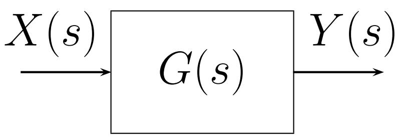
Impulse response

- Linear time invariant systems:

$$\sum_{i=0}^{n} a_i \frac{d^i y(t)}{dt^i} = \sum_{i=0}^{m} b_i \frac{d^i x(t)}{dt^i}$$

- The transfer function $G(s)$ is defined with zero initial conditions:



$$G(s) = \frac{Y(s)}{X(s)} = \frac{\sum_{i=0}^{m} b_i\, s^i}{\sum_{i=0}^{n} a_i\, s^i}$$

- Note that the coefficients a_i and b_i of the transfer function $G(s)$ are the same that characterize the differential equation.

- Possible relations between input and output signals:

If $y(t)$ is the response to the signal $x(t)$, $\Rightarrow \quad Y(s) = G(s)X(s)$

$\int_0^t y(t)dt$ is the response to the signal $\int_0^t x(t)dt \quad \Rightarrow \quad \frac{Y(s)}{s} = G(s)\frac{X(s)}{s}$

and $\frac{d\,y(t)}{dt}$ is the response to the signal $\frac{d\,x(t)}{dt} \quad \Rightarrow \quad sY(s) = G(s)\,sX(s)$

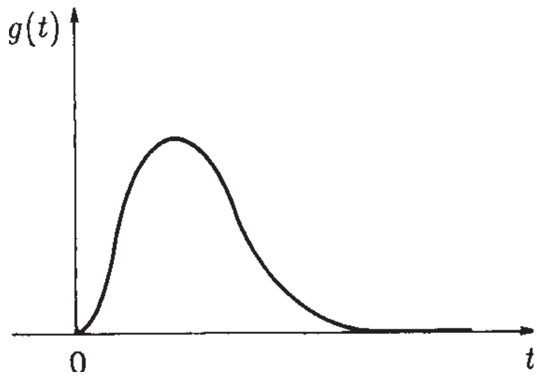
- The response to the unit ramp is the derivative of the response to the unit parabola; the response to the unit step is the derivative of the unit ramp response; etc.

- *Canonical responses*: are the system's responses to the typical signals (impulse, step, ramp, parabola, etc.)

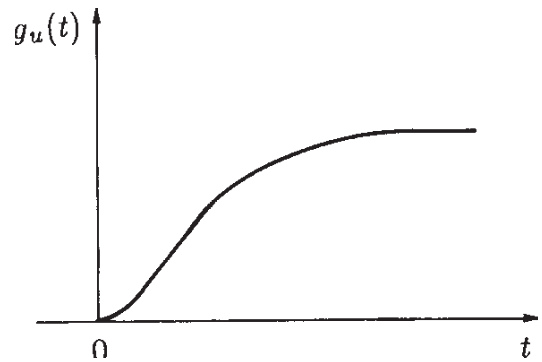
- The *canonical responses* completely characterize the dynamic behavior of the system: from the knowledge of one of them it is possible to trace the response to any other signal.

- The canonical responses which are most frequently used in practice are the *impulse response* (or *impulsive response*) $g(t)$ and the *step response* (o *index response*) $g_u(t)$.

Impulsive response



Step response



- The impulse response $g(t)$ is the inverse Laplace transform of the transfer function $G(s)$:

$$g(t) = \mathcal{L}^{-1}[G(s)] \qquad \leftrightarrow \qquad G(s) = \mathcal{L}[g(t)]$$

- If the impulse response $g(t)$ is known, then it is possible to determine the response of the system (starting from the zero initial conditions) to any other input signal. In fact, starting from the following relation

$$Y(s) = G(s)\,X(s)$$

and applying the *theorem of the integral product* we have that

$$y(t) = \int_0^{\infty} x(\tau)\, g(t-\tau)\, d\tau$$

that is, by calculating this *integral convolution* it is possible to determine the system's response $y(t)$ to any input signal $x(t)$.

- Being $x(t) = 0$ and $g(t) = 0$ for $t < 0$, the integral convolution can be rewritten as follows:

$$y(t) = \int_0^{t} x(\tau)\, g(t-\tau)\, d\tau$$