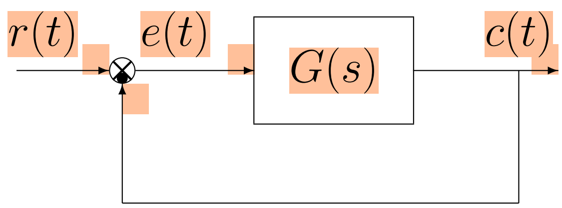
Steady-state errors

- Let us consider the following feedback system:



(1)

If the input signal $r(t)$ is a unitary step, unitary ramp or unitary parabola:

$$r(t) = R_0\, u(t), \qquad r(t) = R_0\, t, \qquad r(t) = \frac{R_0}{2} t^2$$

the corresponding *steady-state errors* are given by the following formulas:

$$\boxed{e_p = \frac{R_0}{1+K_p}}, \qquad \boxed{e_v = \frac{R_0}{K_v}}, \qquad \boxed{e_a = \frac{R_0}{K_a}}$$

where K_p, K_v and K_a:

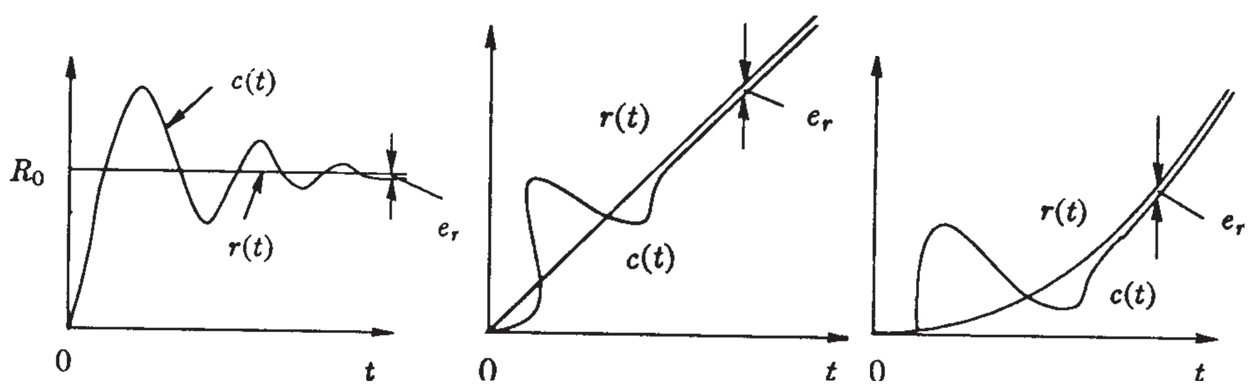
$$K_p = \lim_{s\to 0} G(s), \qquad K_v = \lim_{s\to 0} sG(s), \qquad K_a = \lim_{s\to 0} s^2 G(s)$$

are the position, velocity and acceleration constants.

- Using the Laplace transform method:

$$e_\infty = \lim_{s\to 0} s\, E(s) = \lim_{s\to 0} s \left(\frac{1}{1+G(s)} R(s) \right)$$

- Qualitative time behaviors of the three output responses:



- Step input signal:

$$e_r = \frac{R_0}{1+K_p}$$

If the system is of type 0 the position constant coincides with the static gain ($K_p = K$) and the steady-state error is finite. If the system is of type 1 or 2 the position constant is infinite ($K_p = \infty$) and the steady-state error is zero.

- Ramp input signal:

$$e_r = \frac{R_0}{K_v}$$

If the system is of type 0, the velocity constant is zero ($K_v = 0$) and then the steady-state error is infinite. If the system is of type 1, the velocity constant is finite ($K_v = K$) and the steady-state error is finite (R_0/K_v). If the system is of type 2, the velocity constant is infinite ($K_v = \infty$) and and the steady-state error is zero.

- Parabola input signal:

$$e_r = \frac{R_0}{K_a}$$

If the system is of type 0 or type 1, the acceleration constant is zero ($K_a = 0$) and the steady-state error is infinite. If the system is of type 2, the acceleration constant is finite ($K_a = K$) and and the steady-state error is finite (R_0/K_a).

- Internal model Principle: *referring to system (1), an input signal $r(t)$ can be neutralized with a zero steady-state error if all the poles of its Laplace transform function $R(s)$ are also poles of the $G(s)$ function.*