Ideal sampling

- Consider the following continuous time system:

$$G(s) = \frac{25}{s^2 + 2\,s + 5}$$

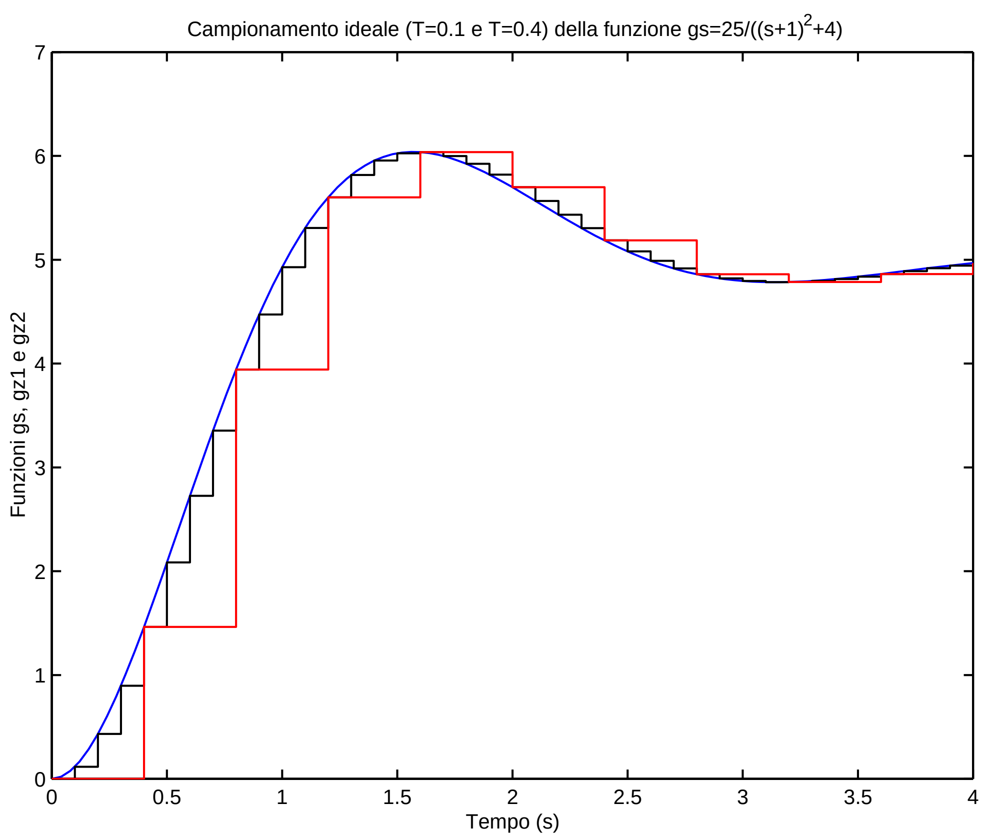
- If you discretize this function using the sampling period $T = 0.1$ you get the function:

$$G_1(z) = \mathcal{Z}[H_0(s)G(s)]_{T=0.1} = \frac{0.1166z + 0.1091}{z^2 - 1.774z + 0.8187}$$

- Instead, by using the sampling period $T = 0.4$, the following function is obtained:

$$G_2(z) = \mathcal{Z}[H_0(s)G(s)]_{T=0.4} = \frac{1.463\,z + 1.114}{z^2 - 0.934\,z + 0.4493}$$

- Response to the unit step of the 3 functions $G(s)$, $G_1(z)$ and $G(2)(z)$:



- Note the exact coincidence of the 3 step responses in the sampling instants.

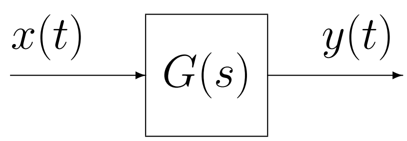
Proportional discrete regulator

- Consider the following system:

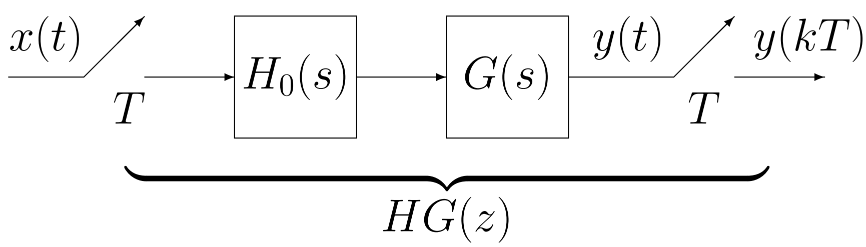
$$G(s) = \frac{25}{s(s+1)(s+10)}$$

Compare the harmonic response function of the following 3 systems:

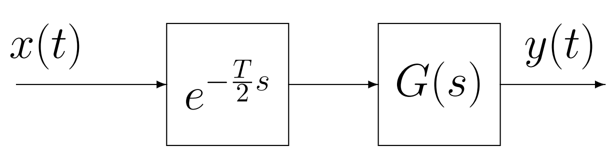
1) Il sistema $G(s)$:



2) The system $HG(z) = \mathcal{Z}[H_0(s)G(s)]$ obtained by inserting a sampler and a zero order rebuilder in cascade to the $G(s)$ system:

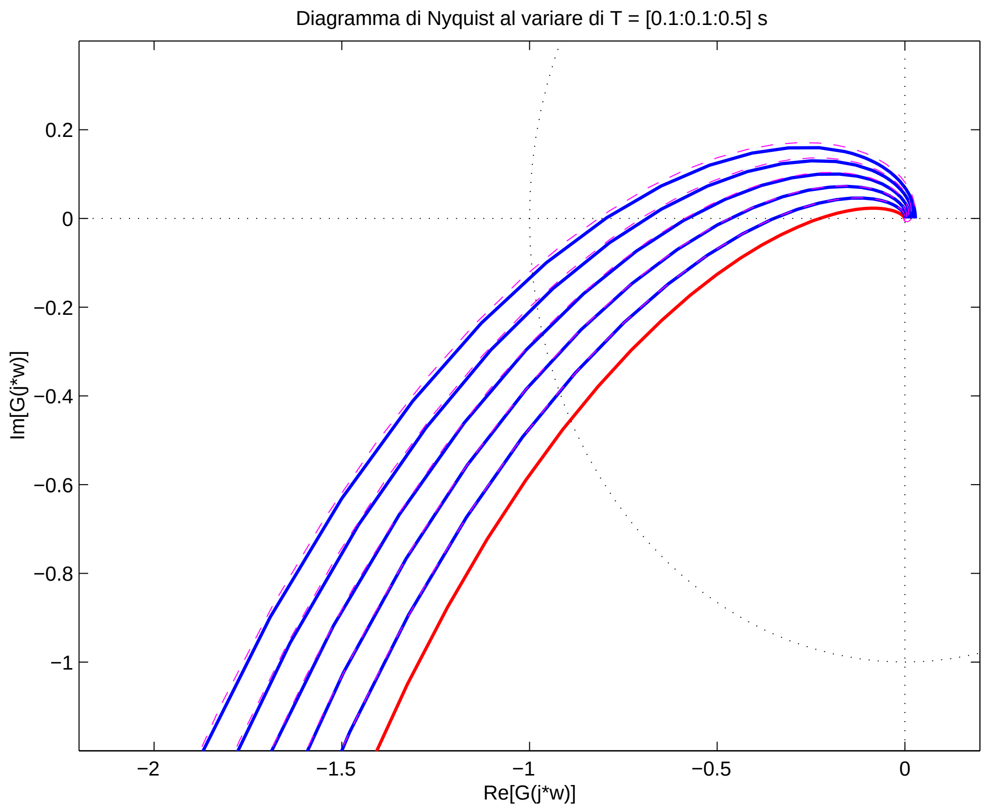


3) The system $G(s)\,e^{-\frac{T}{2}s}$ obtained by replacing the zero-order sampler and rebuilder cascade with a pure delay $e^{-\frac{T}{2}s}$ equal to half of the T sampling period:



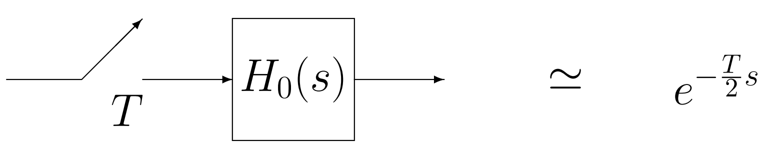
- The function $HG(z,T) = \mathcal{Z}[H_0(s)G(s)]$ which is obtained by discretizing the system $G(s)$ placed in cascade with the zero order rebuilder $H_0(s)$ is a function of the sampling period T.

- Nyquist diagrams of systems $G(s)$, $HG(z,T)$ and $G(s)\,e^{-\frac{T}{2}s}$ for $T \in [0.1,\,0.2,\,0.3,\,0.4,\,0.5]$:

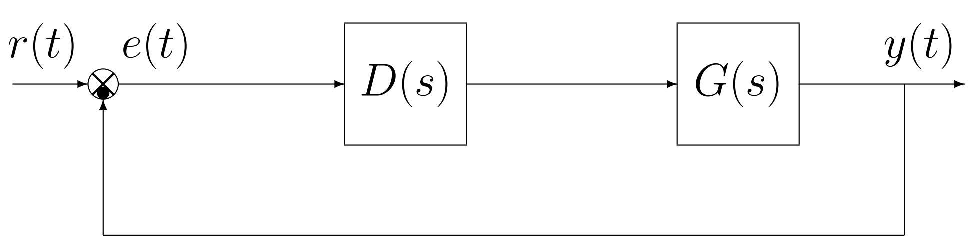


- Notice that as the sampling period increases T, there is an increase in the phase displacement present within the system and therefore a reduction in the stability margins.

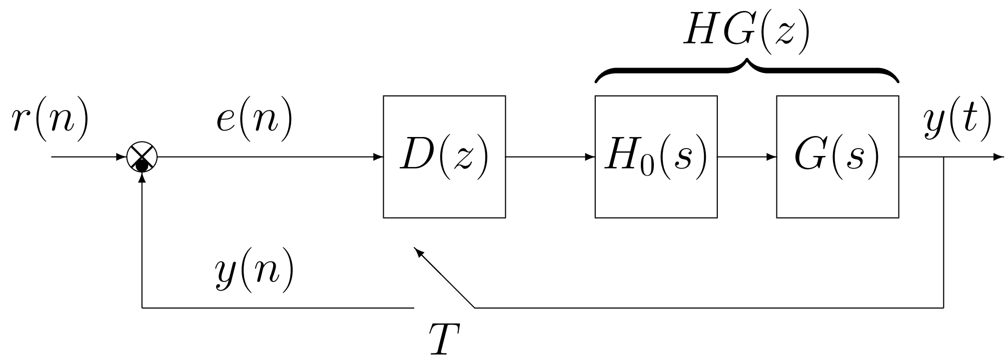
- From the above Nyquist diagrams it is evident that for low pulsations the cascade of the sampler and of the zero order rebuilder can be well approximated by a pure delay:



- Let's now compare the continuous time feedback system:



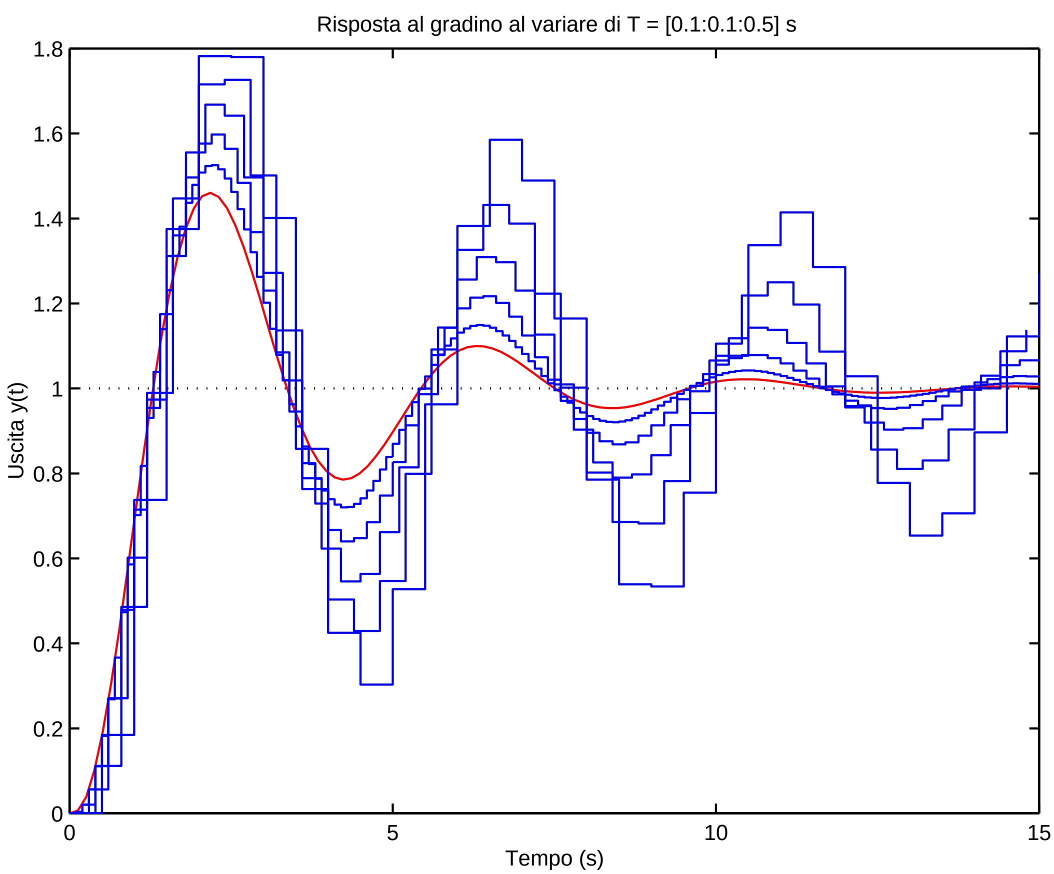
with the corresponding discrete time system:



- Transfer functions $G_0(s)$ and $G_0(z)$ of the two feedback systems:

$$G_0(s) = \frac{D(s)\,G(s)}{1 + D(s)G(s)}, \qquad G_0(z) = \frac{D(z)\,HG(z)}{1 + D(z)\,HG(z)}$$

- Response to the systems step $G(s)$ and $HG(z,T)$ placed in unit feedback for $T \in [0.1,\ 0.2,\ 0.3,\ 0.4,\ 0.5]$ when $D(s) = D(z) = 1$:



- As the sampling period increases, there is less and less dampened step responses and a higher and higher overshoot.

Comparison between different methods of description

- Consider the following system:

$$G(s) = \frac{25}{s(s+1)(s+10)}$$

and for it a suitable corrective network (anticipator) is designed:

$$D(s) = \frac{1+\tau_1\, s}{1+\tau_2\, s} = \frac{1+0.806\, s}{1+0.117\, s}$$

that improve the response to the step of the corresponding feedback system.

- The transfer function $D(s)$ can be discretized using several approximate methods:

1) backwards differences:

$$D_1(z) = D(s)\big|_{s=\frac{1-z^{-1}}{T}} = \frac{T+\tau_1-\tau_1\, z^{-1}}{T+\tau_2-\tau_2\, z^{-1}}$$

2) differences forward:

$$D_2(z) = D(s)\big|_{s=\frac{z-1}{T}} = \frac{\tau_1+(T-\tau_1)\, z^{-1}}{\tau_2+(T-\tau_2)\, z^{-1}}$$

3) bilinear transformation:

$$D_3(z) = D(s)\big|_{s=\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}} = \frac{T+2\,\tau_1+(T-2\,\tau_1)\, z^{-1}}{T+2\,\tau_2+(T-2\,\tau_2)\, z^{-1}}$$

4) poly-zero match:

$$D(s) \qquad \rightarrow \qquad D_4(z) = \frac{(1-\beta)-\alpha\,(1-\beta)\, z^{-1}}{(1-\alpha)-\beta\,(1-\alpha)\, z^{-1}}$$

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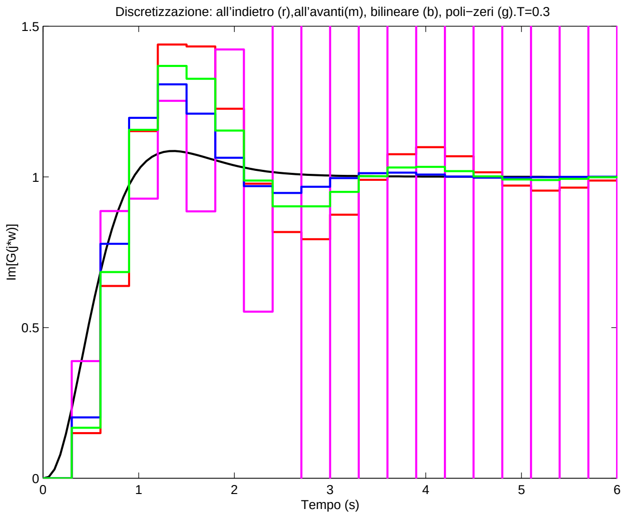
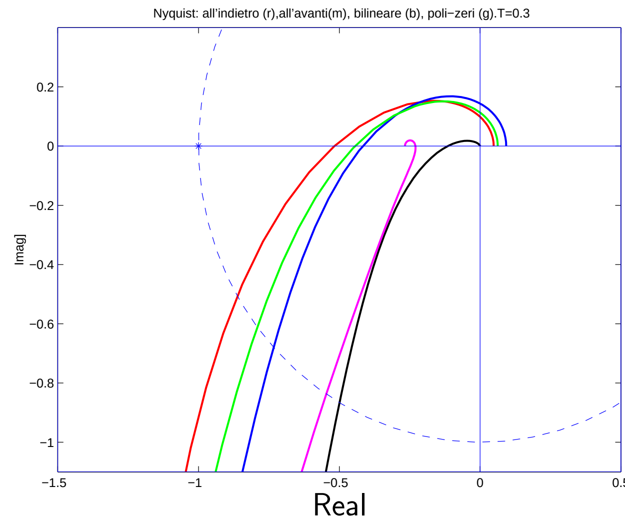
$$\alpha = e^{-\frac{T}{\tau_1}}, \qquad \beta = e^{-\frac{T}{\tau_2}}$$

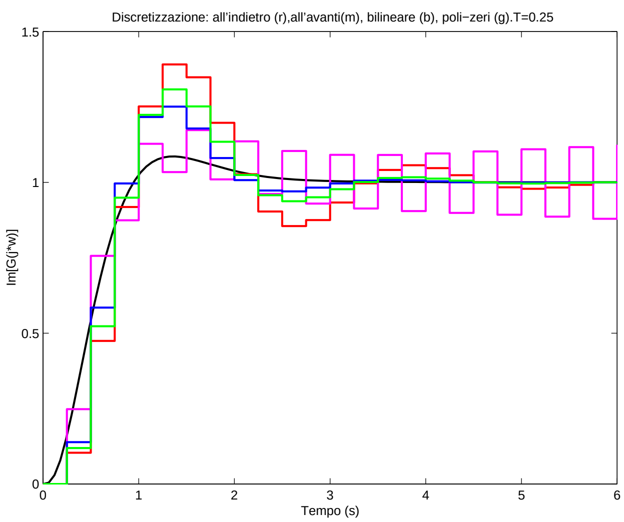
- Note that the $D_2(z)$ regulator is only stable when $T < 2\,\tau_2$ while the other 3 regulators are stable for any value of $T > 0$.

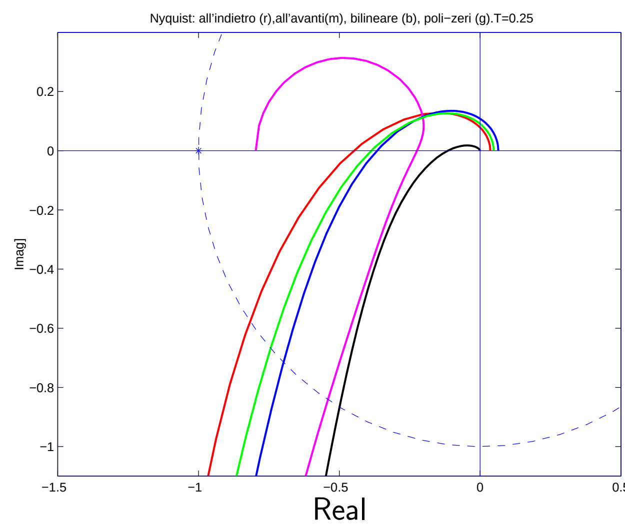
- Answers to the step of the feedback system and Nyquist diagrams when $T = 0.3$ and using the various methods of discretization:



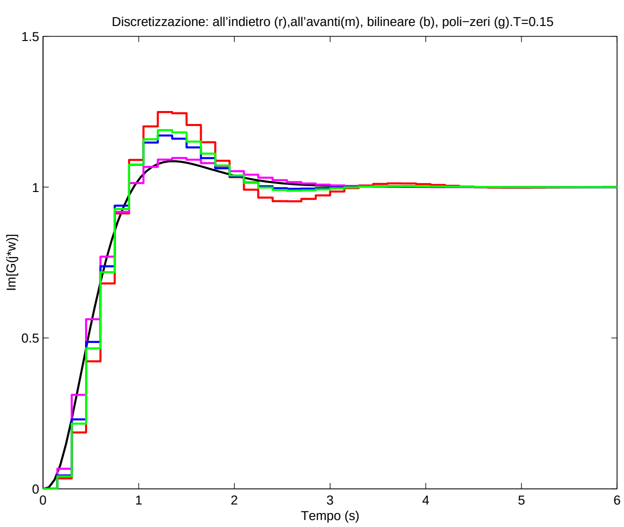


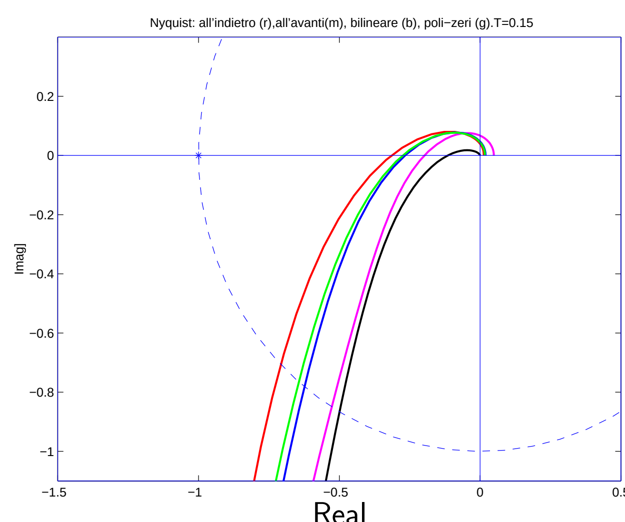
- Answers to the step and Nyquist diagrams when $T = 0.25$:



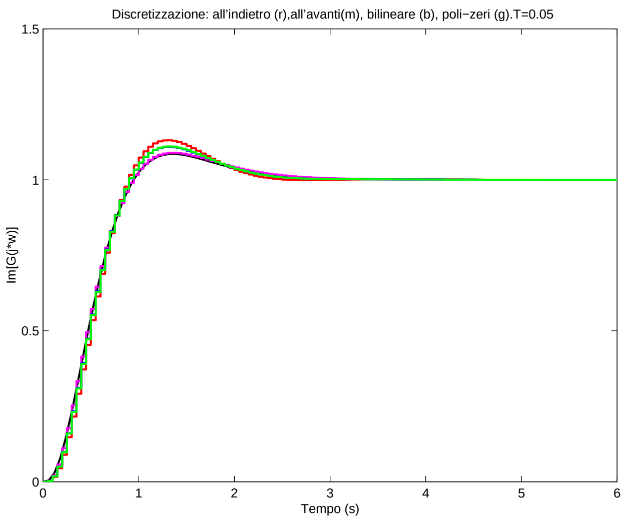


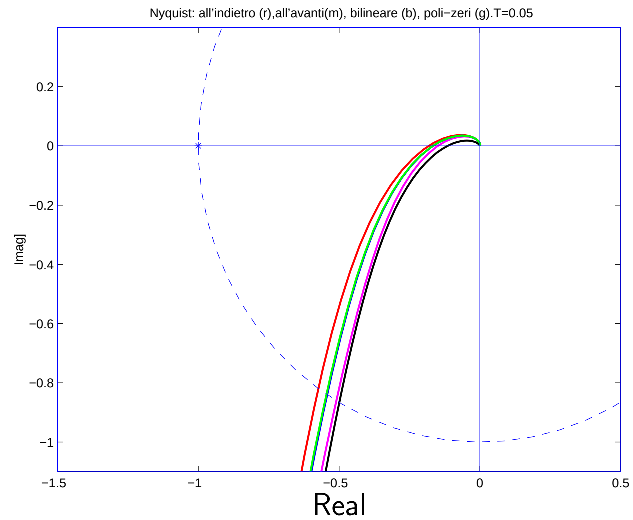
- Answers to the step and Nyquist diagrams when $T = 0.15$:



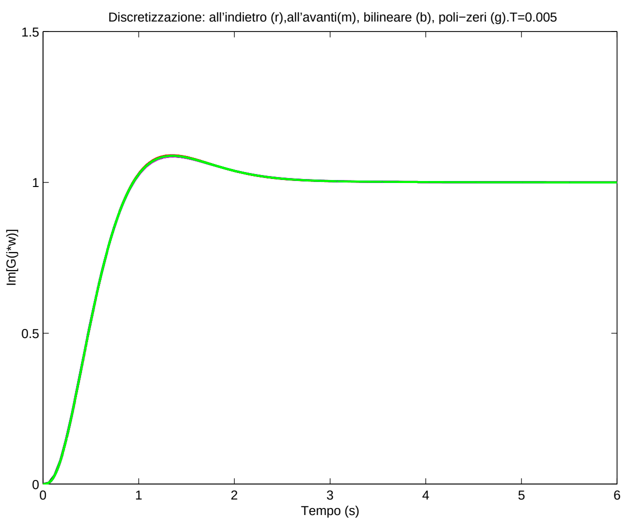


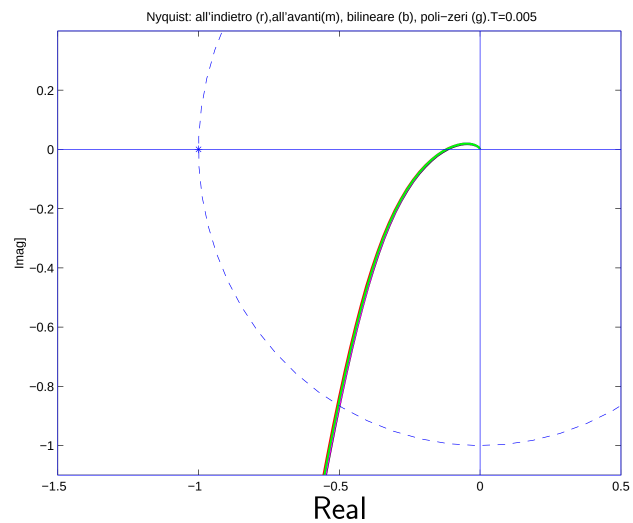
- Answers to the step and Nyquist diagrams when $T = 0.05$:





- Answers to the step and Nyquist diagrams when $T = 0.005$:

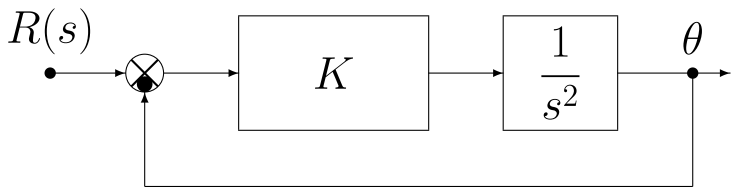




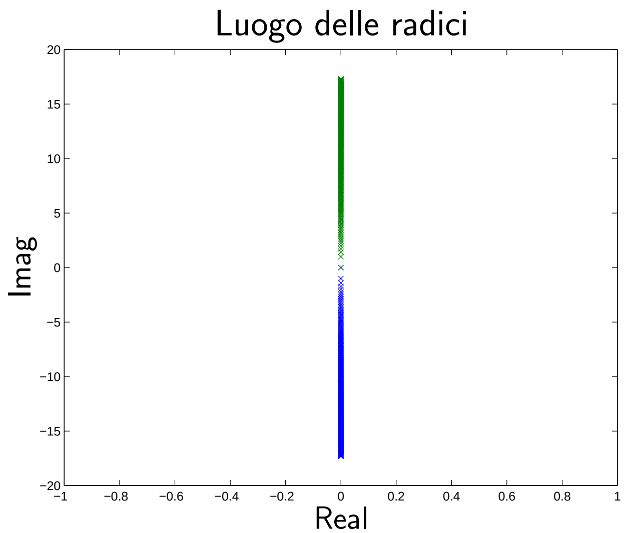
- For sampling periods so small the discrete controllers all have a substantially equivalent behavior.

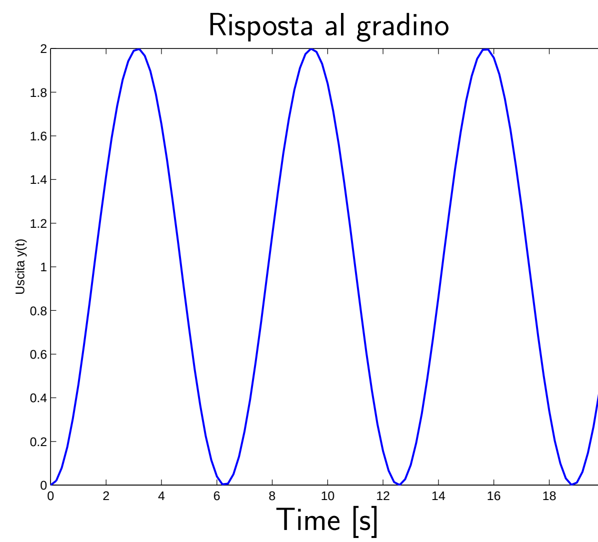
Exercise: control of a double supplement

- Place of the roots to vary the gain K:

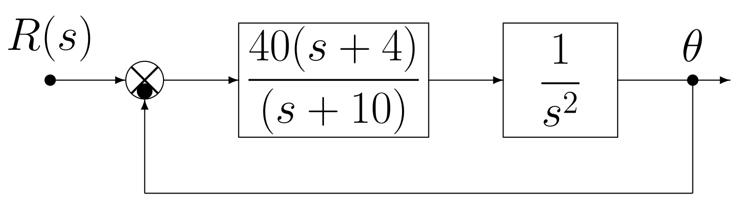


- The feedback system is simply stable for all values of $K > 0$.

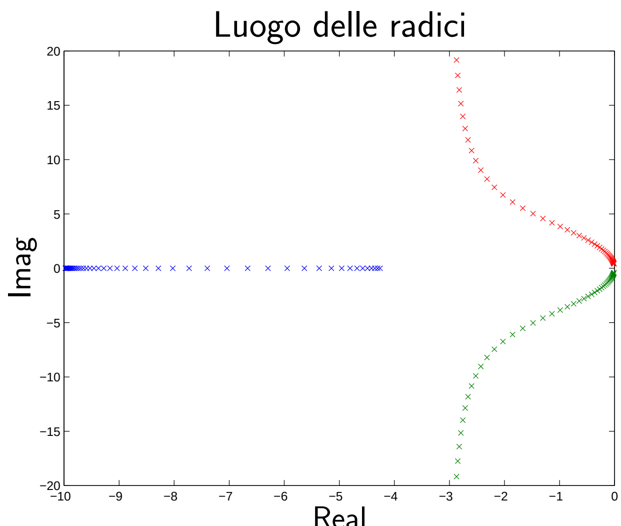


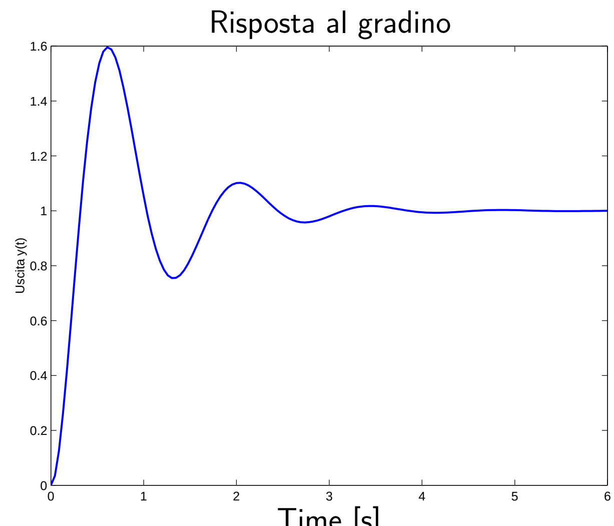


- The system can be stabilized using an anticipatory network:



- Place of the roots and response to the step of the feedback system:





- Let $T = 0.03\,s$. The discretization of the regulator

$$D(s) = \frac{40(s+4)}{(s+10)}$$

It can be done using various methods:

1) method of backwards differences:

$$D_1(z) = D(s)|_{s=\frac{1-z^{-1}}{T}} = \frac{4.48 - 4\,z^{-1}}{0.13 - 0.1\,z^{-1}}$$

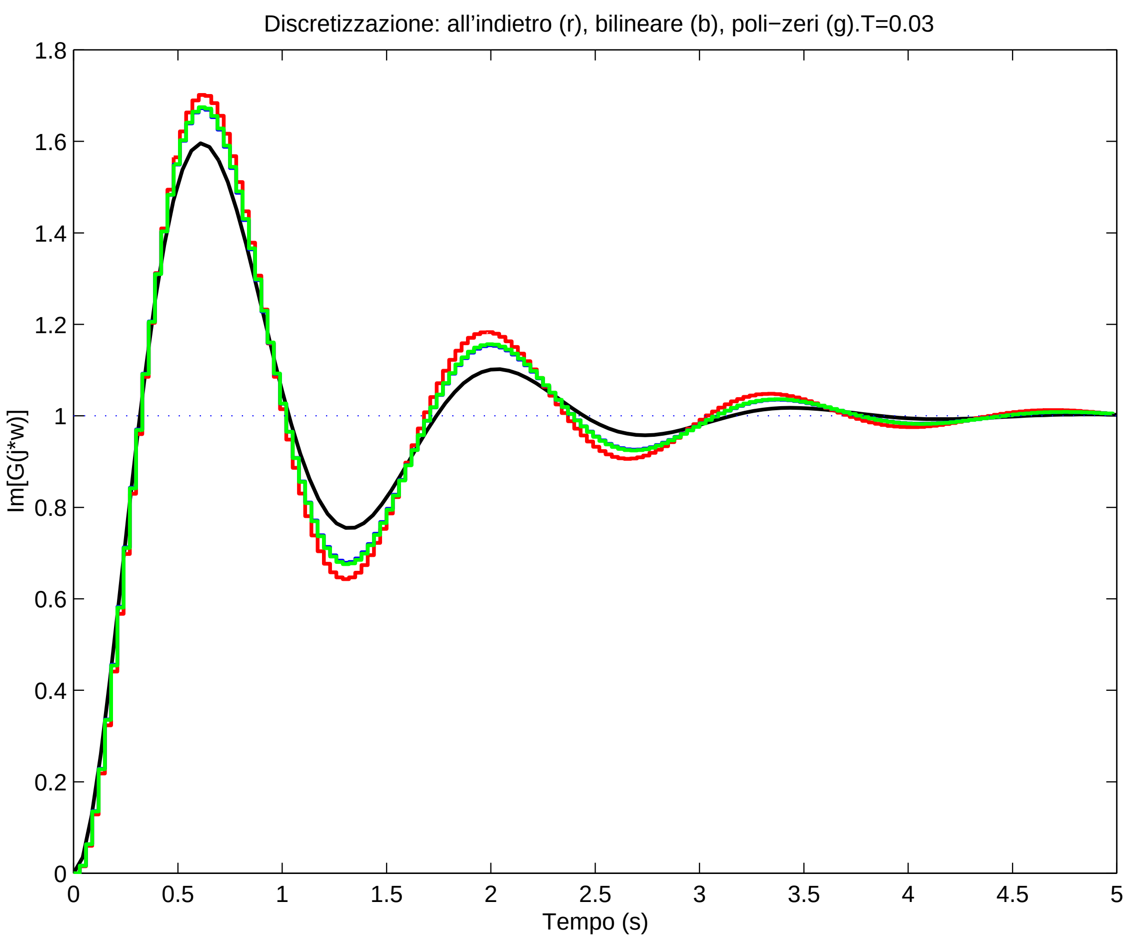
2) method of bilinear transformation:

$$D(z)_2 = D(s)|_{s=\frac{2}{T}\frac{1-z^{-1}}{1-z^{-1}}} = \frac{8.48 - 7.52\,z^{-1}}{0.23 - 0.17\,z^{-1}}$$

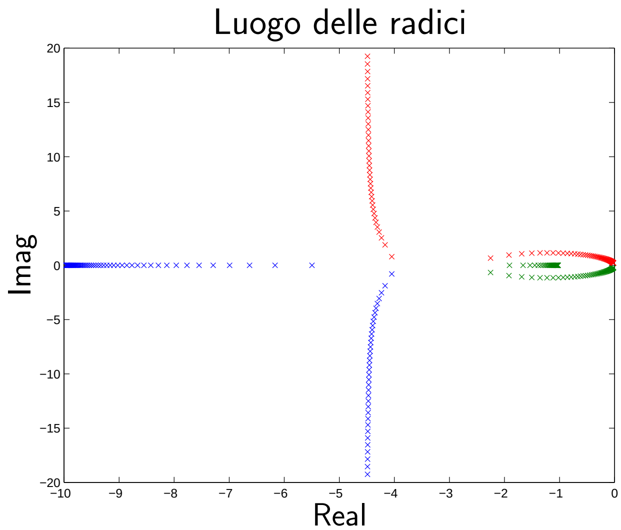
3) method of poly-zero matching:

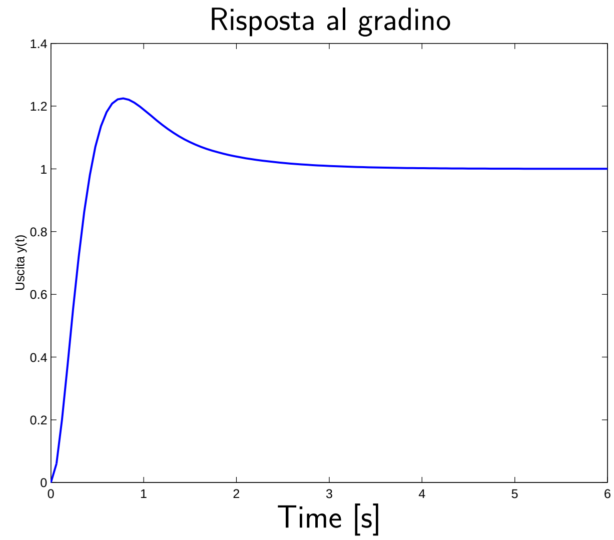
$$D(z)_3 = k\,\frac{1 - e^{-aT}z^{-1}}{1 - e^{-bT}z^{-1}} = \frac{36.67 - 32.53\,z^{-1}}{1 - 0.7408\,z^{-1}}$$

- System response retroazoined to the unit step:



- Place of the roots and response to the step of the feedback system if you use the corrector network $D(s) = \frac{40(s+1)}{(s+10)}$:





- Answers to the step that are obtained by discretizing the corrector network $D(s)$ and using the sampling period $T = 0.03$ s: