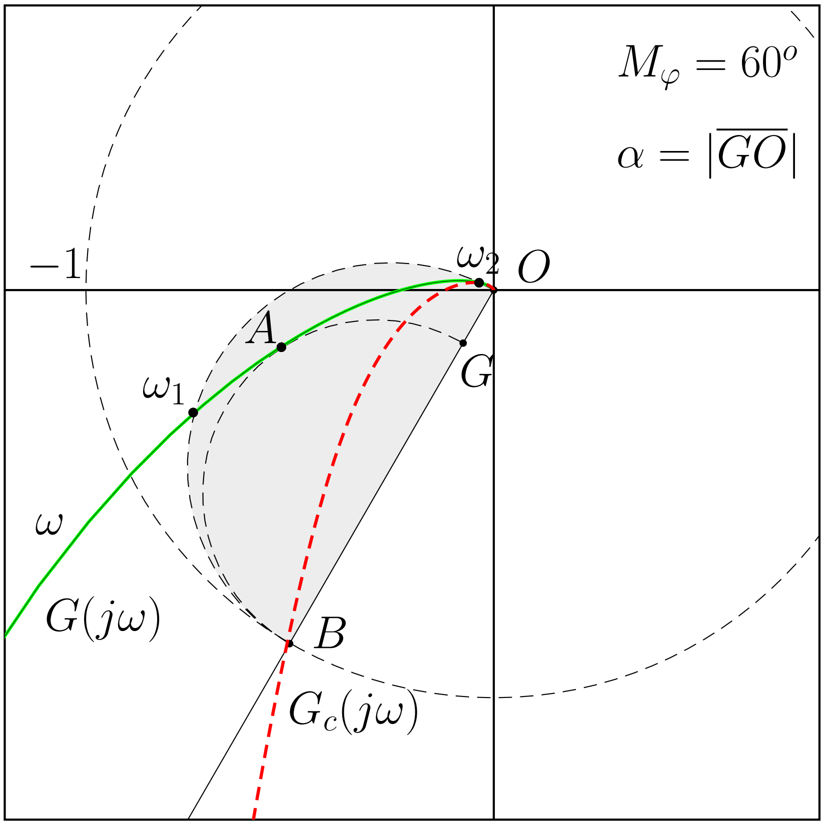
Lead/lag networks: design on the Nyquist plane

a) Design of a lead network. Specification on the phase margin $M_\varphi = 60^o$.



- System $G(s)$ and lead network $C(s)$:

$$G(s) = \frac{25}{s(s+1)(s+10)}, \qquad \rightarrow \qquad C(s) = \frac{(1+0.806s)}{(1+0.117s)}$$

- Point A:

$$A = G(j\omega_A) = M_A e^{j\varphi_A}, \qquad \rightarrow \qquad M_A = 0.538 \qquad \varphi_A = 194.9^o$$

can be moved in B:

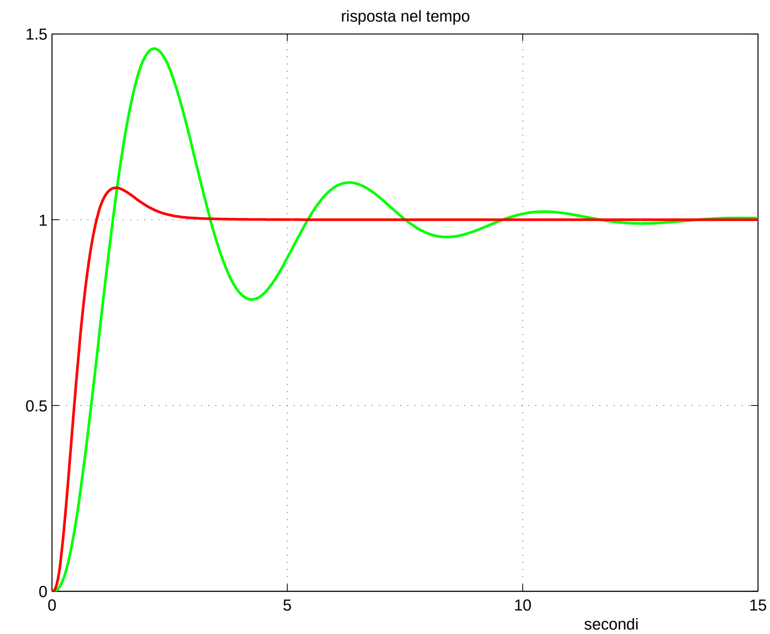
$$B = e^{j(\pi + M_\varphi)} \qquad \rightarrow \qquad M_B = 1, \qquad \varphi_B = \pi + M_\varphi = 240^o$$

if the lead network amplifies M and introduces a phase shift φ as follows:

$$M = \frac{M_B}{M_A} = \frac{1}{M_A} = 1.8587, \qquad \varphi = \varphi_B - \varphi_A = 45.1^o$$

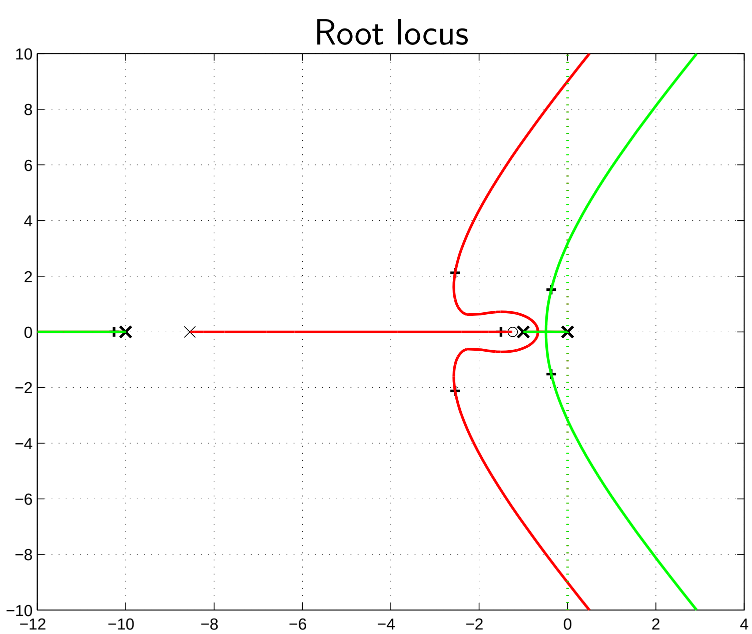
- Substituting the parameters M, φ and $\omega_A = 2.02$ in the inversion formulas, one obtains the parameters: $\tau_1 = 0.806$ and $\tau_2 = 0.117$.

- Time responses of systems $G(s)$ and $C(s)G(s)$:



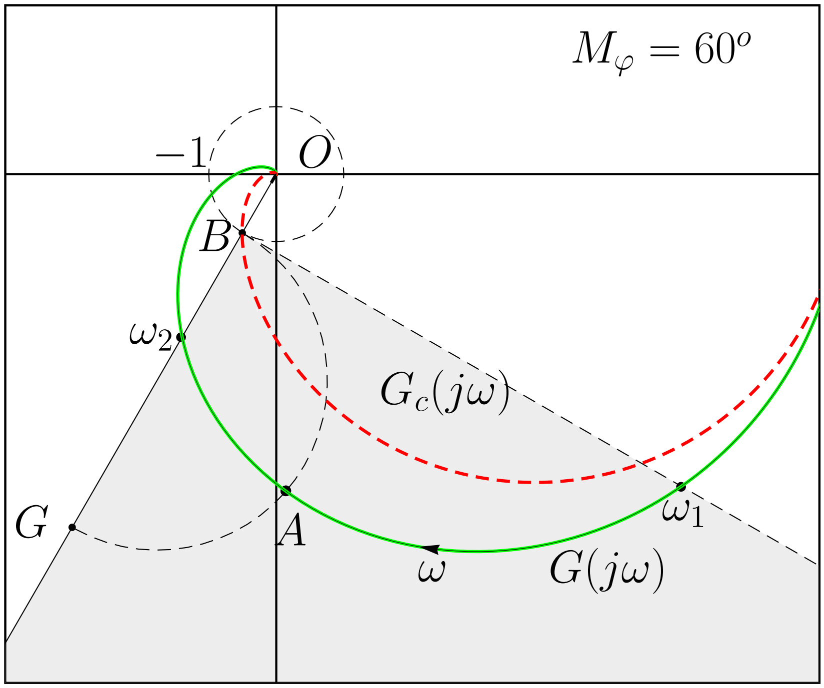
The use of a lead network has improved both the transient (the maximum overshoot is decreased) and the speed of the system (the rise time is lower).

- The root locus of the two systems:



The presence of the lead network has moved the dominant poles of the feedback system towards the negative half-plane.

b) Design of a lag network. Specification on phase margin: $M_\varphi = 60^o$.



- System $G(s)$ and lag network $C(s)$:

$$G(s) = \frac{5000}{(s+1)(s+2)(s+10)(s+30)}, \qquad \to \qquad C(s) = \frac{(1+1.04s)}{(1+6.25s)}$$

- Point A:

$$A = G(j\omega_A) = M_A e^{j\varphi_A}, \quad \to \quad M_A = 4.672, \quad \varphi_A = 271.82^o$$

can be moved in B:

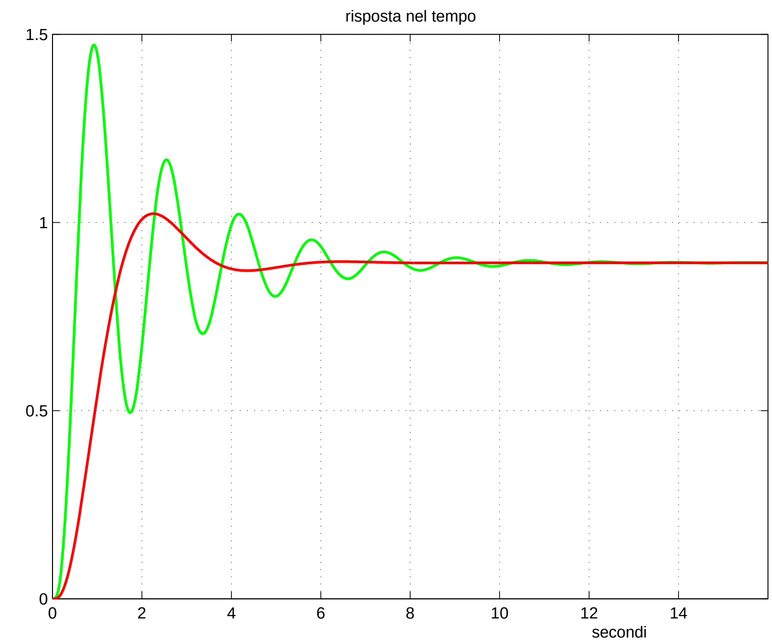
$$B = e^{j(\pi + M_\varphi)} \qquad \to \qquad M_B = 1, \qquad \varphi_B = \pi + M_\varphi = 240^o$$

if the lag network amplifies M and shifts the phase φ as follows:

$$M = \frac{M_B}{M_A} = \frac{1}{M_A} = 0.214, \qquad \varphi = \varphi_B - \varphi_A = -31.82^o$$

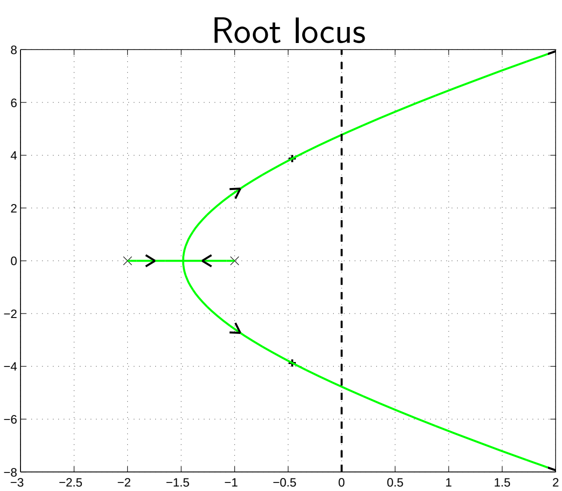
- Substituting the parameters M, φ and $\omega_A = 2.02$ in the inversion formulas, one obtains the parameters: $\tau_1 = 1.04$ and $\tau_2 = 6.25$.

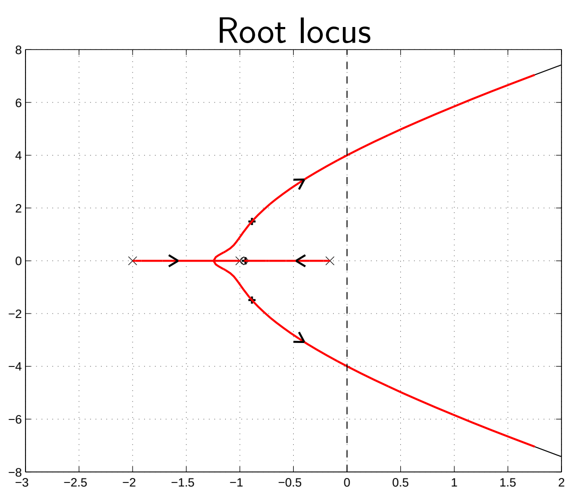
- Time responses of systems $G(s)$ and $C(s)G(s)$:



The use of a lag network has improved the transient by decreasing the maximum overshoot, but the speed of the system is lower (the rise time is higher).

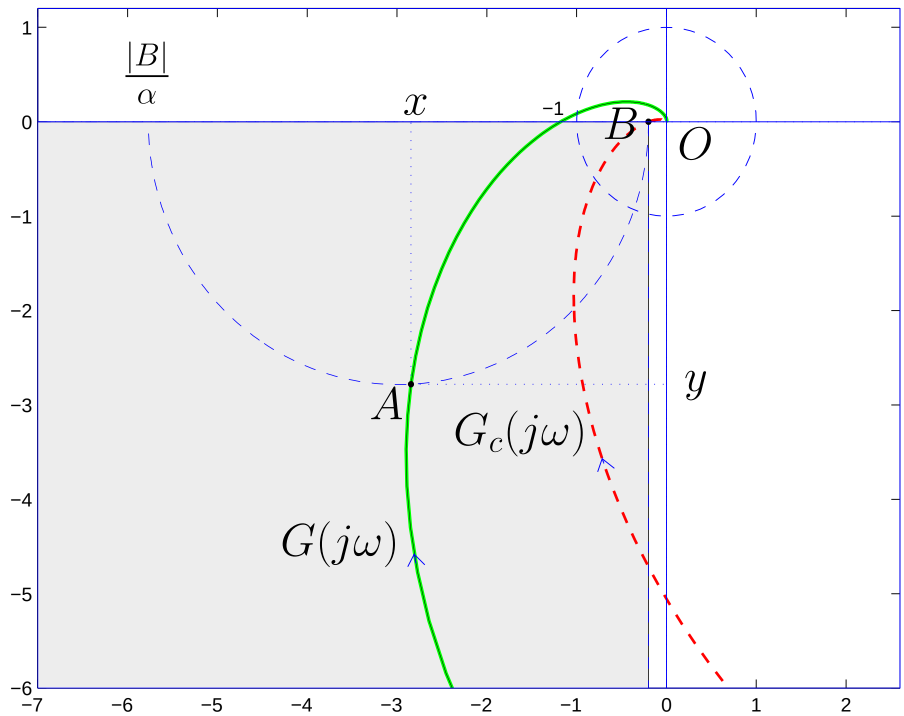
- Root locus of the two systems:





The presence of the lag network has moved the dominant poles of the feedback system towards the negative half-plane.

c) Design a lag network. Specification on the gain margin $M_\alpha = 5$.



- System $G(s)$ and lag network $C(s)$:

$$G(s) = \frac{10000}{(s+1)(s+2)(s+10)(s+30)}, \qquad \rightarrow \qquad C(s) = \frac{(1+0.396s)}{(1+11.42s)}$$

- Point A:

$$A = G(j\omega_A) = M_A e^{j\varphi_A} \qquad \rightarrow \qquad M_A = 3.978, \quad \varphi_A = 224.4^o$$

can be moved in B

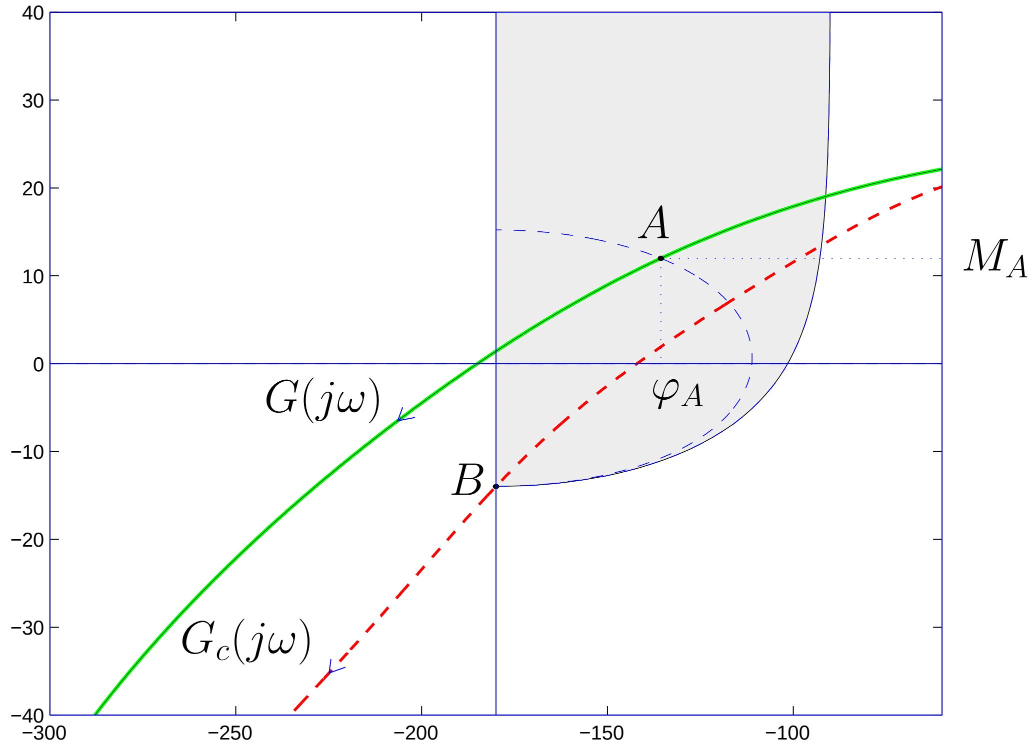
$$B = -\frac{1}{M_\alpha} \qquad \rightarrow \qquad M_B = \frac{1}{M_\alpha} = \frac{1}{5}, \qquad \varphi_B = -\pi$$

by using the following parameters:

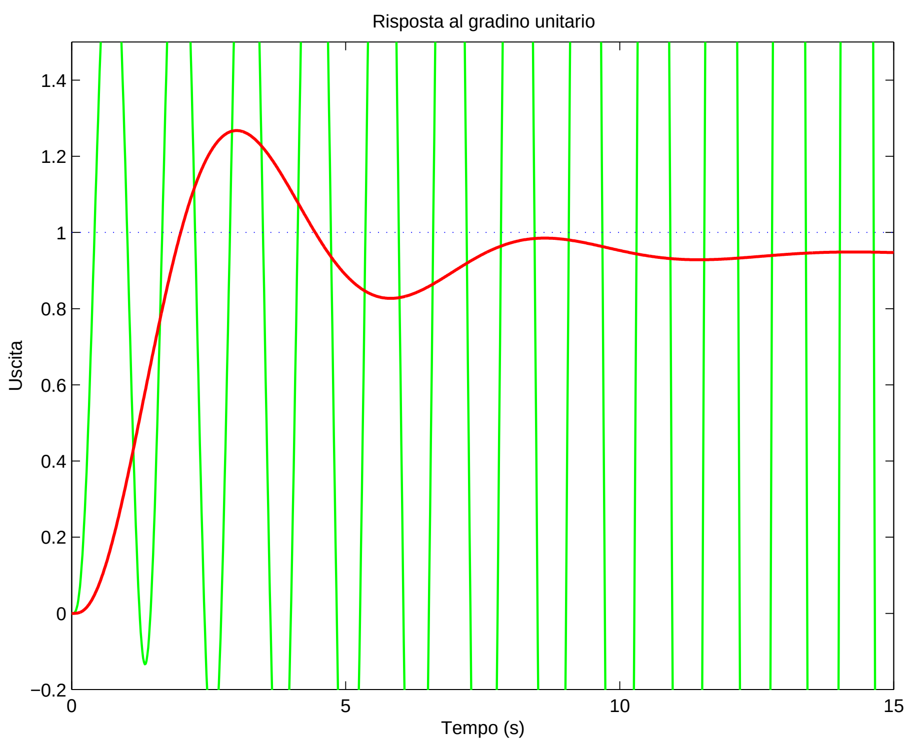
$$M = \frac{M_B}{M_A} = \frac{1}{M_A M_\alpha} = 0.0503, \qquad \varphi = \varphi_B - \varphi_A = -44.4^o$$

- Substituting the parameters M, φ and $\omega_A = 2.02$ in the inversion formulas, one obtains the parameters: $\tau_1 = 0.396$ and $\tau_2 = 11.42$.

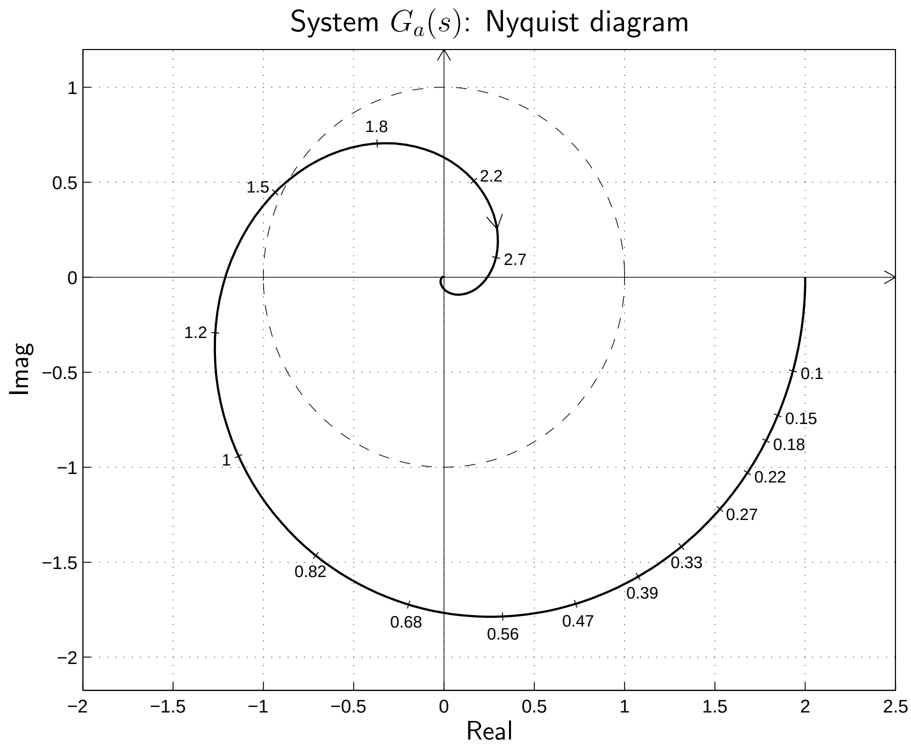
- The design of the same lag network could also be done on the Nichols plane:



- The step response of the feedback system without and with the lag network are the following:



Example. Let us consider the following Nyquist diagram of system $G_a(s)$:

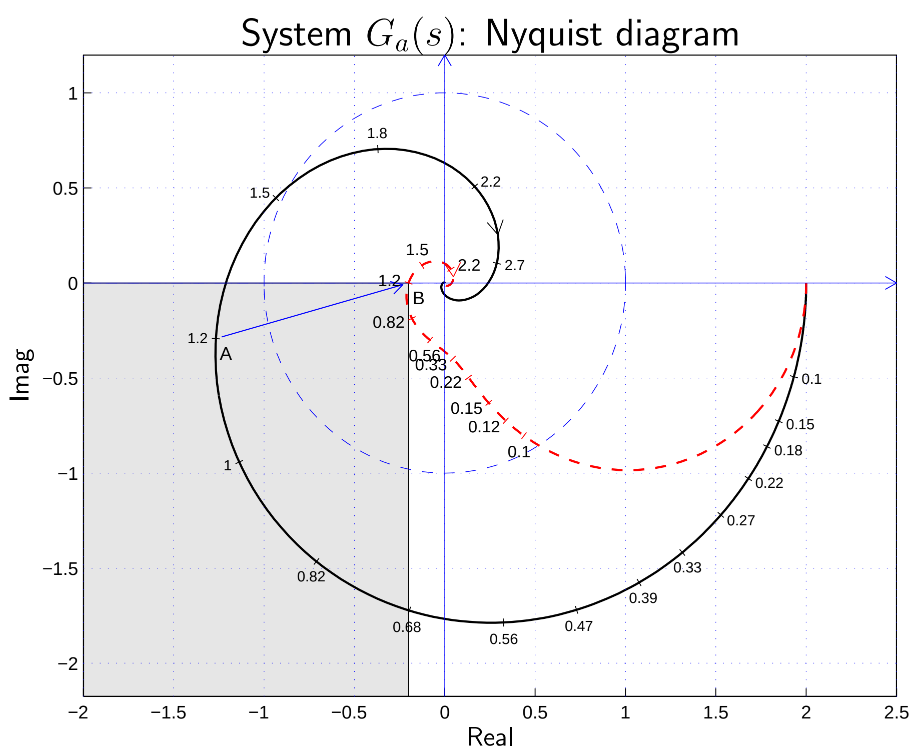


- Design a lead/lag network $C(s)$ in order to obtain compensated system with an gain margin $M_a = 5$. Choose for ω one of the values shown in the picture.

- *Solution.* The specification on gain margin $M_a = 5$ completely defines the position of point $B = M_B\, e^{j\varphi_B}$:

$$M_B = \frac{1}{M_a} = 0.2, \quad \varphi_B = -180^\circ.$$

The admissible region for the design of a lag network is shown in gray in this figure:



Let us choose as point $A = G_b(j\omega_A)$ the point characterized by frequency $\omega_A = 1.2$:

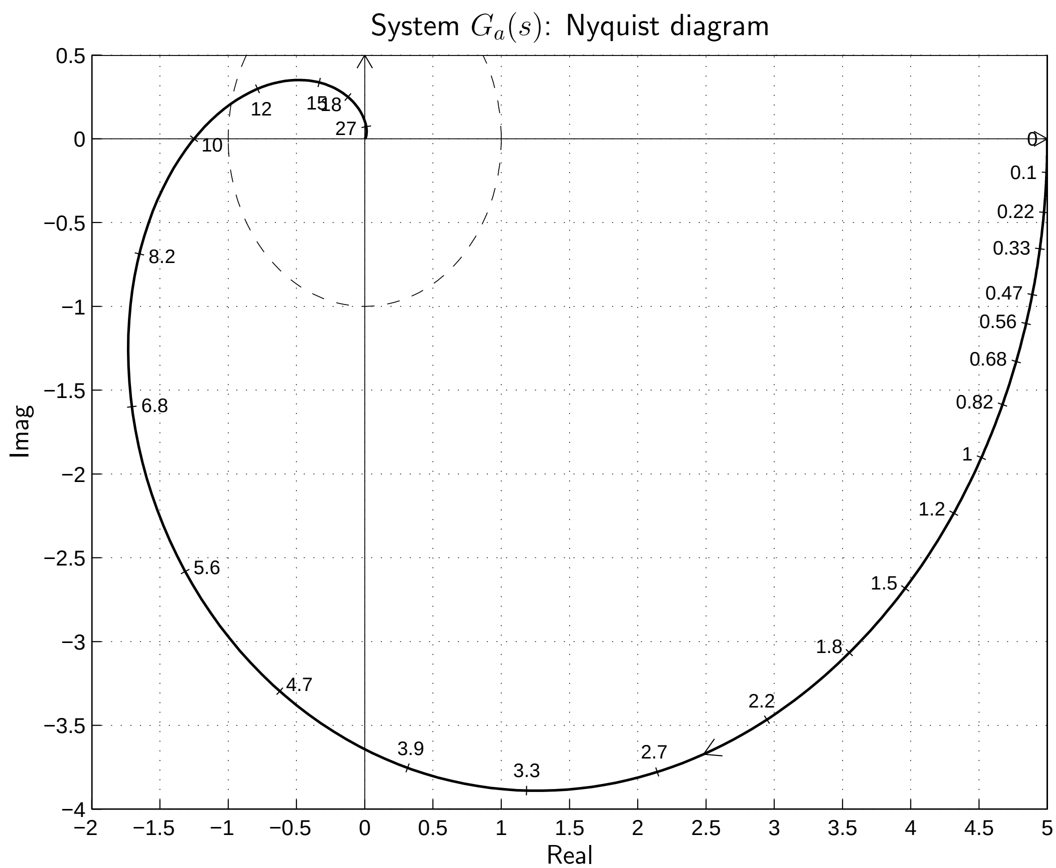
$$M_A = |G(j\omega_A)| = 1.3, \qquad \varphi_A = \arg[G(j\omega_A)] = -167^\circ.$$

Computing parameters M, φ and $\omega = \omega_A$ and using the inversion formulas, one obtains $\tau_1 = 3.038$ and $\tau_2 = 20.46$ and the following lag network:

$$M = \frac{M_B}{M_A} = 0.1539, \qquad \varphi = \varphi_B - \varphi_A = -13^\circ \quad \rightarrow \quad C_1(s) = \frac{(1 + 3.038\, s)}{(1 + 20.46\, s)}.$$

The Nyquist diagram of function $C_1(s)G_a(s)$ is shown in red in the previous figure.

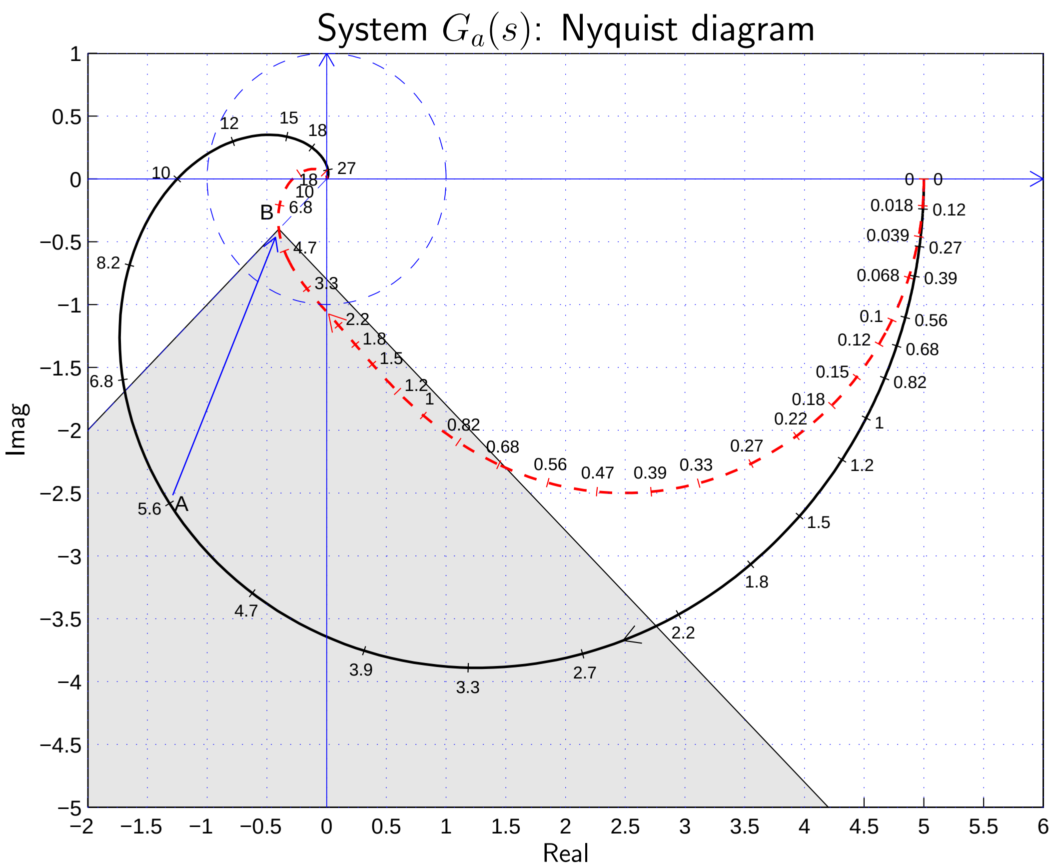
Example. Let us consider the following Nyquist diagram of system $G_a(s)$:



- Design a lead/lag network $C(s)$ such that the compensated system $C(s)G_a(s)$ passes through the point $B = (-0.4,\ -0.4)$.
- *Solution.* The design specification provides the position of point $B = M_B\, e^{j\varphi_B}$:

$$M_B = \sqrt{0.4^2 + 0.4^2} = 0.5657, \qquad \varphi_B = -135^o$$

In this case the design specification can be satisfied only by a lag network. The admissible region for the design of a lag network is shown in gray in the following figure.



In this case the point $A = G_b(j\omega_A)$ chosen for the synthesis of the lag network is the point characterized by frequency $\omega_A = 5.6$:

$$M_A = |G(j\omega_A)| = 2.898, \qquad \varphi_A = \arg[G(j\omega_A)] = -117^\circ.$$

Computing parameters M, φ and $\omega = \omega_A$ and using the inversion formulas, one obtains $\tau_1 = 0.4366$ and $\tau_2 = 2.41$ and the following lag network:

$$M = \frac{M_B}{M_A} = 0.1952, \quad \varphi = \varphi_B - \varphi_A = -18^\circ \quad \rightarrow \quad C_1(s) = \frac{(1 + 0.4366\, s)}{(1 + 2.41\, s)}.$$

The Nyquist diagram of function $C_1(s)G_a(s)$ is shown in red in the previous figure.