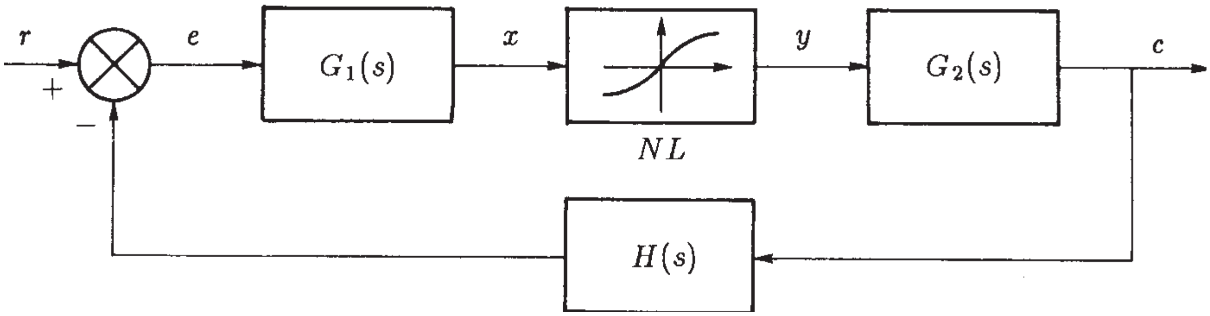
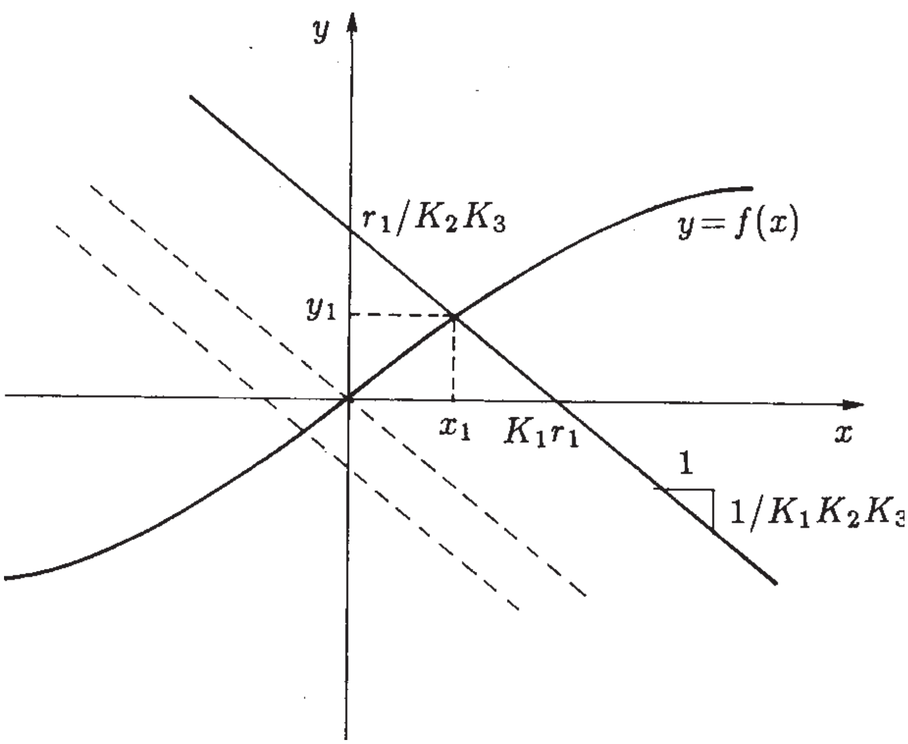
Nonlinear systems: equilibrium points

• Let us consider the following feedback nonlinear system:



• Let us suppose that the reference signal r_1 is constant.

• The *equilibrium points* (x_i, y_i) of the considered nonlinear system can easily be determined graphically:



• The equilibrium point (x_1, y_1) is the intersection of the non linear function $y = F(x)$ with the following straight line which describes the steady-state behavior of the linear part of the considered feedback system:

$$x = K_1\, r - K_1\, K_2\, K_3\, y$$

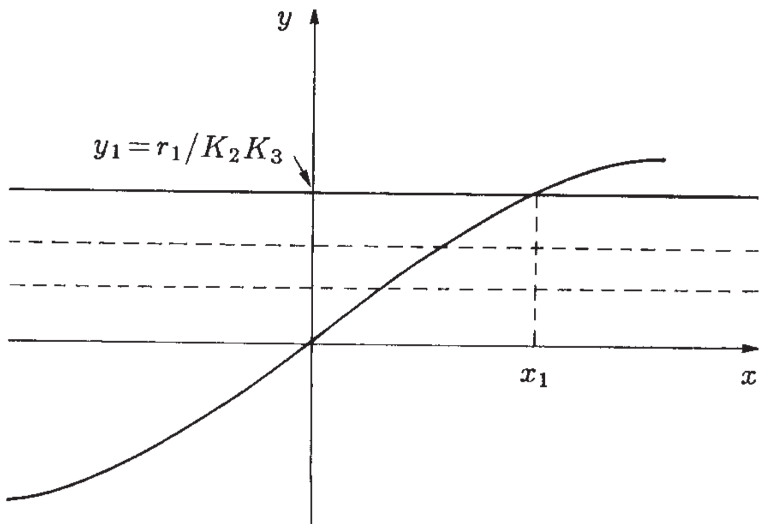
where $K_1 := G_1(0)$, $K_2 := G_2(0)$, $K_3 := H(0)$ are the static gains of the system.

Special cases:

1) If the system $G_1(s)$ is of type 1 (i.e. $G_1(s)$ has a pole in the origin), the corresponding static gain is $K_1 = \infty$ and the load line becomes:

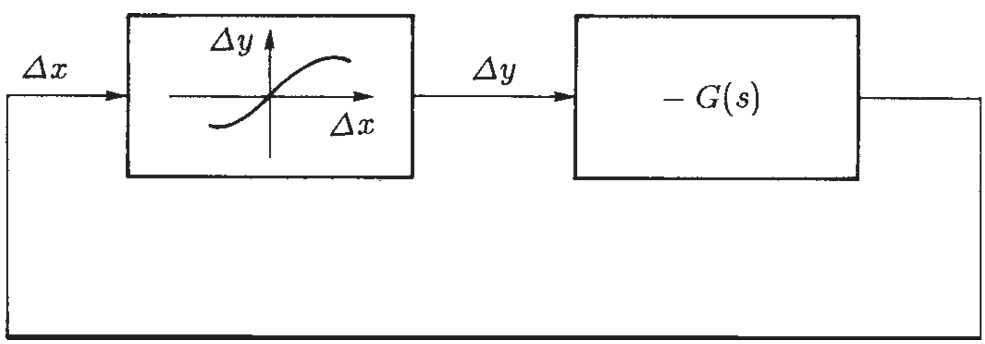
$$r = K_2 K_3 y \qquad \rightarrow \qquad y = \frac{1}{K_2 K_3} r$$

The corresponding graphic solution is:



2) If system $G_2(s)$ [or system $H(s)$] is of type 1, the corresponding static gain is $K_2 = \infty$ ($K_3 = \infty$) and the load line is $y = 0$ and the equilibrium point is given by the intersection of function $y = f(x)$ with the horizontal axis $y = 0$.

• Using the following new variables $\Delta x := x - x_1$, $\Delta y := y - y_1$ and $\Delta r := r - r_1$, the previous feedback system can be represented (equivalently) as follows:



where you place $G(s) := G_1(s)\, G_2(s)\, H(s)$. The origin of the new coordinate system $(\Delta x, \Delta y)$ coincides with the equilibrium point x_1, y_1.