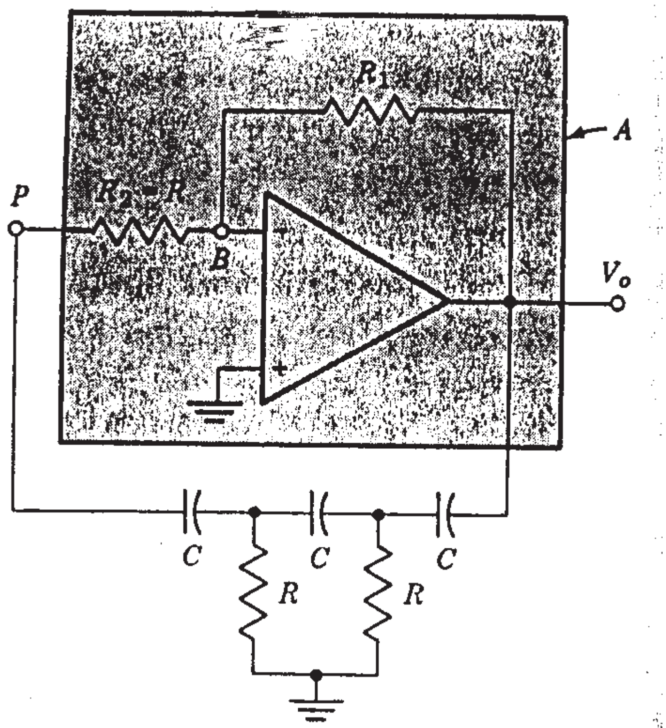
Phase shift oscillator

Example. Let us consider the following **phase shift oscillator**:



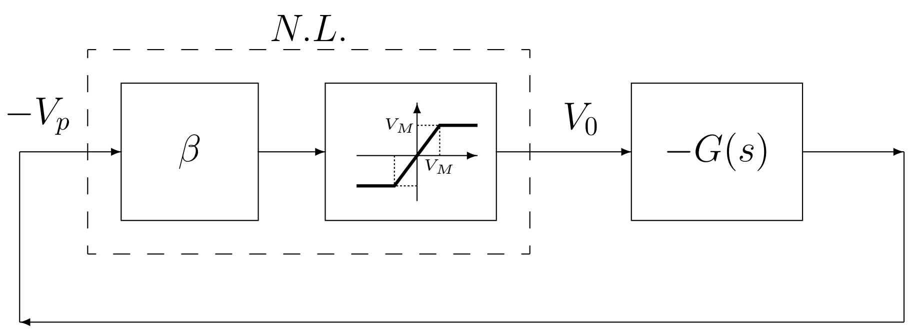
The gain of the amplifier is:

$$A = -\beta$$

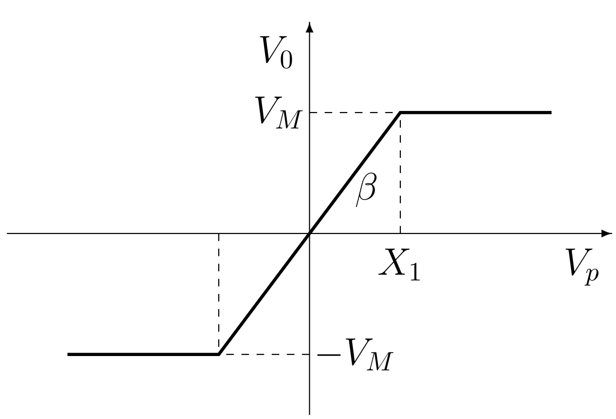
where

$$\beta = \frac{R_1}{R}$$

- The feedback system can be described by the following block scheme:



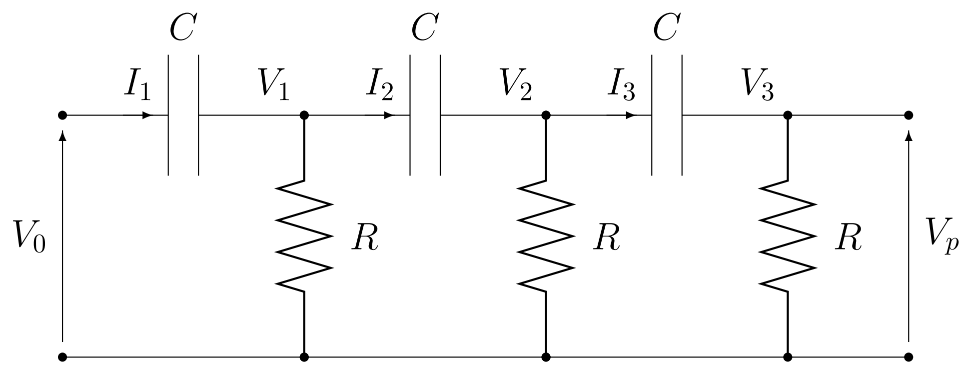
- The nonlinear element (N.L.) is a saturation with a central slope β:



$$X_1 = \frac{R\,V_M}{R_1}$$

$$\beta = \frac{R_1}{R}$$

• $G(s) = \frac{V_p(s)}{V_0(s)}$ is the transfer function of the following electrical network:



• The electrical network is described by the following differential and static equations:

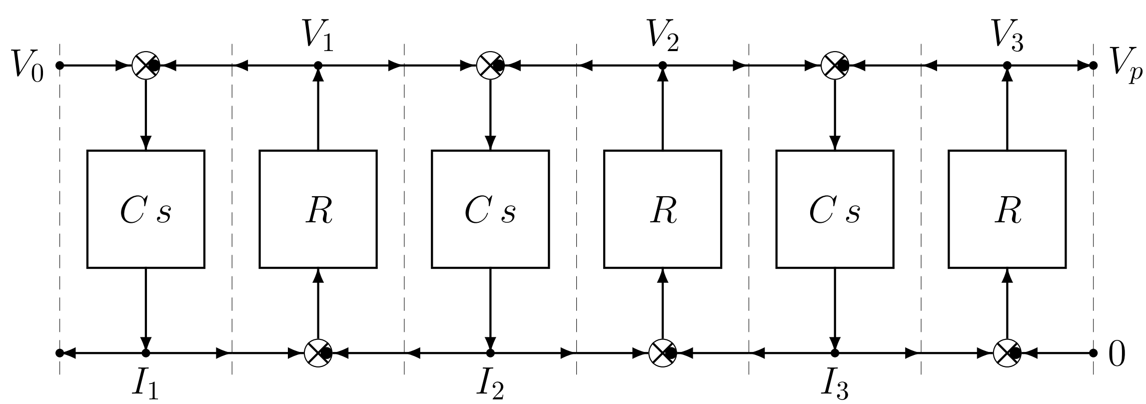
$$I_1 = C\,\frac{d(V_0 - V_1)}{dt}, \qquad I_2 = C\,\frac{d(V_1 - V_2)}{dt}, \qquad I_3 = C\,\frac{d(V_2 - V_3)}{dt}$$

$$V_1 = R(I_1 - I_2), \qquad V_2 = R(I_2 - I_3), \qquad V_3 = RI_3$$

• Using the Laplace transforms, the three differential equations can be expressed as follows:

$$I_1 = C\,s\,(V_0 - V_1), \qquad I_2 = C\,s\,(V_1 - V_2), \qquad I_3 = C\,s\,(V_2 - V_3)$$

• The mathematical model of the considered physical system can be expressed as follows:

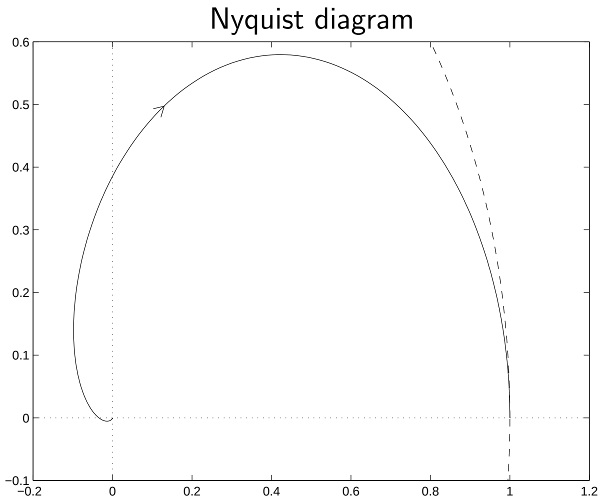


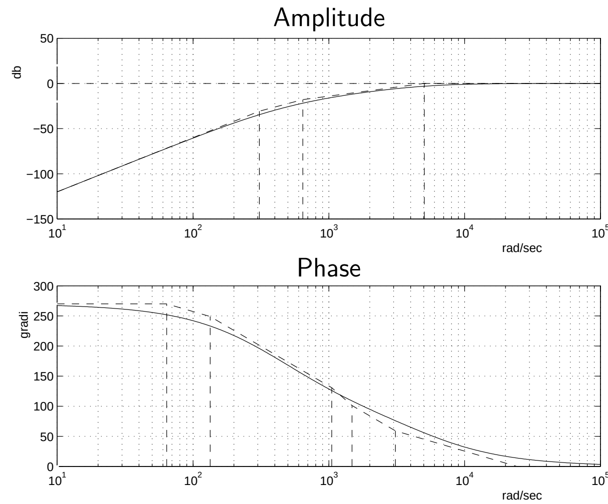
• The transfer function $G(s)$ which links the input $V_0(s)$ to the output $V_p(s)$ can be easily obtained using the Mason formula:

$$G(s) = \frac{V_p(s)}{V_0(s)} = \frac{R^3C^3s^3}{1 + 5\,R\,C\,s + 6\,R^2\,C^2\,s^2 +\, R^3\,C^3\,s^3}$$

In fact, the block diagram shows 5 rings with gain $-\,R\,C\,s$, 6 pairs of rings that do not touch each other, and one set of rings that do not touch three to three. Moreover, the only path starting from V_0 and arriving at V_p crosses all the blocks.

- Nyquist Diagram and Bode Diagrams of function $G(s)$:





The Bode diagram intersects the real negative axis at point $\sigma^* = -1/K^*$ characterized by frequency ω^*. The values of parameters K^* and ω^* can be easily calculated using the Routh criterion. The characteristic equation $1 + K\,G(s) = 0$ of the system is:

$$R^3C^3(K+1)s^3 + 6\,R^2\,C^2\,s^2 + 5\,R\,C\,s + 1 = 0$$

From the Routh table:

$$\begin{array}{c|cc} 3 & R^3C^3(K+1) & 5\,R\,C \\ 2 & 6R^2C^2 & 1 \\ 1 & (30-1-K)R^3C^3 & \\ 0 & 1 & \end{array}$$

one obtains that the system is stable for $-1 < K < K^* = 29$.

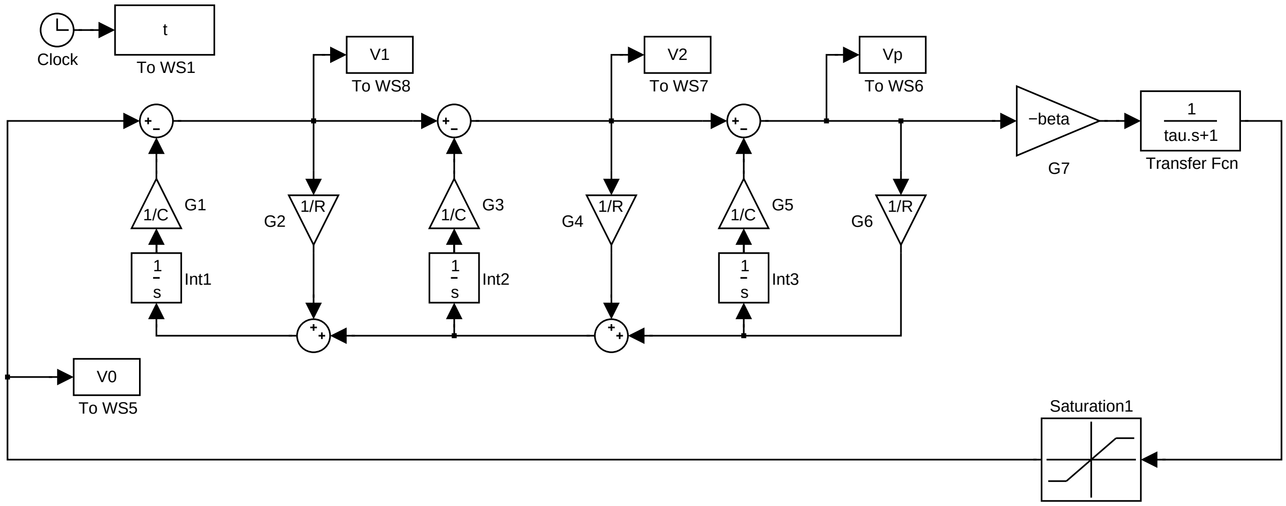
- A limit cycle appears in the system if:

$$\boxed{\beta > K^*} \qquad \text{dove} \qquad \beta = \frac{R_1}{R} \quad \text{e} \quad K^* = 29$$

- The frequency ω^* can be obtained solving the following auxiliary equation:

$$6\,R^2\,C^2\,s^2 + 1 = 0 \qquad \rightarrow \qquad \boxed{\omega^* = \frac{1}{R\,C\sqrt{6}}}$$

- Simulink block scheme of the considered nonlinear system (Phase_Shift_Oscillator_mdl.mdl):



- Simulation parameters (Phase_Shift_Oscillator_m):

```
R=1;                                     % Resistance
C=0.001;                                 % Capacity
beta=60;                                 % Gain
VM=12;                                   % Maximum voltage
V30=0.1;                                 % Initial condition
Q30=C*V30;                               % Initial condition
wstar=1/(R*C*sqrt(6));                   % Oscillation pulse
tau=0.000001;                            % Amplifier time constant
Tfin=10*2*pi/wstar;                      % Duration of the simulation
sim('Phase_Shift_Oscillator_mdl',Tfin)   % Simulation
figure(1)                                % Opening of the figure nr. 1
subplot(211); plot(t,V0);                % Plot of voltage V0
subplot(212); plot(t,Vp);                % Plot of voltage Vp
```

- Simulation results (variables V_p and V_0):

Let us choose:

$$R = 1, \quad C = 0.001$$

$$\beta = 60, \quad V_M = 12$$

One obtains:

$$\omega^* = 408.2$$

$$T = 15.4 \text{ ms}$$