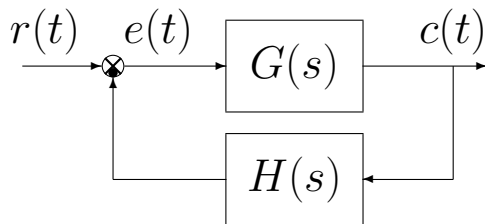


The Nyquist criterion

- Given the frequency response of a linear system $F(s)$, the Nyquist criterion provides a necessary and sufficient condition to determine the stability of the corresponding closed loop system $G_0(s)$:



$$F(s) = G(s)H(s)$$

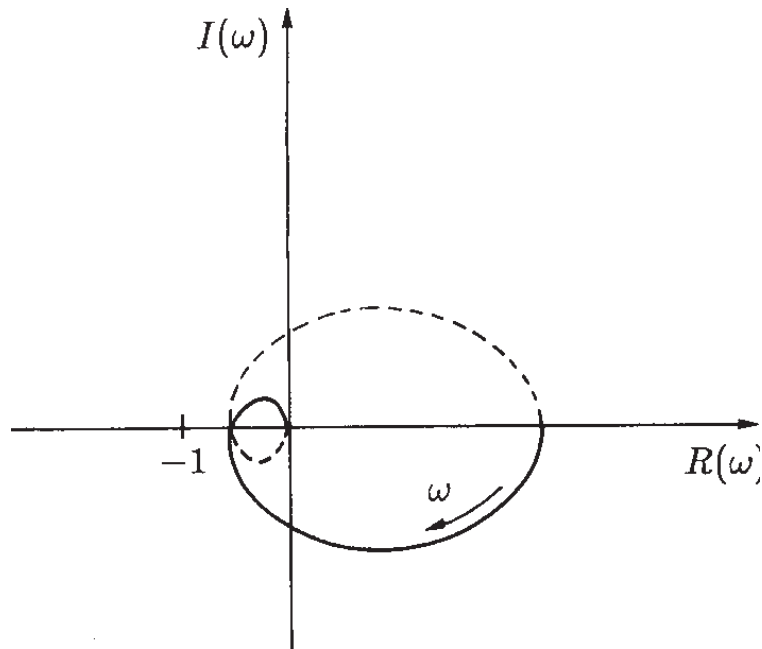
$$G_0(s) = \frac{G(s)}{1 + F(s)}$$

- Due to its graphical nature, the Nyquist criterion is useful for the design of the controller because it provides information: 1) on the stability and robustness of the controlled system; 2) on the type of controller which is useful to use to improve the dynamic behavior of the controlled system.
- Nyquist criterion (systems stable in open loop).** *If all the poles of the closed loop gain $F(s)$ are asymptotically stable, except for one or two poles in the origin, a necessary and sufficient condition for the feedback system $G_0(s)$ to be asymptotically stable is that the complete Nyquist diagram of function $F(s)$ does not touch nor encircle the critical point $-1+j0$.*
- The Nyquist criterion refers to the "complete Nyquist diagram", i.e. the Nyquist diagram must be plotted for $\omega \in [-\infty, +\infty]$. Since:

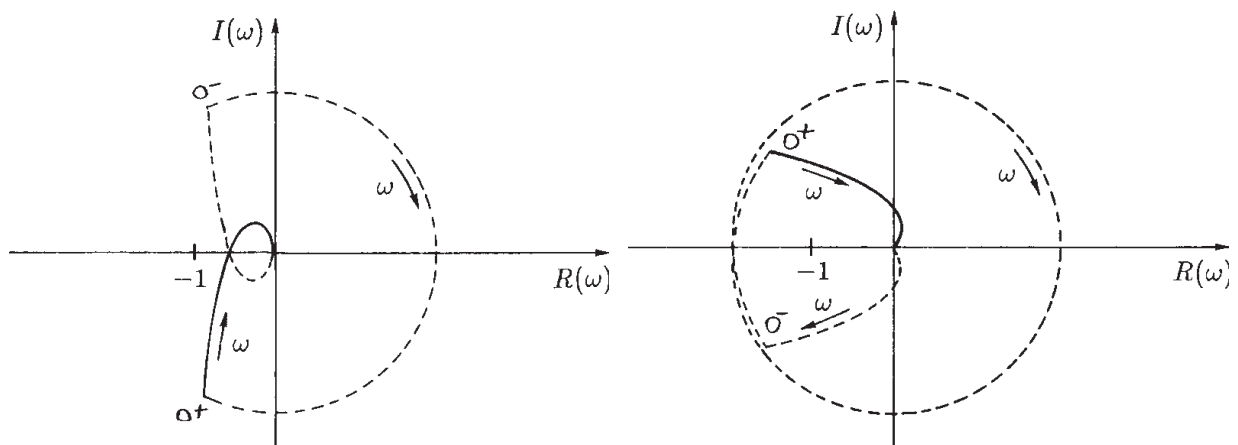
$$F(-j\omega) = F^*(j\omega)$$

the Nyquist diagram for negative frequencies can easily be obtained by flipping upsidedown, around the real axis, the Nyquist diagram plotted for positive frequencies.

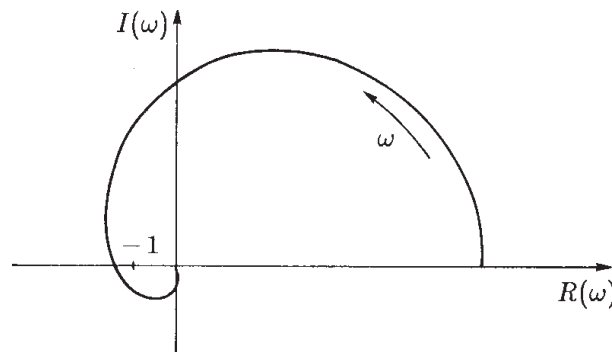
- For systems of “type 0” the “complete Nyquist diagram” is a closed curve:



- For systems of “type 1” or “type 2” (which have branches which pass through the infinity) the Nyquist diagram must be closed at the infinity with a semi or full circumference, respectively, plotted in the clockwise direction starting from $\omega = 0^-$ and arriving at $\omega = 0^+$

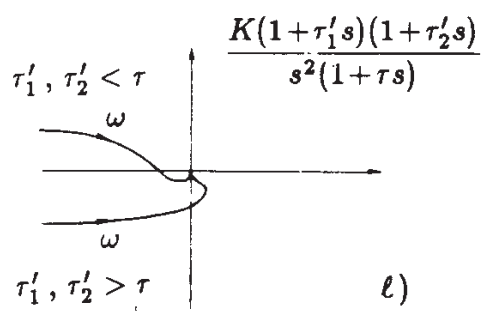
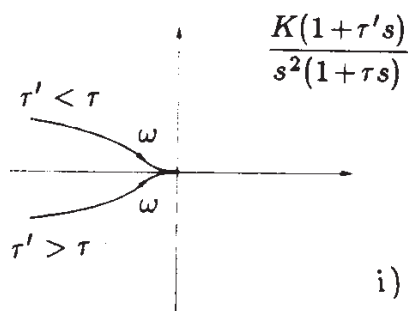
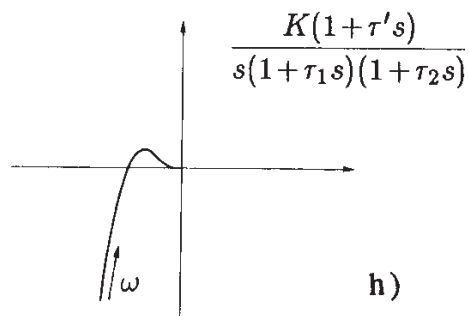
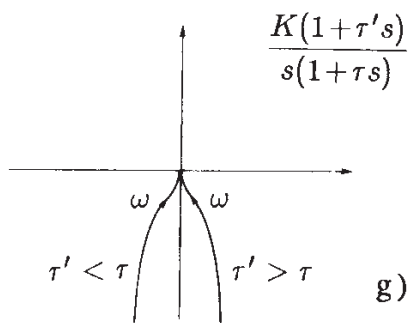
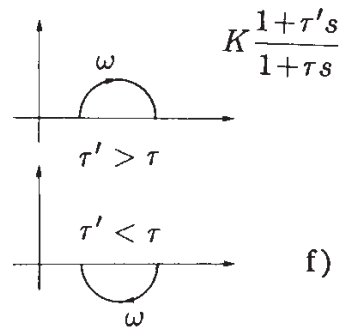
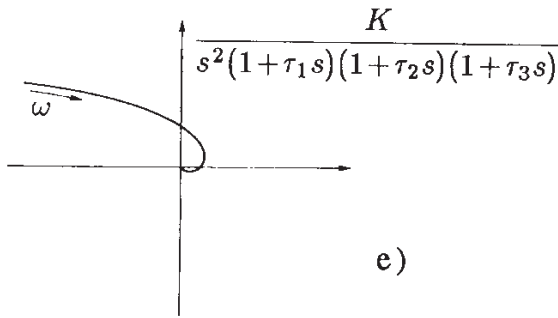
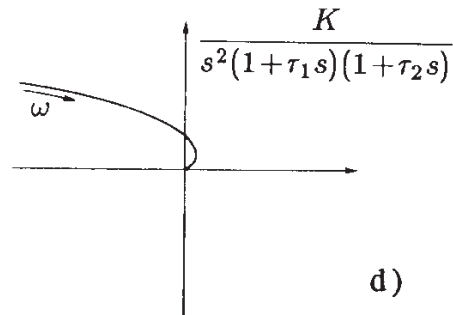
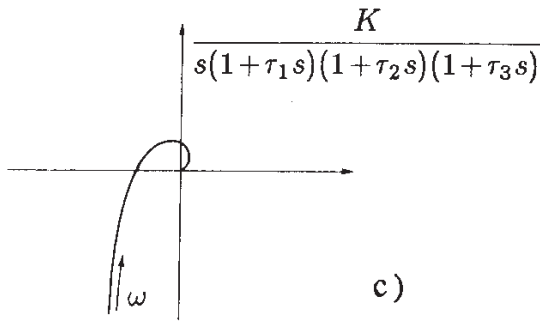
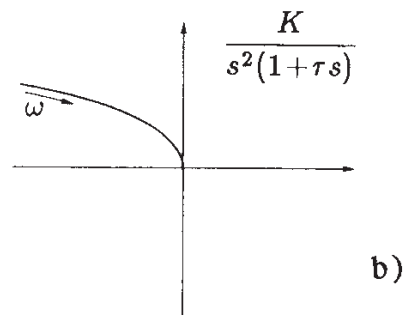
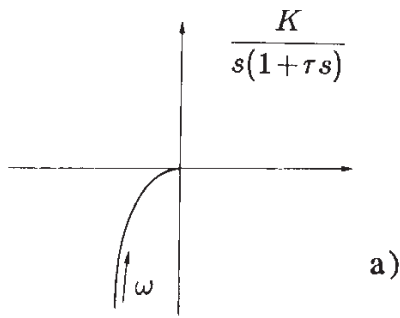


- The Nyquist criterion for the open loop stable systems covers most of the cases of interest. However, the Nyquist criterion can be given in a more general version which applies also to the systems which are unstable in open loop.
- **Nyquist criterion (for systems unstable in open loop).** If the closed loop gain $F(s)$ does not have poles on the imaginary axis, except for one or two poles in the origin, *necessary and sufficient condition* for the feedback system $G_0(s)$ to be asymptotically stable is that the *complete Nyquist diagram* of function $F(s)$ encircles the critical point $-1 + j0$ as many times in counterclockwise direction as is the number of unstable poles of function $F(s)$.
- **Example** Consider the following system:



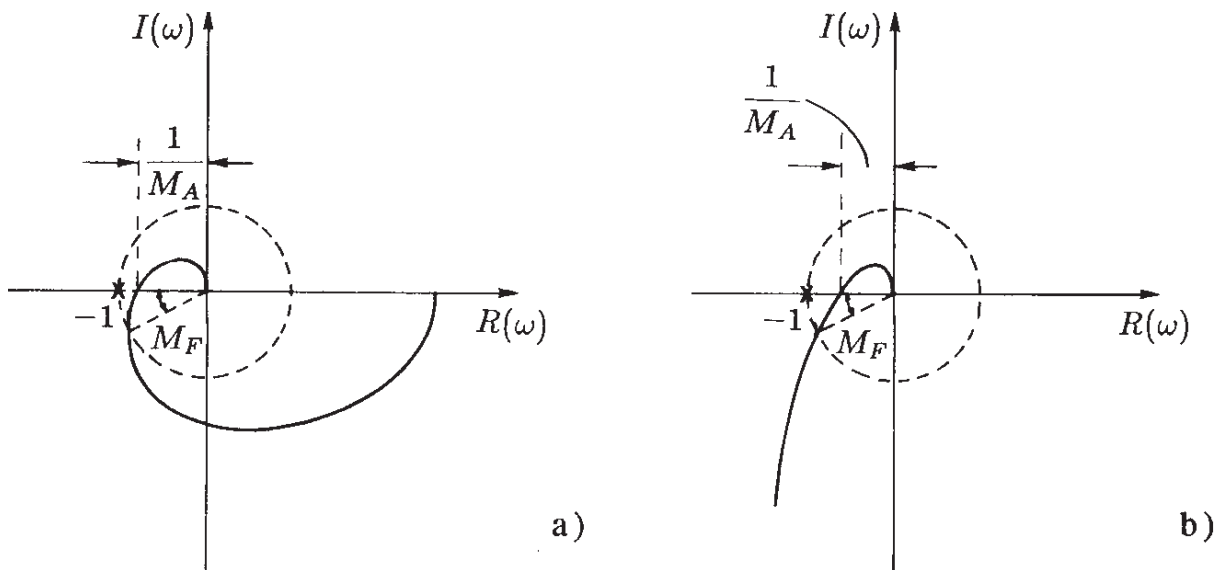
$$F(s) = \frac{K(1 + \tau s)}{(1 - \tau_1 s)(1 - \tau_2 s)}$$

The feedback system is stable for high values of constant K because, in this case, the Nyquist diagram of function $F(s)$ encircles two times counterclockwise the critical point $-1 + j0$ and function $F(s)$ has two unstable poles. On the contrary, for small values of constant K , the Nyquist diagram of function $F(s)$ does not encircle the critical point $-1 + j0$ and therefore the feedback system is unstable.



Gain and Phase Margins

- The Nyquist diagram of a given function $G(s)$ is useful also because it provides a measure of the “robustness” of the feedback systems, that is the distance of the Nyquist diagram $G(j\omega)$ from the critical point $-1 + j0$.
- If the Nyquist diagram $G(j\omega)$ is far from the critical point (and the Nyquist criterion is satisfied) then the feedback system is stable and far from the instability.



- The “measure of the distance” of the Nyquist diagram $G(j\omega)$ from the critical point is usually given by means of the *Gain and Phase Margins*:

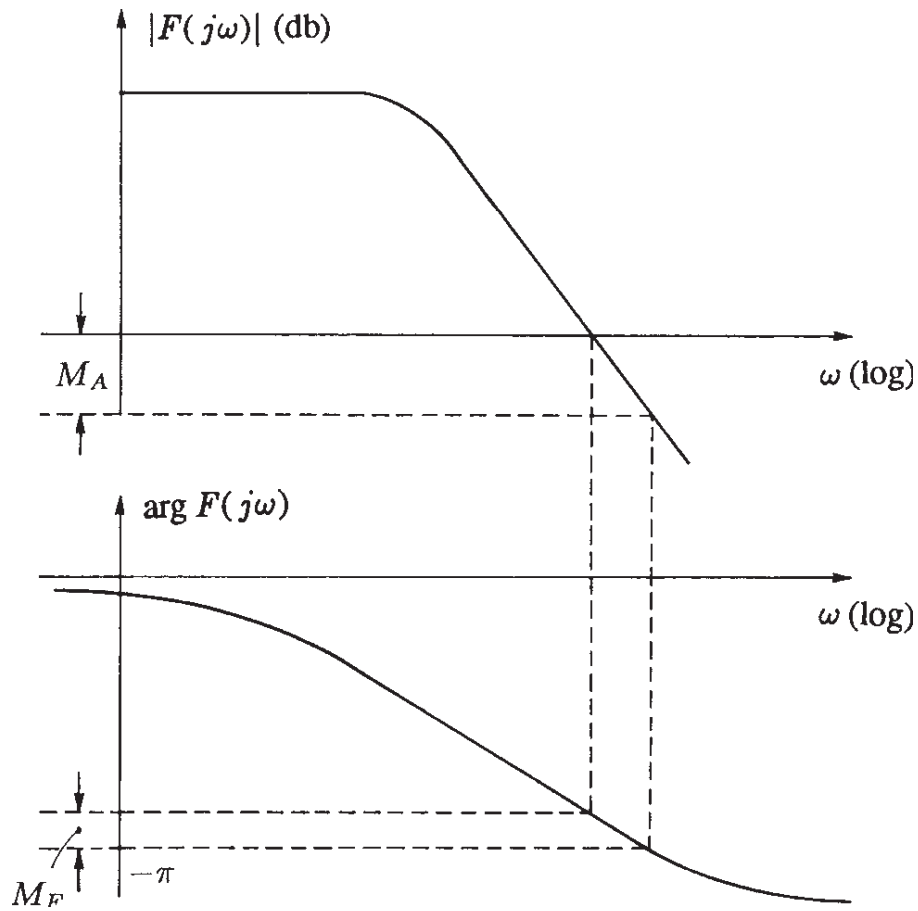
Gain Margin M_A : is the inverse of the gain of function $G(j\omega)$ at the frequency $\bar{\omega}$ for which $\arg G(j\bar{\omega}) = -\pi$:

$$M_A = \frac{1}{|G(j\bar{\omega})|} \quad \text{where} \quad \arg G(j\bar{\omega}) = -\pi.$$

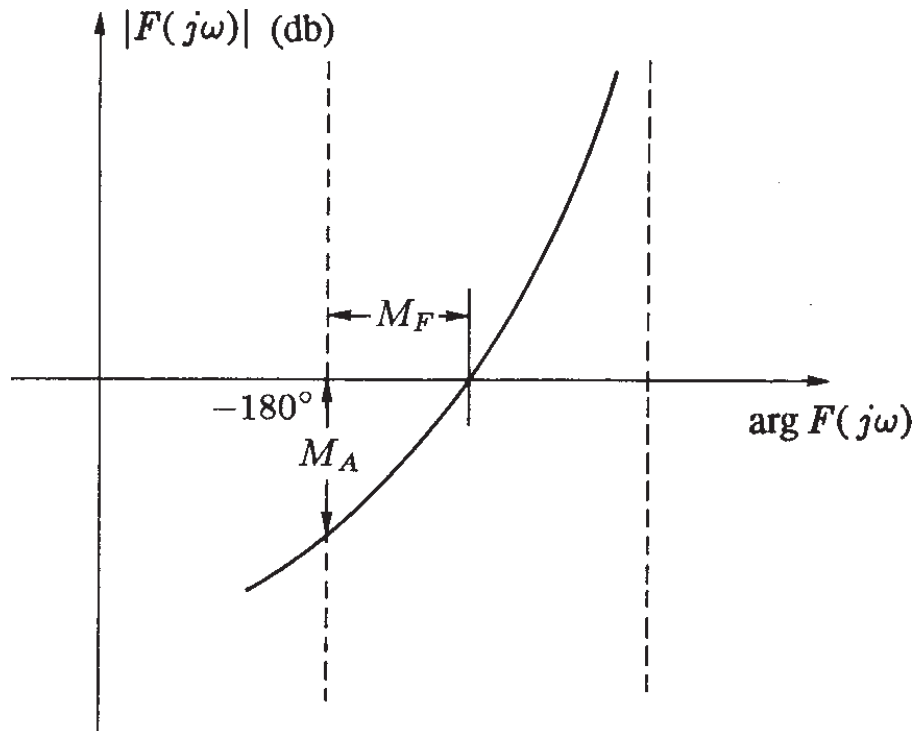
Phase Margin M_F : is the phase of function $G(j\omega)$ at the frequency ω^* for which $|G(j\omega^*)| = 1$ (i.e. when $G(j\omega)$ intersects the unitary circle) minus π (i.e. the phase of the critical point $-1 + j0$).

$$M_F = \arg G(j\omega^*) - \pi \quad \text{where} \quad |G(j\omega^*)| = 1.$$

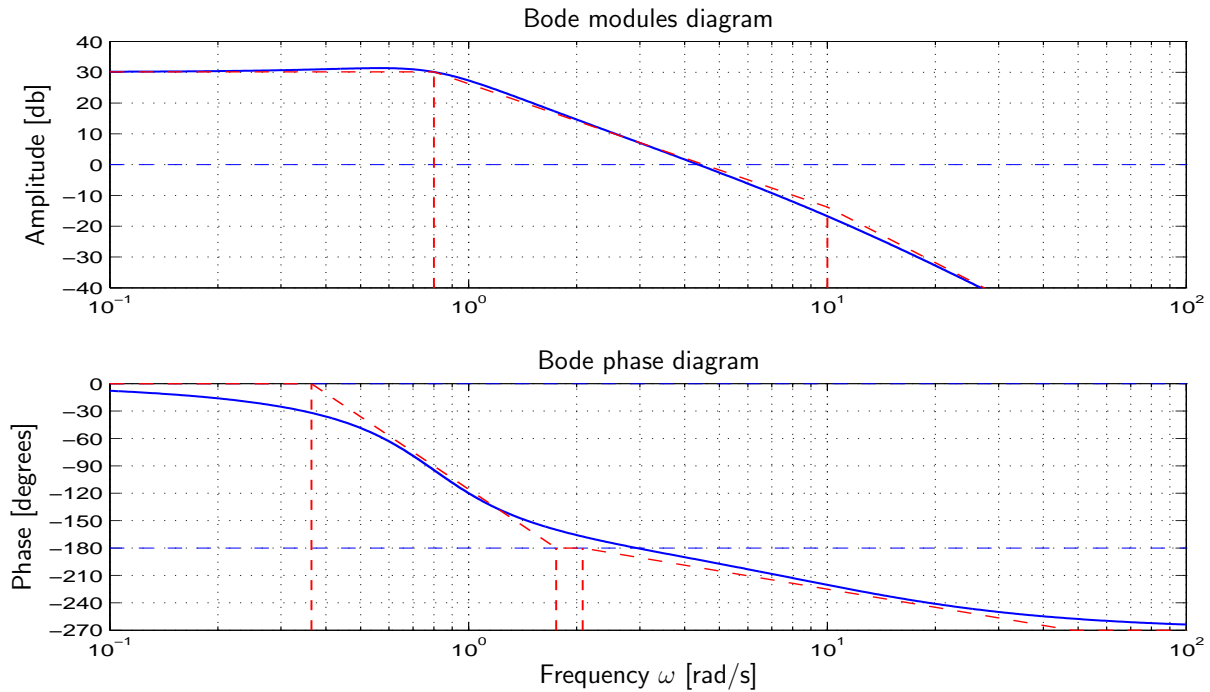
- Gain and Phase margins on Bode diagrams:



- Gain and Phase margins on Nichols diagrams:



- Given the following system:



Compute:

- a) The position of the two dominant pole $p_{1,2}$ of system $G(s)$:

$$p_{1,2} \simeq -0.4 \pm j0.693.$$

- b) The settling time T_a of the step response of system $G(s)$:

$$T_a \simeq \frac{3}{0.4} \text{ s} = 7.5 \text{ s}.$$

- c) The stability margins of system $G(s)$:

$$M_\varphi \simeq -10^\circ, \quad M_a \simeq 0.5,$$