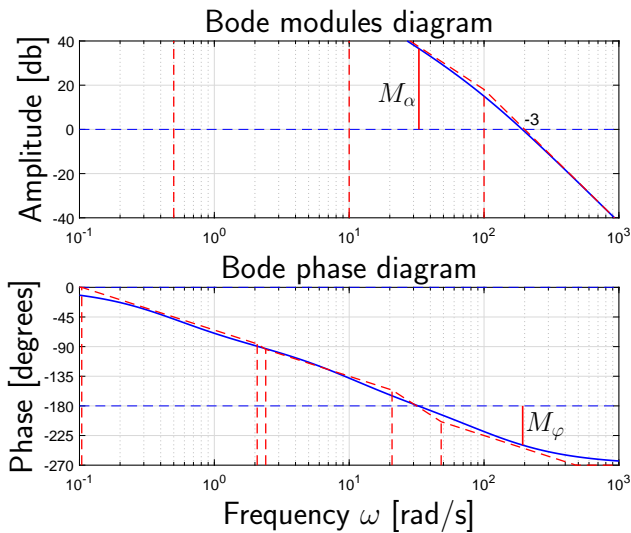
- Affinché un sistema  $G(s)$  retroazionato sia stabile con un ampio margine si fase é bene che il diagramma di Bode della funzione  $G(s)$  intersechi la retta a guadagno unitario con pendenza  $p \simeq -1$ .



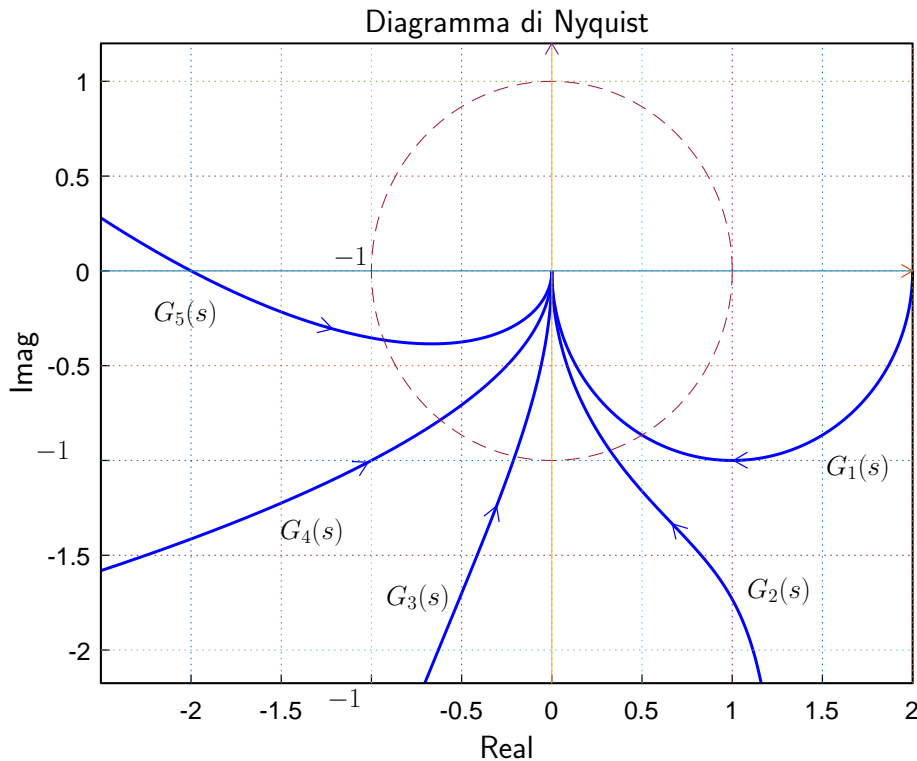
- Se il diagramma di Bode di una funzione  $G(s)$  interseca la retta a guadagno unitario con pendenza  $p \simeq -2$ , il corrispondente sistema retroazionato sarà instabile o molto vicino all’instabilità.



- Se il diagramma di Bode di una funzione  $G(s)$  interseca la retta a guadagno unitario con pendenza  $p < -2$ , il corrispondente sistema retroazionato sarà sicuramente instabile.



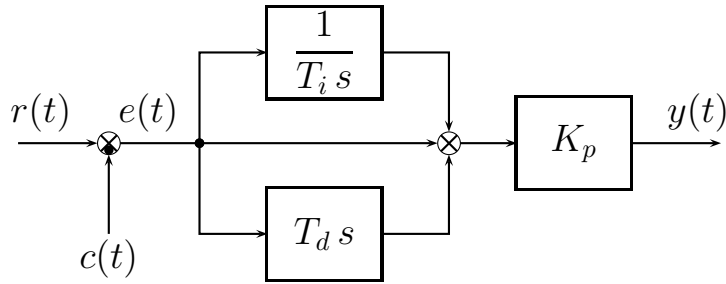
- Tutti i sistemi  $G(s)$  a fase minima e con grado relativo  $r = 1$  possono essere efficacemente stabilizzati in retroazione, utilizzando un guadagno  $K$  sufficientemente elevato.



Funzioni  $G(s)$  a grado relativo  $r = 1$  visualizzate sul diagramma di Nyquist:

$$G_1(s) = \frac{20}{s + 10}, \quad G_2(s) = \frac{6(s + 1)}{s(s + 3)}, \quad G_3(s) = \frac{16(s + 2)}{s(s + 1)}, \quad G_4(s) = \frac{(s + 1)}{s^2}, \quad G_5(s) = \frac{(s + 1)^2}{s^3}$$

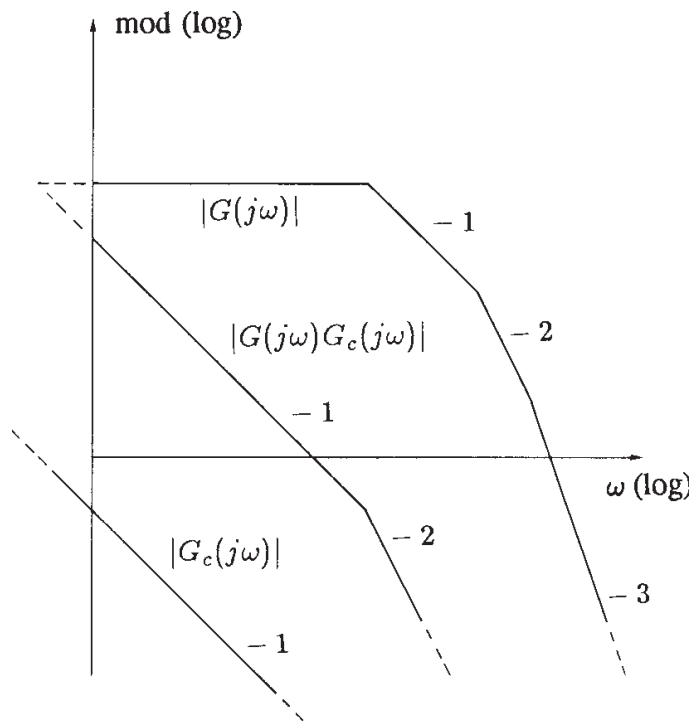
- General structure of the PID regulator:



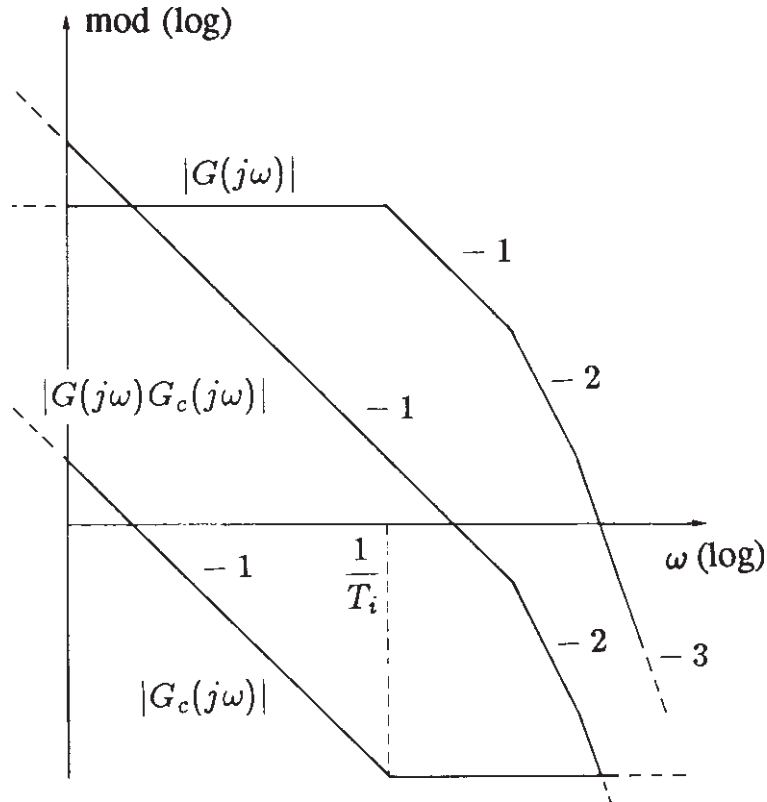
$$G_c(s) = K_p \left( 1 + T_d s + \frac{1}{T_i s} \right)$$

For what concerns the relative advantages and the particularity of use of the various types of regulators previously mentioned, similar considerations apply to those developed for the corrector networks.

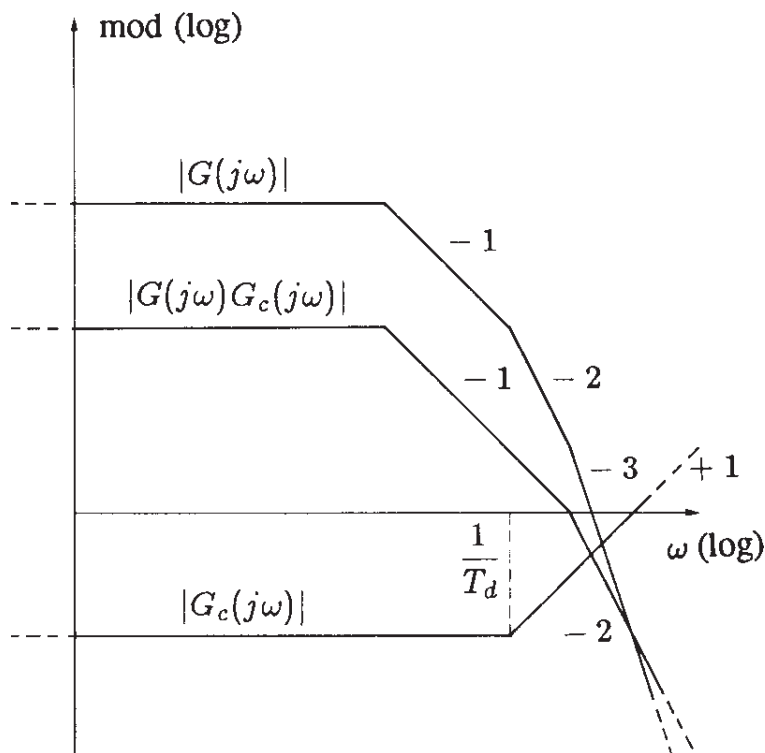
- The  $P$  regulator is used when the process allows a high ring gain constant without compromising stability, such as systems having the dynamic behavior of an integrator (for example controlling a level by varying a flow) or characterized by the presence of a single predominant time constant.
- The  $I$  regulator is used for systems of type 0 difficult to stabilize and for systems with dominant ending delays: in this last case a  $P$  regulator can not be used because it corresponds to an error at unacceptable regime, while an integral control guarantees a stable behavior with null static error.



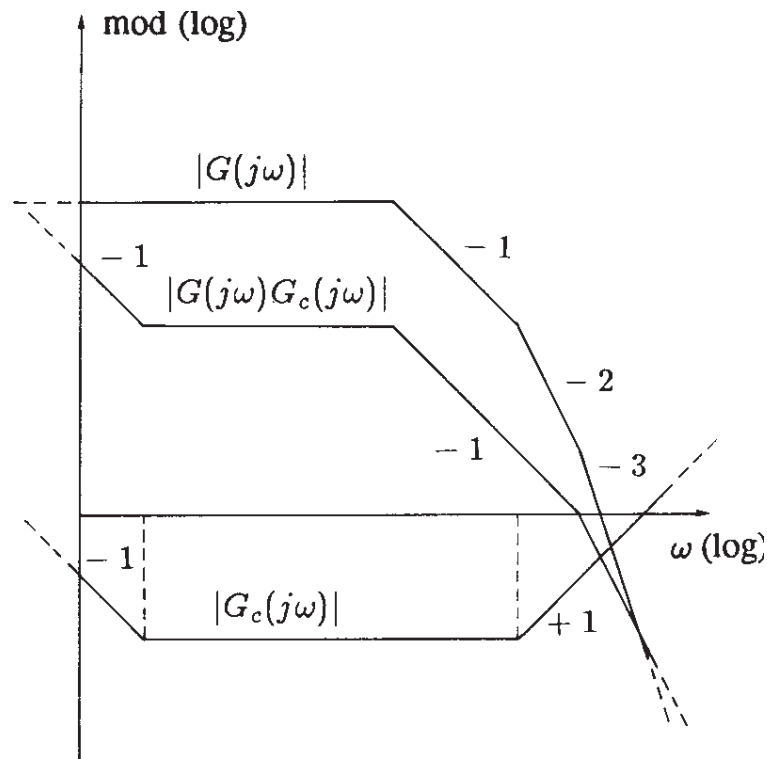
- The  $PI$  regulator compared to the  $I$  regulator allows to keep a greater bandwidth and therefore a greater promptness of response.



- The  $PD$  regulator is used instead for intrinsically type 1 systems or for type 0 systems to improve the response speed: its intervention is completely analogous to that of an anticipating network.



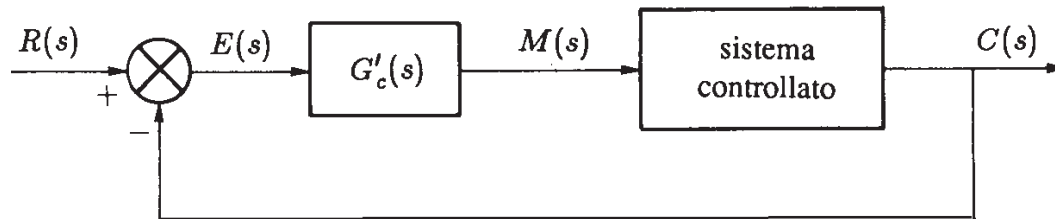
- The *PID* regulator can be used, as an alternative to the *PD*, for type 0 systems and has the advantage of allowing, in addition to a good response speed, also a null static error.



The triple action regulator is therefore the most general: by appropriately choosing the values of the three parameters that characterize its dynamic behavior, it is possible to obtain, as special cases, the actions of all the types of regulators previously taken into consideration.

## Calibration of standard regulators

Many industrial systems (eg chemical and petrochemical) are characterized by strongly nonlinear models. In these cases the choice of the parameters of the regulators is carried out in many cases with semiempirical methods.

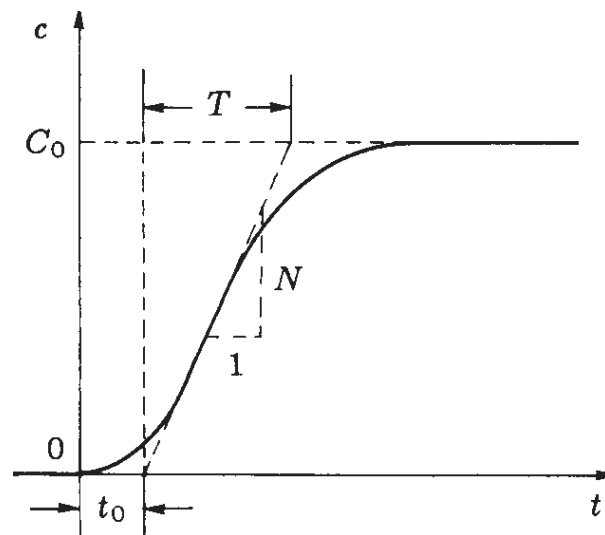


### Ziegler-Nichols method

It provides the values of the first attempt of the parameters of the regulator according to some parameters of the step response (often aperiodic) of the controlled system.

The physical system can be approximated with a system of the first order:

$$G(s) \simeq \frac{K e^{-t_0 s}}{1 + T s}$$



The values of the characteristic parameters are obtained from the curve of response to the step at the inflection point:

$t_0$ : time lag;

$T$ : constant of time;

$R = t_0/T = N t_0/C_0$ : delay report;

$N = C_0/T$ : response speed;

$K = C_0/M_0$ : static gain ( $M_0$  is the width of the step applied).

• Place  $G'_c(s) = M(s)/E(s)$ , the parameter values recommended by Ziegler and Nichols, modified by Cohen and Coon, are:

1.  $P$  control :

$$G'_c(s) = K'_p, \quad \text{con} \quad K'_p = \frac{M_0}{N t_0} \left(1 + \frac{R}{3}\right);$$

2. Check  $I$ :

$$G'_c(s) = \frac{K'_p}{T_i s}, \quad \text{con} \quad \frac{K'_p}{T_i} = \frac{4 M_0}{N t_0^2} \frac{R^2}{1 + 5 R};$$

3. Control  $PI$ :

$$G'_c(s) = K'_p \left(1 + \frac{1}{T_i s}\right), \quad \text{con}$$

$$K'_p = \frac{M_0}{N t_0} \left(\frac{9}{10} + \frac{R}{12}\right), \quad T_i = t_0 \frac{30 + 3 R}{9 + 20 R};$$

4. Check  $PD$ :

$$G'_c(s) = K'_p (1 + T_d s), \quad \text{con}$$

$$K'_p = \frac{M_0}{N t_0} \left(\frac{5}{4} + \frac{R}{6}\right), \quad T_d = t_0 \frac{6 - 2 R}{22 + 3 R};$$

5. Controllo  $PID$ :

$$G'_c(s) = K'_p \left(1 + \frac{1}{T_i s} + T_d s\right), \quad \text{con}$$

$$K'_p = \frac{M_0}{N t_0} \left(\frac{4}{3} + \frac{R}{4}\right), \quad T_i = t_0 \frac{32 + 6 R}{13 + 8 R}, \quad T_d = t_0 \frac{4}{11 + 2 R}.$$

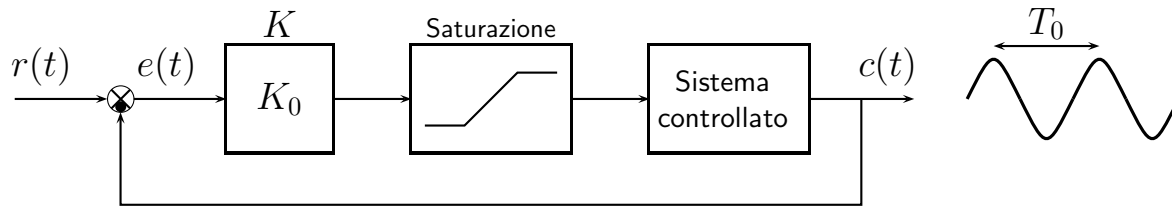
• I valori  $K'_p$ ,  $T_i$  e  $T_d$  proposti da queste tabelle sono da considerarsi “*valori di primo tentativo*” e non “*valori ottimali*”.

• L'utilizzo di queste tabelle é sempre seguito da una “*taratura sul campo*” durante la quale i singoli parametri del controllore PID vengono leggermente modificati rispetto ai valori forniti dalle tabelle e si verifica “*sul campo*” se queste modifiche determinano o meno un miglioramento del comportamento dinamico del sistema retroazionato.

• In letteratura é possibile trovare molti modi diversi di tarare in modo qualitativo i parametri di un regolatore PID. Se la funzione di trasferimento  $G(s)$  del sistema controllato non é nota, tutti questi metodi sono da considerarsi “*qualitativi*” e non necessariamente “*sicuri*” per quanto riguarda la stabilitá del sistema retroazionato.

## Proportional pendulum band method

- La banda proporzionale di pendolazione è definita come il valore della banda proporzionale  $1/K_0$  che, in assenza delle azioni integrale e derivativa, porta il sistema di regolazione in condizione di stabilità limite, cioè in oscillazione permanente.
- Il valore del guadagno limite  $K_0$  si determina sperimentalmente utilizzando il seguente schema retroaziante:



- Il valore del guadagno  $K$  viene aumentato fino al valore limite  $K_0$  in corrispondenza del quale il segnale di uscita  $c(t)$  inizia a oscillare in modo periodico con periodo  $T_0$ . La presenza della saturazione evita che il sistema retroazionato diverga se il valore limite  $K_0$  del guadagno  $K$  viene superato.
- Determinati i valori  $K_0$  e  $T_0$ , per la taratura del regolatore PID é possibile utilizzare le seguenti formule, pure dovute a Ziegler e Nichols:
  1. Check  $P$ :  $K'_p = 0.5 K_0$ ;
  2. Control  $PI$ :  $K'_p = 0.45 K_0$ ,  $T_i = 0.85 T_0$ ;
  3. Check  $PD$ :  $K'_p = 0.5 K_0$ ,  $T_d = 0.2 T_0$ ;
  4.  $PID$  check:  $K'_p = 0.6 K_0$ ,  $T_i = 0.5 T_0$ ,  $T_d = 0.12 T_0$ .
- Se fosse nota la funzione di trasferimento  $G(s)$  del “Sistema controllato”, i valori  $K_0$  e  $T_0$  potrebbero essere calcolati esattamente nel seguente modo:

$$K_0 = K^*, \quad T_0 = \frac{2\pi}{\omega^*}$$

dove  $K^*$  e  $\omega^*$  sono, rispettivamente, il guadagno limite e la pulsazione limite che si ottengono applicando il criterio di Routh alla funzione  $G(s)$ .

- La prova sperimentale descritta sopra ha proprio come obiettivo quello di determinare la posizione  $-1/K_0$  e la frequenza  $\omega^* = 2\pi/T_0$  in corrispondenza della quale la funzione  $G(j\omega)$  interseca il semiasse reale negativo.