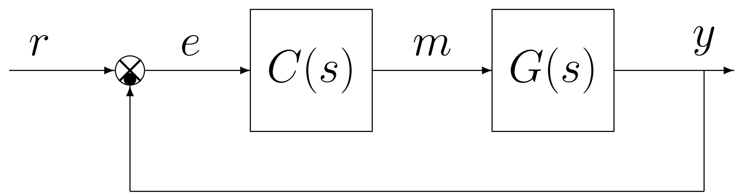
Design of Lead and Lag networks

- Consider the following feedback system:



- The design of the controller $C(s)$ is usually based on the following specifications:
  - **precision**: the required steady-state error in response to a given signal and the steady-state behavior in the presence of external disturbances or parametric variations;
  - **stability** ("a satisfactory dynamic behavior"): maximum overshoot in the step response, the resonance peak, the amplitude and phase margins, the damping coefficient of the dominant poles;
  - **response speed**: delay time, rise time, settling time, bandwidth.

- The controller is usually designed referring the the frequency response $G(j\omega)$ of the system. The specifications on "temporal parameters" are usually converted to "frequency" specifications: this conversion is usually done supposing that the feedback system behaves approximately as a second-order system with dominant poles.

- Design of the controller: the first parameter to be determined is the **static gain K of the system** in order to satisfy the given precision specifications.

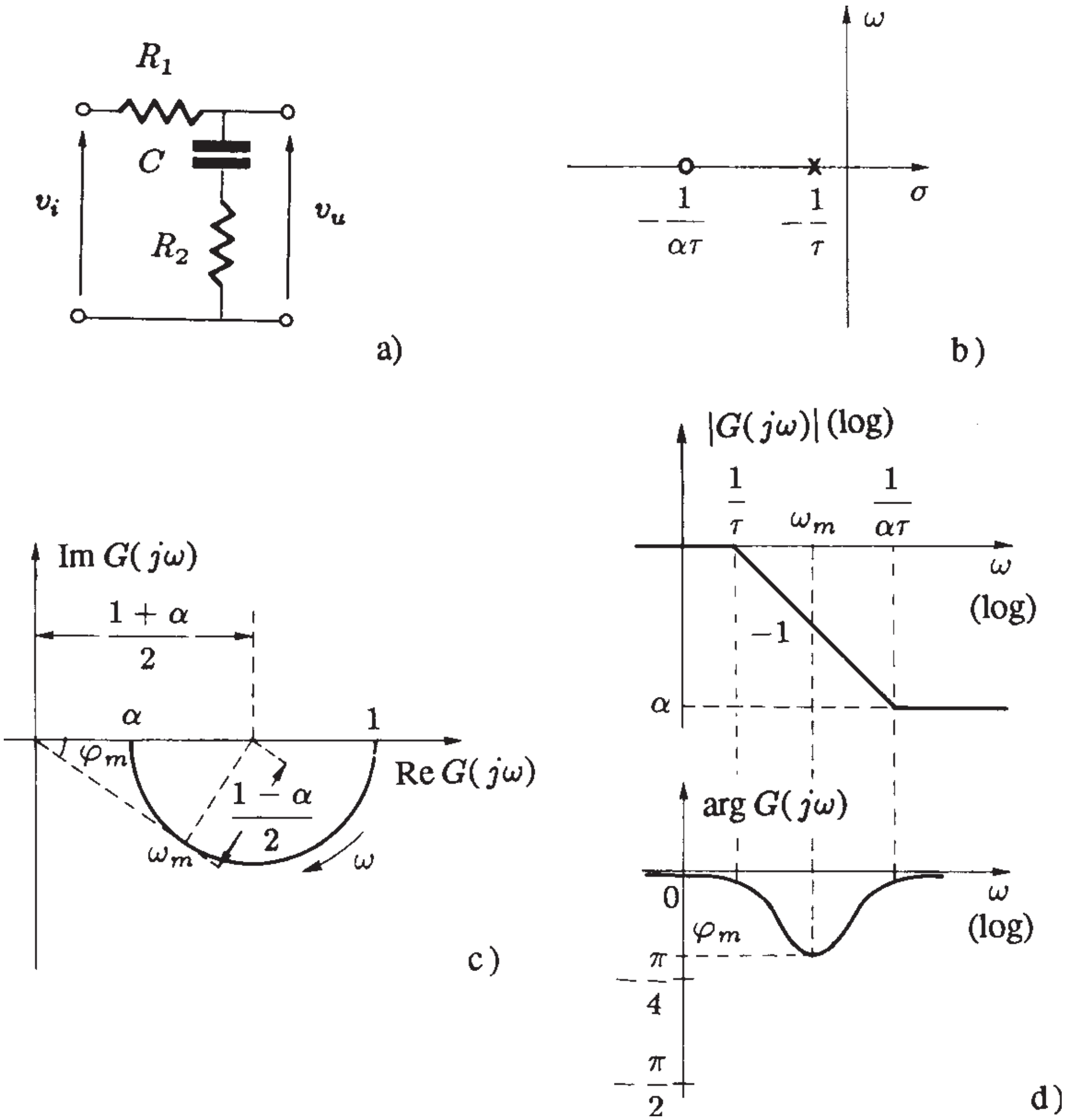
- The next step is to design the controlled in order to **guarantee the stability of the feedback system** with the required specifications on the stability margins. Usually the stability specifications are satisfied by adding a Lead/Lag network to the original proportional controller.

Lag network

The transfer function of a **Lag network** is the following:

$$G(s) = \frac{1 + \alpha\,\tau\, s}{1 + \tau\, s} \qquad \text{or} \qquad G(s) = \frac{1 + \tau_1\, s}{1 + \tau_2\, s}$$

where $\alpha < 1$ and $\tau_1 < \tau_2$. The Nyquist and Bode diagrams:



The gain and the phase of a lag network are always negative for all the frequencies $\omega \in [0,\ \infty]$. The maximum phase delay φ_m is obtained at the following frequency ω_m:

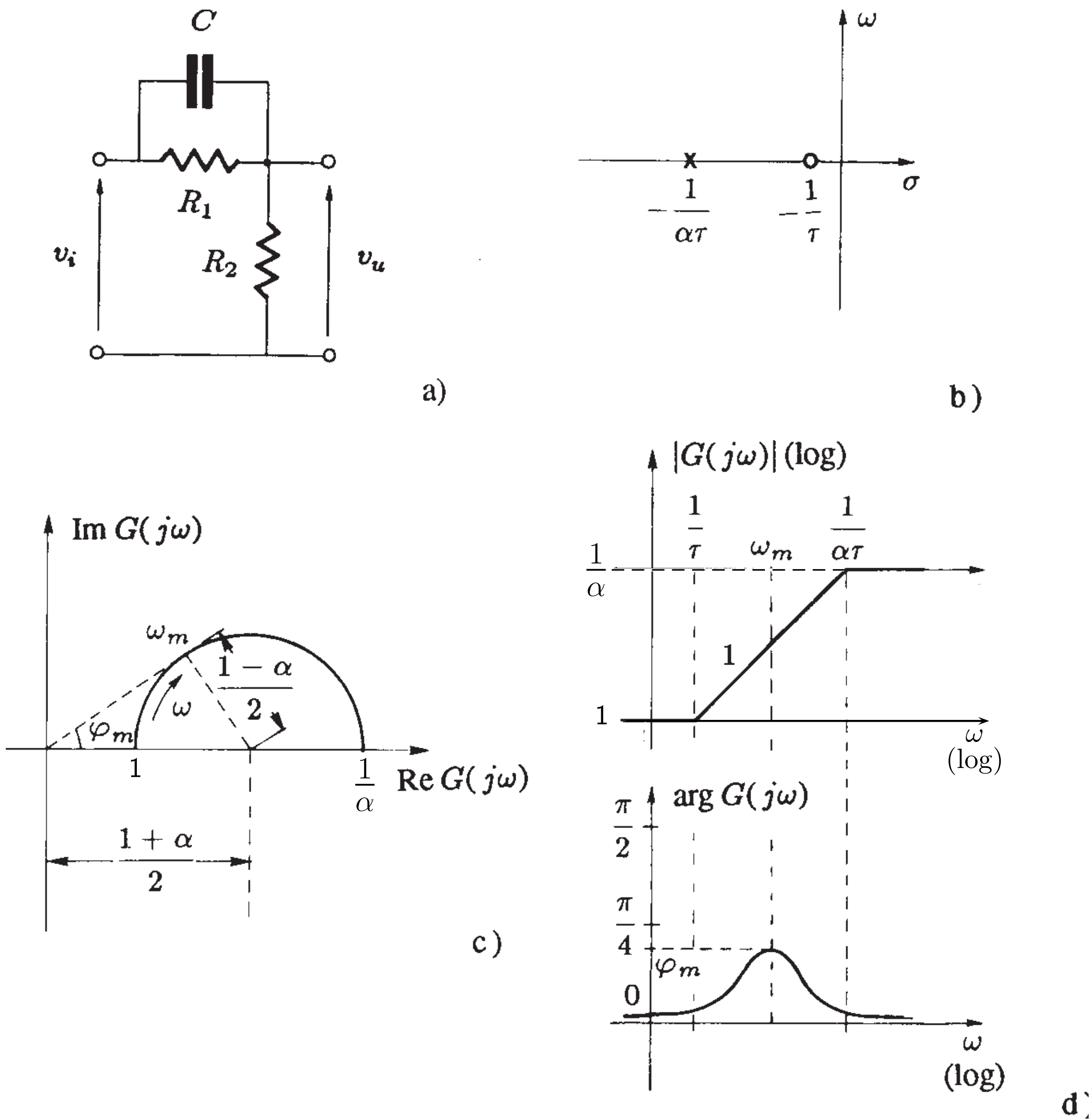
$$\varphi_m = -\text{arcsen}\,\frac{1-\alpha}{1+\alpha}, \qquad \omega_m = \frac{1}{\tau\sqrt{\alpha}}$$

Lead network

The transfer function of a **lead network** is the following:

$$G(s) = \frac{1+\tau\, s}{1+\alpha\,\tau\, s} \qquad \text{or} \qquad G(s) = \frac{1+\tau_1\, s}{1+\tau_2\, s}$$

where $\alpha < 1$ and $\tau_1 > \tau_2$. The Nyquist and Bode diagrams:



The gain and the phase of a lead network are always positive for all the frequencies $\omega \in [0,\ \infty]$. The maximum phase lead φ_m is obtained at the following frequency ω_m:

$$\varphi_m = \operatorname{arcsen} \frac{1-\alpha}{1+\alpha}, \qquad \omega_m = \frac{1}{\tau\sqrt{\alpha}}$$

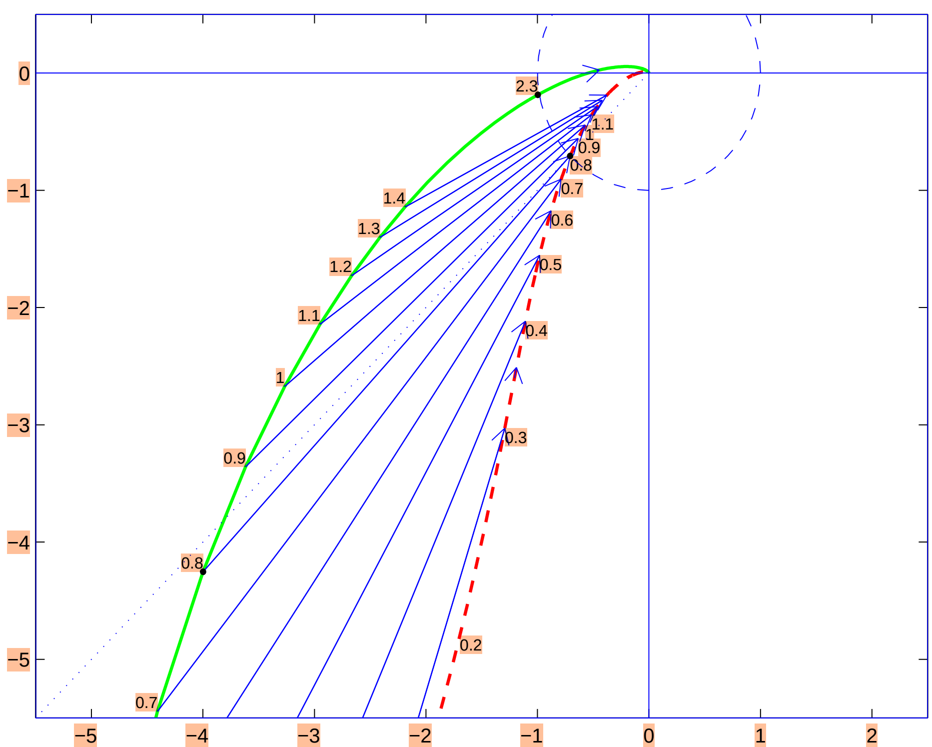
Stabilizing action of a lag network

- A lag network has negative gains and negative phase shifts for all the frequencies $\omega \in [0, \infty]$.

$$C(s) = \frac{1+\tau_1\, s}{1+\tau_2\, s} \qquad \text{with} \qquad \tau_1 < \tau_2.$$

- The stabilizing action of a lag network is essentially due to the **attenuation at the high frequencies.**

- Stabilizing action of a lag network on the Nyquist plane:



- The attenuation at high frequencies has the negative effect of reducing the bandwidth of the system.

- An advantage of the lag network compared to the lead one is its ability to stabilize systems with strongly negative phase margins.

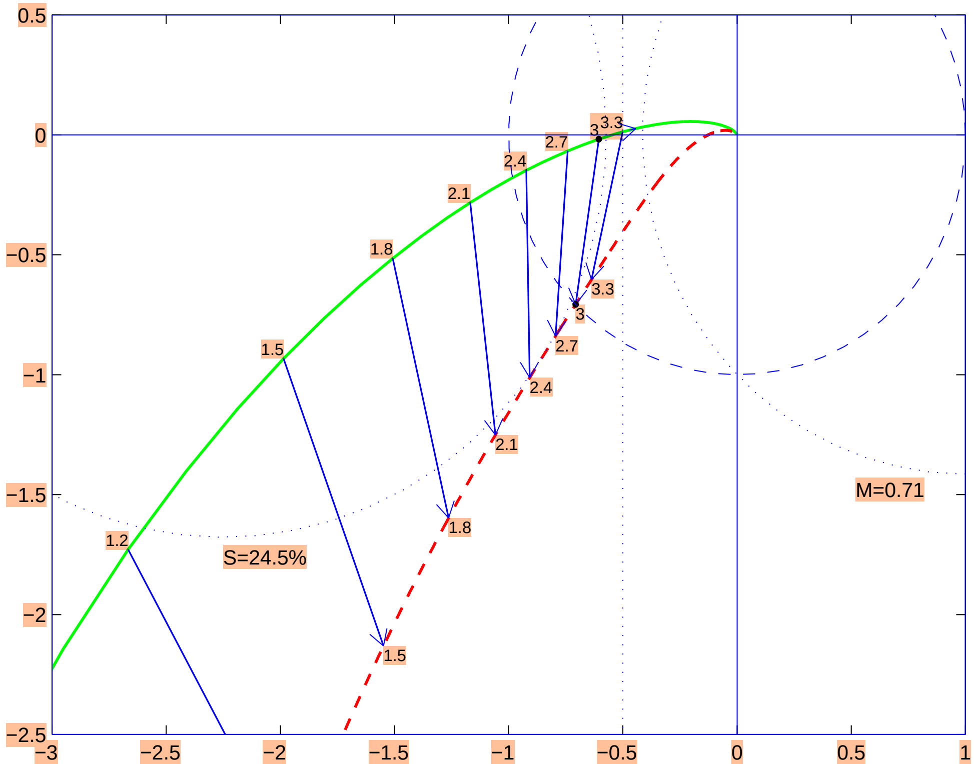
Stabilizing action of an anticipatory network

- A lead network has positive gains and positive phase shifts for all the frequencies $\omega \in [0, \infty]$

$$C(s) = \frac{1 + \tau_1 s}{1 + \tau_2 s} \qquad \text{with} \qquad \tau_1 > \tau_2.$$

- The stabilizing action of an lead network is essentially due to its positive phase shift.

- Stabilizing action of a lead network on the Nyquist plane:



- The amplifying action has a positive effect on the bandwidth.

- The maximum positive phase shift φ_m that can be obtained by a lead network is $\varphi_m = \frac{\pi}{2}$. This network is usually used to improve the initial transient of systems that are already stable or to stabilize systems with negative but small phase margins.