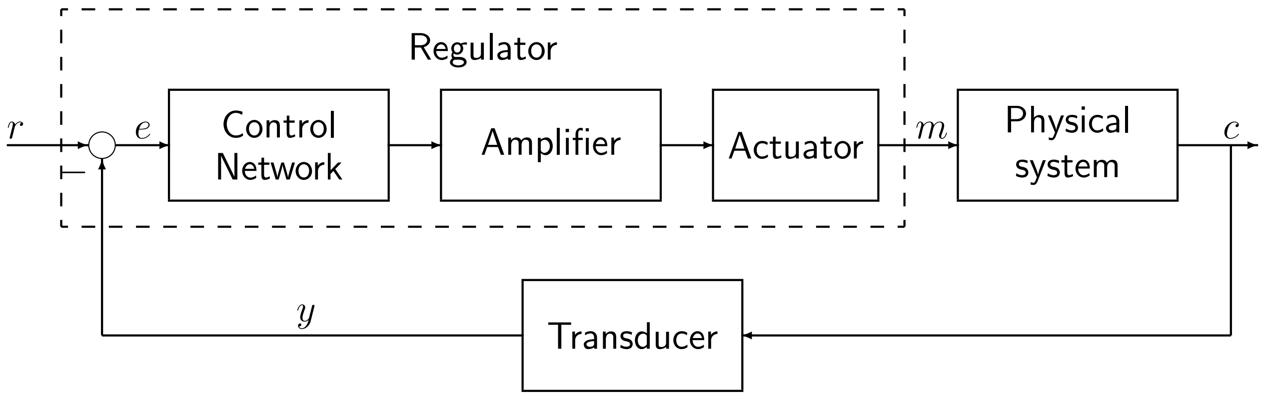
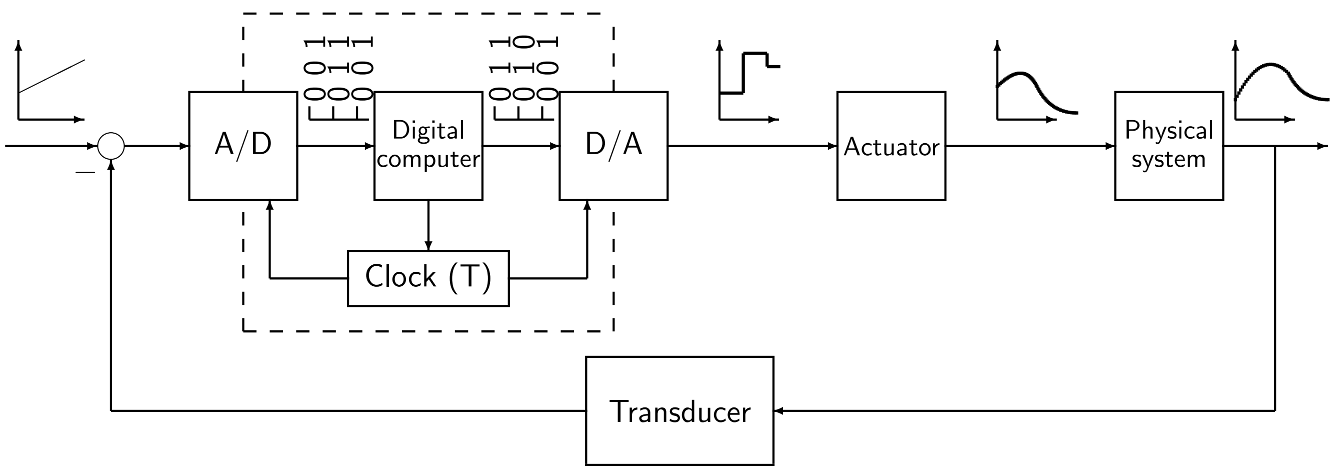
DIGITAL CONTROL

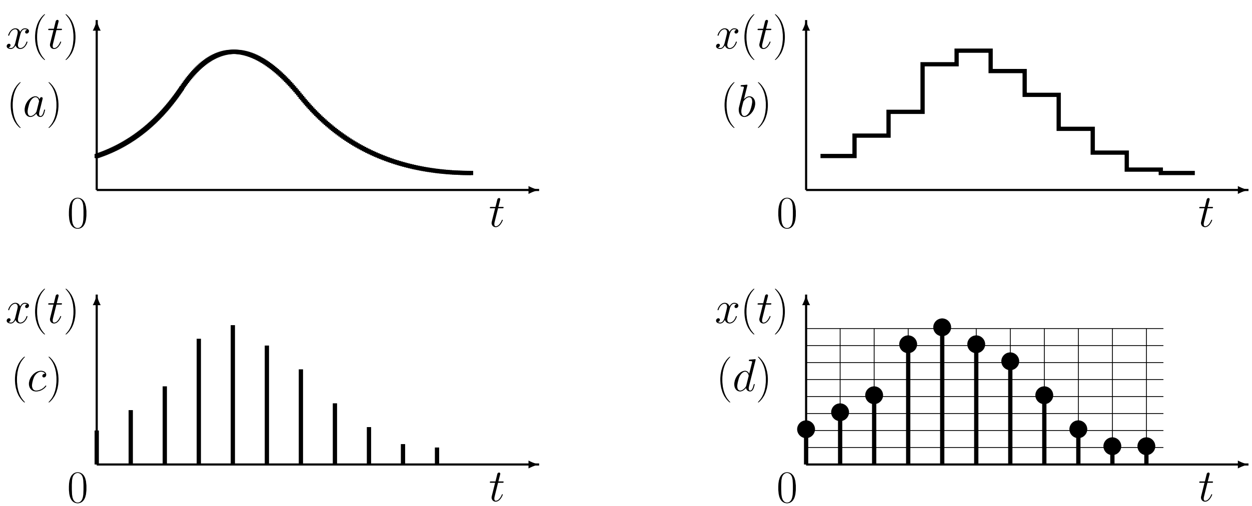
- ANALOG CONTROL SYSTEM:



- DIGITAL CONTROL SYSTEM:



a) Continuous signal; b) Reconstructed continuous signal; c) Sampled signal; d) Sampled and quantized signal. ;



- Analog to Digital converter:

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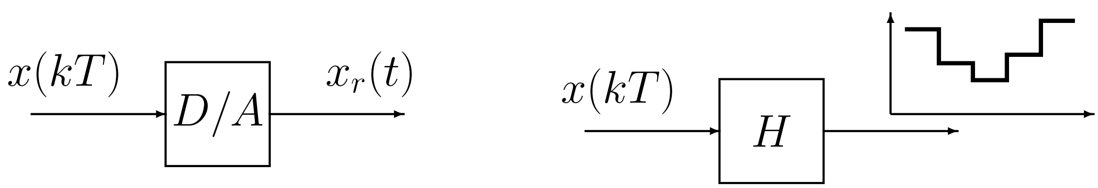


- Zero Order Reconstructor:

$$G_r(s) = \frac{1 - e^{-sT}}{s}$$

- The discrete systems can be described by using difference equations.

- Difference equations: linear or nonlinear static equations which provides a constraint between the input and output sequences at the instants k, $k-1$, $k-2$, etc. (i.e. e_k, e_{k-1}, e_{k-2}, ..., u_k , u_{k-1} , u_{k-2}, ...):

$$u_k = f(e_0, e_1, \ldots, e_k; u_0, u_1, \ldots, u_{k-1})$$

The difference equation is linear if function $f(\cdot)$ is linear:

$$u_k = -a_1 u_{k-1} - \ldots - a_n u_{k-n} + b_0 e_k + \ldots + b_m e_{k-m}$$

- Example of a first order differential equation:

$$y(n) - 0.5\, y(n-1) = u(n)$$

- To solve the difference equations one can use the $\mathcal{Z}$-transform method.

$\mathcal{Z}$-transform method

- Let x_k, for $k = 0, 1, 2, \ldots$, be a sequence of real numbers. The $\mathcal{Z}$-transform of the sequence x_k is a complex function $X(z)$ defined as follows:

$$X(z) = \mathcal{Z}[x_k] = \sum_{k=0}^{\infty} x_k\, z^{-k} = x_0 + x_1\, z^{-1} + \cdots + x_k\, z^{-k} + \cdots$$

- The $\mathcal{Z}$-transform creates a biunivocal correspondence between the sequence x_k and the complex function $X(z)$:

$$x_k \qquad \Leftrightarrow \qquad X(z)$$

Sequence x_k always satisfies the following condition: $x_k = 0$ for $k < 0$.

- If sequence x_k is obtained by sampling a continuous-time signal $x(t)$, then it is $x_k = x(kT) = x(k)$. The following notation:

$$X(z) = \mathcal{Z}[X(s)]$$

will be used to describe the following mathematical relation:

$$X(z) = \mathcal{Z}\Big[\big\{\mathcal{L}^{-1}[X(s)]\big|_{t=kT}\big\}\Big]$$

- $X(z)$ is always a rational function of the complex variable z:

$$X(z) = \frac{b_0\, z^m + b_1\, z^{m-1} + \cdots + b_m}{z^n + a_1\, z^{n-1} + \cdots + a_n}$$

- The same function $X(z)$ can also be expressed as follows:

$$X(z) = \frac{b_0\, z^{-(n-m)} + b_1\, z^{-(n-m+1)} + \cdots + b_m\, z^{-n}}{1 + a_1\, z^{-1} + \cdots + a_n z^{-n}}$$

- Example:

$$X(z) = \frac{z(z+0.5)}{(z+1)(z+2)} = \frac{1 + 0.5\, z^{-1}}{(1+z^{-1})(1+2\, z^{-1})}$$

$\mathcal{Z}$-transform of basic signals

- Unitary discrete impulse (also called Kronecker function $\delta_0(k)$):

$$x(k) = \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases} \qquad \leftrightarrow \qquad X(z) = 1$$

- Unitary step:

$$x(k) = \begin{cases} 1 & k = 0, 1, 2, \ldots \\ 0 & k < 0 \end{cases} \qquad \leftrightarrow \qquad X(z) = \frac{z}{z-1}$$

- Unitary ramp:

$$x(k) = \begin{cases} k & k \geq 0 \\ 0 & k < 0 \end{cases} \qquad \leftrightarrow \qquad X(z) = \frac{z}{(z-1)^2}$$

- Power function a^k:

$$x(k) = \begin{cases} a^k & k = 0, 1, 2, \ldots \\ 0 & k < 0 \end{cases} \qquad \leftrightarrow \qquad X(z) = \frac{z}{z-a}$$

- Sinusoidal function:

$$x(k) = \begin{cases} \sin(\omega kT) & t \geq 0 \\ 0 & t < 0 \end{cases} \qquad \leftrightarrow \qquad X(z) = \frac{z \sin \omega T}{z^2 - 2z \cos \omega T + 1}$$

- Cosinusoidal function:

$$x(k) = \begin{cases} \cos(\omega kT) & t \geq 0 \\ 0 & t < 0 \end{cases} \qquad \leftrightarrow \qquad X(z) = \frac{z(z - \cos \omega T)}{z^2 - 2z \cos \omega T + 1}$$

PROPRIETIES AND THEOREMS OF THE $\mathcal{Z}$-TRANSFORM METHOD

- Linearity: Given $x(k) = a\,f(k) + b\,g(k)$, the following property holds:

$$\boxed{X(z) = a\,F(z) + b\,G(z)}$$

- Multiplication for a^k. Let $X(z)$ be the $\mathcal{Z}$-transform of signal $x(k)$ and let a be a constant. The following property holds:

$$\boxed{\mathcal{Z}\left[a^k x(k)\right] = X(a^{-1}z)}$$

- Time shift theorem. Let $X(z)$ be the $\mathcal{Z}$-transform of signal $x(k)$ and let n be a positive integer constant. The following property holds:

$$\boxed{\mathcal{Z}[x(k-n)] = z^{-n}X(z)} \qquad \text{(delay)}$$

$$\boxed{\mathcal{Z}[x(k+n)] = z^n\left[X(z) - \sum_{k=0}^{n-1} x(kT)z^{-k}\right]} \qquad \text{(advance)}$$

- Initial value theorem. Let $X(z)$ be the $\mathcal{Z}$-transform of signal $x(k)$. The following property holds:

$$\boxed{x(0) = x(k)|_{k=0} = \lim_{z\to\infty} X(z)}$$

- Final value theorem. Let all the poles of $X(z)$ be inside the unit circle, with at most one simple pole for $z = 1$. The following property holds:

$$\boxed{\lim_{k\to\infty} x(k) = \lim_{z\to 1}\left[(1-z^{-1})X(z)\right]}$$

Discretization of continuous time signals

- Unitary step:

$$\mathcal{Z}\left[\frac{1}{s}\right] = \mathcal{Z}[h(t)|_{t=kT}] = \frac{z}{z-1}$$

- Unitary ramp:

$$\mathcal{Z}\left[\frac{1}{s^2}\right] = \mathcal{Z}[t|_{t=kT}] = \mathcal{Z}[k\,T] = T\frac{z}{(z-1)^2}$$

- Exponential function:

$$\mathcal{Z}\left[\frac{1}{s+b}\right] = \mathcal{Z}\left[e^{-b\,t}\big|_{t=kT}\right] = \mathcal{Z}\left[e^{-b\,k\,T}\right] = \mathcal{Z}\left[(e^{-b\,T})^k\right] = \frac{z}{z-e^{-b\,T}}$$

- Let us consider the following function:

$$X(z) = \frac{T\,z(z+1)}{2(z-0.5)(z-1)}$$

The initial value of sequence $x(k) = \mathcal{Z}^{-1}[X(z)]$ can be obtained as follows:

$$x(0) = \lim_{k\to 0} x(k) = \lim_{z\to\infty} \frac{T\,z(z+1)}{2(z-0.5)(z-1)} = \frac{T}{2}$$

The final value of sequence $x(k) = \mathcal{Z}^{-1}[X(z)]$ can be obtained as follows:

$$\begin{aligned}\lim_{k\to\infty} x(k) &= \lim_{z\to 1}(1-z^{-1})\frac{T\,z(z+1)}{2(z-0.5)(z-1)}\\ &= \lim_{z\to 1}\frac{T(z+1)}{2(z-0.5)}\\ &= 2T\end{aligned}$$

Inverse $\mathcal{Z}$-transform

- Let us consider the following function $X(z)$:

$$X(z) = \frac{b_0 z^m + b_1 z^{m-1} + \cdots + b_{m-1} z + b_m}{(z-p_1)(z-p_2)\cdots(z-p_n)}$$

- Using the simple fraction decomposition, function $X(z)$ can be rewritten as follows:

$$X(z) = \frac{\bar{c}_1}{z-p_1} + \frac{\bar{c}_2}{z-p_2} + \cdots + \frac{\bar{c}_n}{z-p_n} = \sum_{i=1}^{n} \frac{\bar{c}_i}{z-p_i}$$

where coefficients $\bar{c}_i$ can be computed as follows:

$$\bar{c}_i = \Big[(z-p_i)X(z)\Big]_{z=p_i}$$

- The inverse $\mathcal{Z}$-transform function $x(k)$ can be written as follows:

$$X(z) = z^{-1}\sum_{i=1}^{n} \frac{\bar{c}_i\, z}{z-p_i} \qquad \Leftrightarrow \qquad x(k) = \begin{cases} 0 & k=0 \\ \displaystyle\sum_{i=1}^{n} \bar{c}_i\, p_i^{k-1} & k \geq 1 \end{cases}$$

- If in the expression of function $X(z)$ there is at least one zero in the origin, then it is useful to decompose function $X(z)/z$ as follows:

$$\frac{X(z)}{z} = \frac{c_1}{z-p_1} + \cdots + \frac{c_n}{z-p_n} \qquad c_i = \left[(z-p_i)\frac{X(z)}{z}\right]_{z=p_i}$$

- In this case, the inverse $\mathcal{Z}$-transform function can be written as follows:

$$X(z) = \sum_{i=1}^{n} \frac{c_i\, z}{z-p_i} \qquad \Rightarrow \qquad x(k) = \sum_{i=1}^{n} c_i\, p_i^k$$

Example. Solve the following difference equation:

$$c(n+1) = c(n) + i\,c(n)$$

with initial condition $c(0) = c_0$.

[Solution.] Applying the $\mathcal{Z}$-transform to the previous equation one obtains:

$$z\,C(z) - z\,c_0 = (i+1)C(z) \qquad \rightarrow \qquad [z-(1+i)]C(z) = z\,c_0$$

from which it follows:

$$C(z) = \frac{z\,c_0}{z-(1+i)} = \frac{c_0}{1-(1+i)z^{-1}} \qquad \rightarrow \qquad c(n) = c_0(1+i)^n$$

Example. Solve the following difference equation:

$$y(n) = 0.5\,y(n-1) + u(n)$$

when the input signal $u(n) = 1$ is the the unitary step and the initial condition is $y(0) = 0$.

[Solution.] The discrete transfer function $G(z)$ associated to the assigned difference equation is:

$$G(z) = \frac{1}{1-0.5\,z^{-1}}$$

The $\mathcal{Z}$-transform of the input signal $u(n) = 1$ is:

$$U(z) = \frac{1}{1-z^{-1}}$$

The $\mathcal{Z}$-transform of the output signal $y(n)$ is

$$Y(z) = G(z)U(z) = \frac{1}{(1-0.5\,z^{-1})}\,\frac{1}{(1-z^{-1})} = \frac{z^2}{(z-0.5)(z-1)}$$

Applying the simple fraction decomposition to function $Y(z)/z$ one obtains:

$$\frac{Y(z)}{z} = \frac{2}{z-1} - \frac{1}{z-0.5}$$

The inverse $\mathcal{Z}$-transform $y(n)$ is:

$$Y(z) = \frac{2\,z}{z-1} - \frac{z}{z-0.5} \qquad \rightarrow \qquad y(n) = 2 - 0.5^n$$

Correspondence between s and z planes

- The Laplace and $\mathcal{Z}$-transform variables s and z are bound by the following relation:

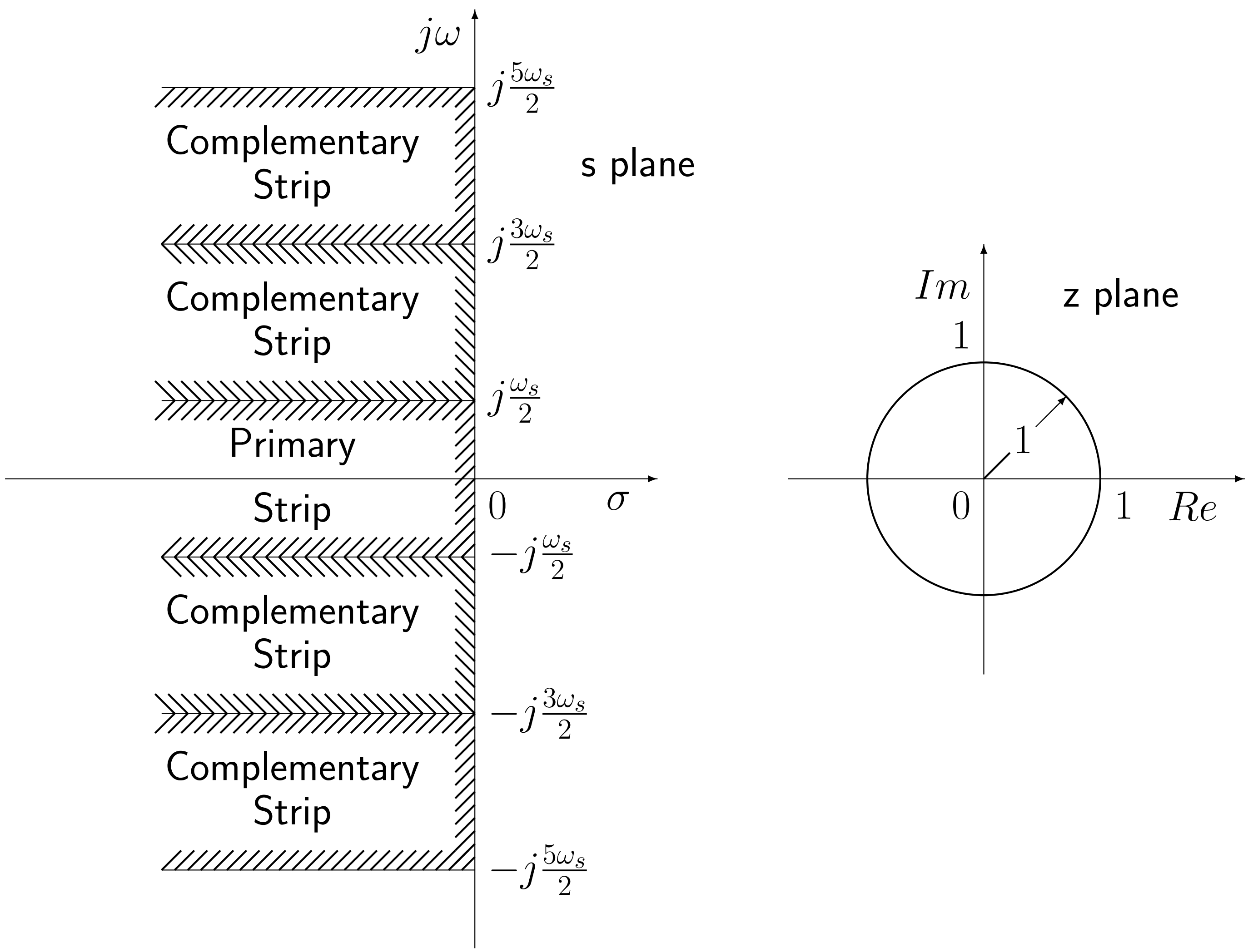
$$\boxed{z = e^{sT}}$$

- Substituting $s = \sigma + j\omega$ one obtains:

$$z = e^{T(\sigma + j\omega)} = e^{T\sigma} e^{jT\omega} = e^{T\sigma} e^{jT(\omega + \frac{2k\pi}{T})}$$

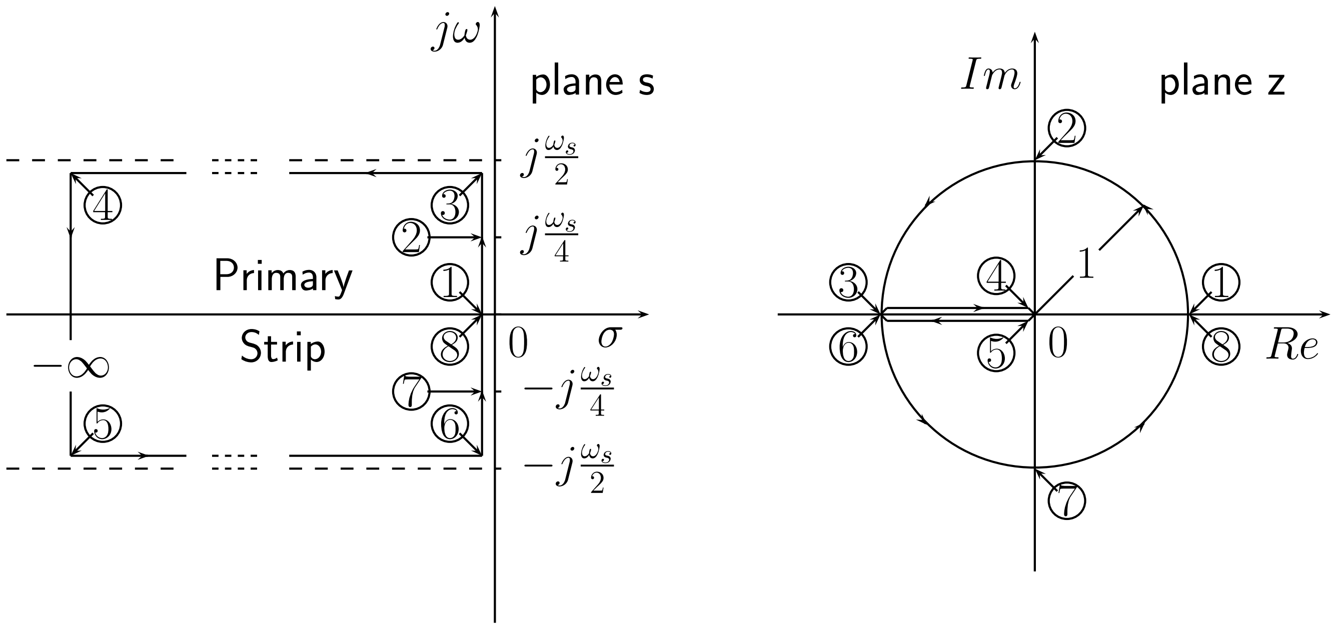
- Function $z = e^{sT}$ does NOT define a biunivocal correspondence between the s and z plane.

- Primary strip and complementary stripes:

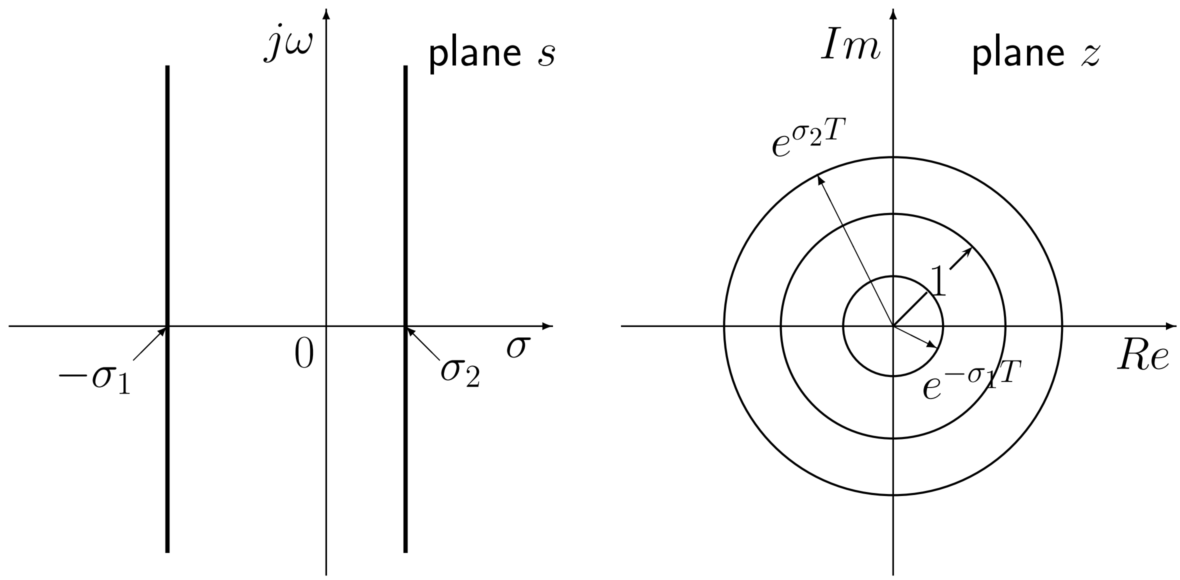


- The z plane is in biunivocal correspondence with the Primary Strip.

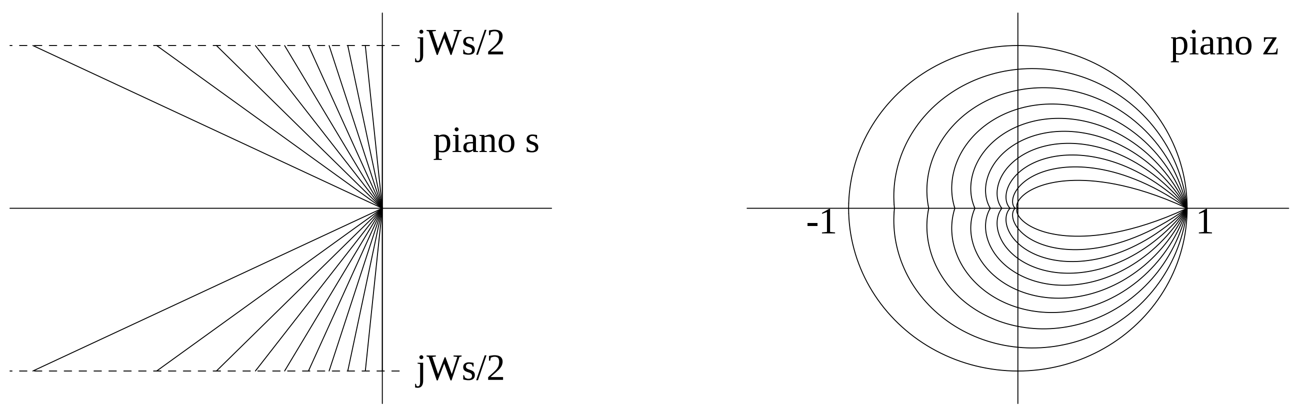
- Mapping between the Primary Strip and the z plane:



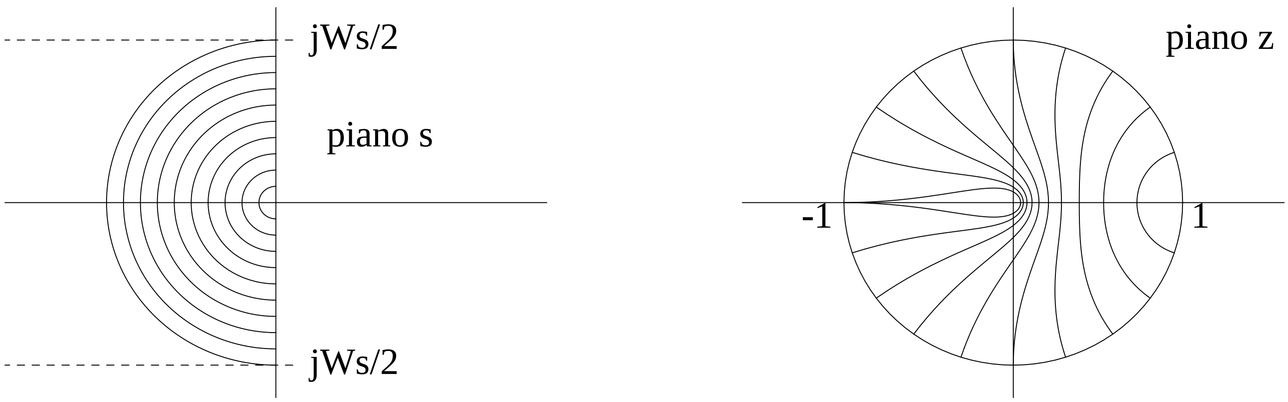
- Places with constant exponential decay:



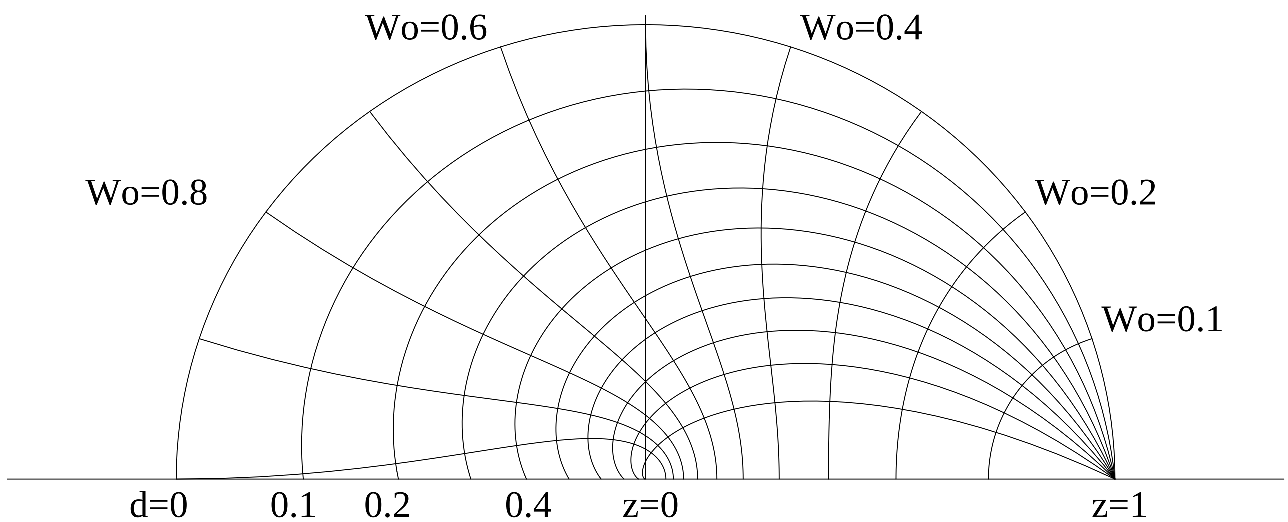
- Places with a constant damping coefficient δ:



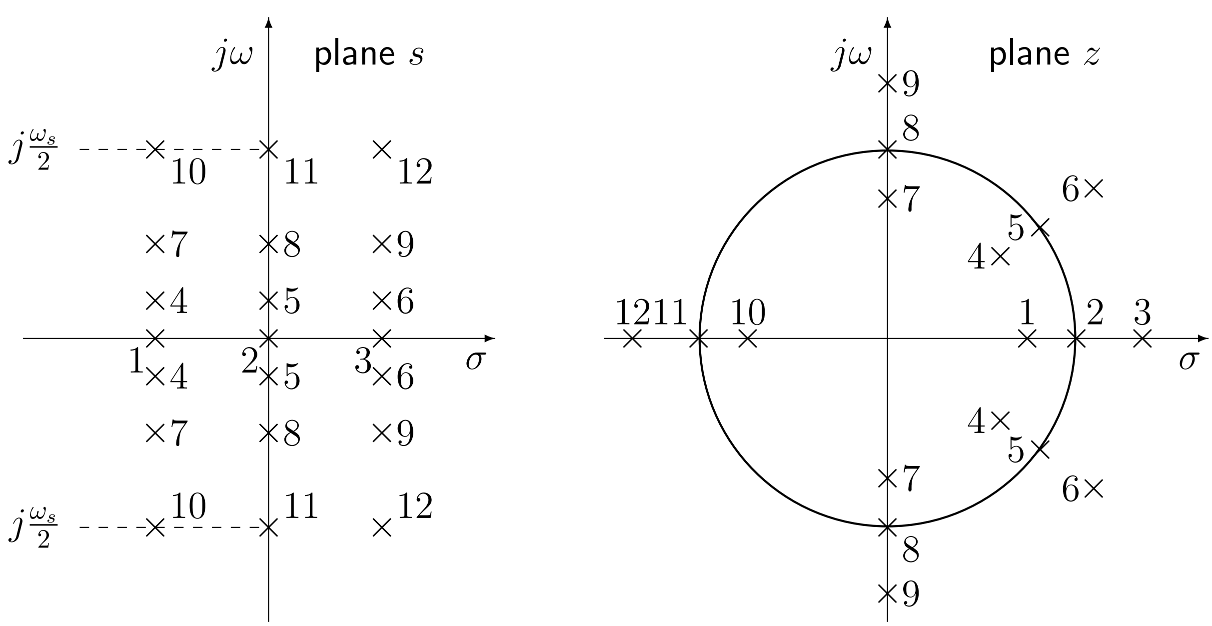
- Places with constant natural frequency ω_n:



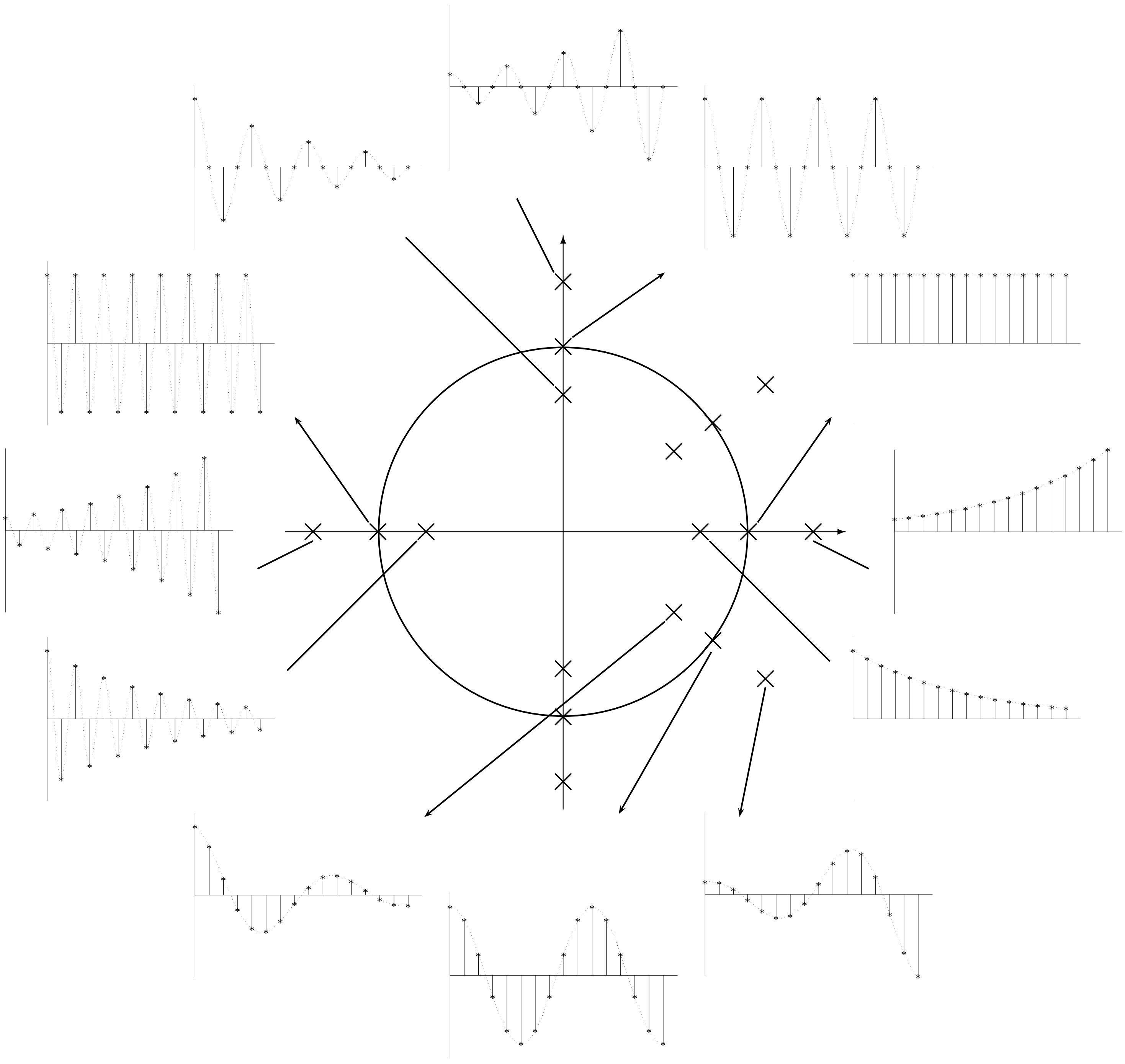
- Places of the z plane where δ and ω_n are constant:



- Correspondences between the s and z poles of the continuous and discrete transfer functions $G(s)$ and $G(z)$ due to the theoretical relation $z = e^{sT}$.



POSITION OF THE POLES ON THE Z PLANE AND THE CORRESPONDING TIME BEHAVIORS



Stability of discrete systems

- Let us consider a discrete transfer function:

$$G(z) = \frac{B(z)}{A(z)}$$

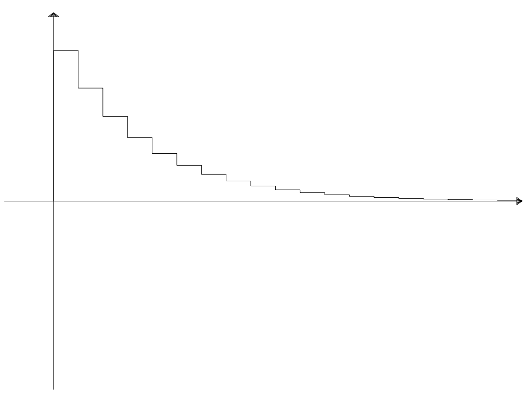
The poles of function $G(z)$ are the roots of the polynomial $A(z)$ at the denominator of function $G(z)$.

Asymptotic Stability: function $G(z)$ is asymptotically stable if all its poles p_i are inside the unit circle: $|p_i| < 1$.

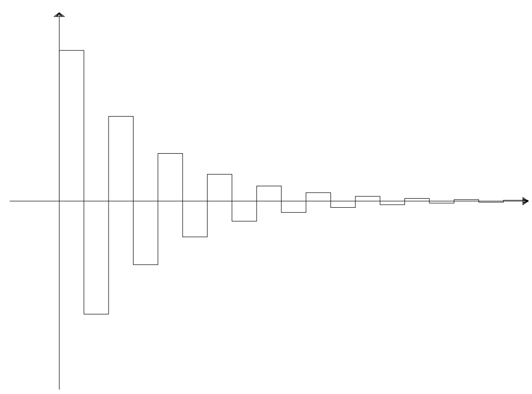
Simple Stability: function $G(z)$ is simply stable if all its poles p_i belong to the unit disk ($|p_i| \leq 1$) and if the poles on the unit circle ($|p_i| = 1$) have unitary multiplicity.

- Time behaviors obtained when the pole is located in $z = 0.75$, $z = -0.75$, $z = 1.25$ and $z = -1.25$:

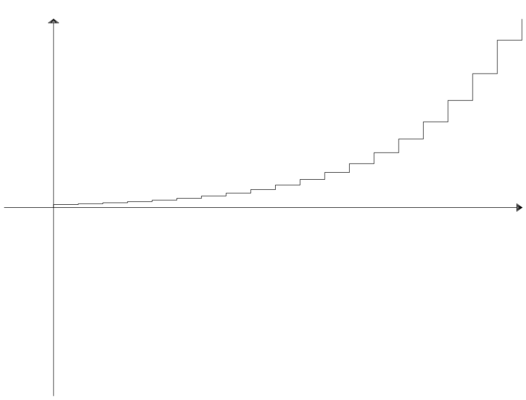
polo in z=0.75



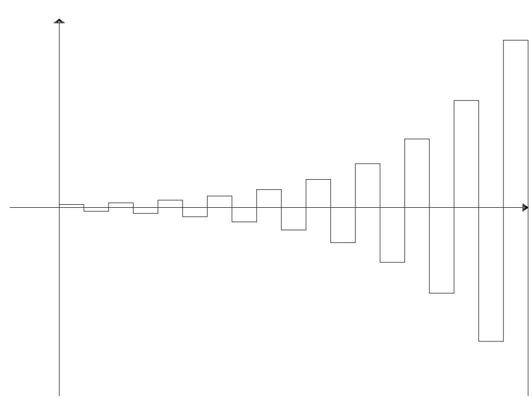
polo in z=-0.75



polo in z=1.25



polo in z=-1.25

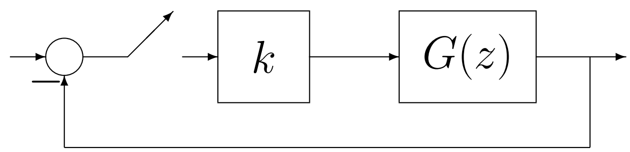


- The frequency response of a discrete function $G(z)$ can be obtained as follows:

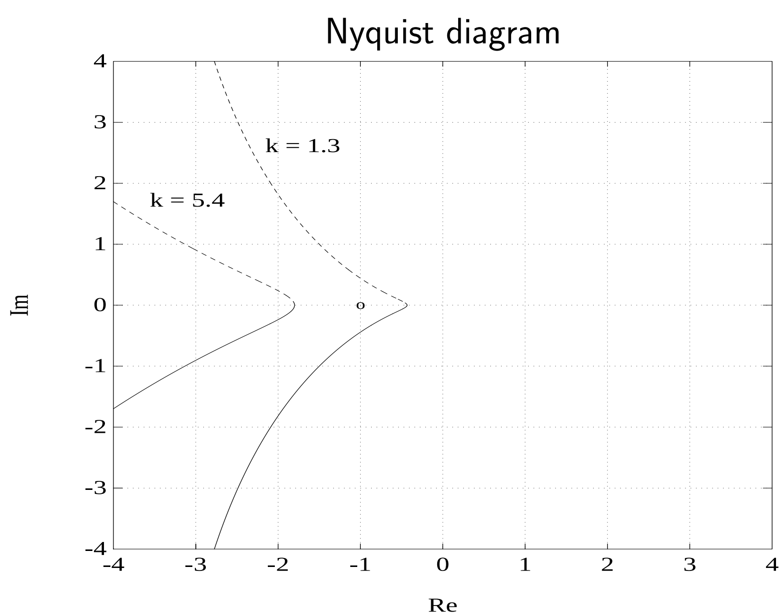
$$G(e^{j\omega T}) = G(z)|_{z=e^{j\omega T}} \qquad \text{for} \qquad -\frac{\pi}{T} \leq \omega \leq \frac{\pi}{T}$$

- The Nyquist Criterion can also be used for discrete feedback system:

  Let $G(z)$ be the ring gain of a discrete feedback system with all its poles asymptotically stable (module smaller than one), except for a simple or double pole in $z = 1$. A necessary and sufficient condition for the feedback system to be asymptotically stable is that the complete Nyquist diagram of the function $G(e^{j\omega T})$, traced for $-\pi/T \leq \omega \leq \pi/T$, does not touch or surround the critical point $-1 + j0$.



- Example: $G(z) = \dfrac{z}{(z-1)(z-0.5)}$



The feedback system is stable for $k = 1.3$ and unstable for $k = 5.4$

- Root Locus. The root locus can be used also for discrete feedback systems. In this case the root locus shows how the roots of the following characteristic equation

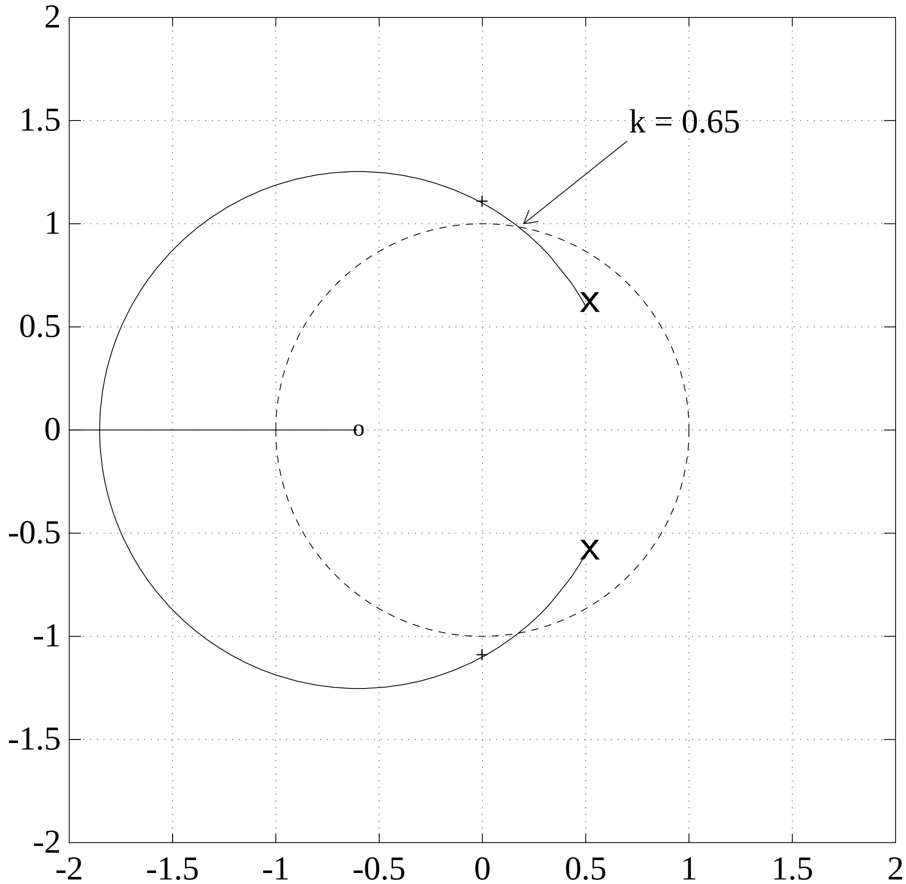
$$1 + k\, G(z) = 0$$

move on the complex plane when parameter k varies from 0 to infinity: $k \in [0,\ \infty]$. The feedback system is asymptotically stable of all the poles which move on the root locus belong to the unitary circle.

- Example. Let us consider the following discrete system:

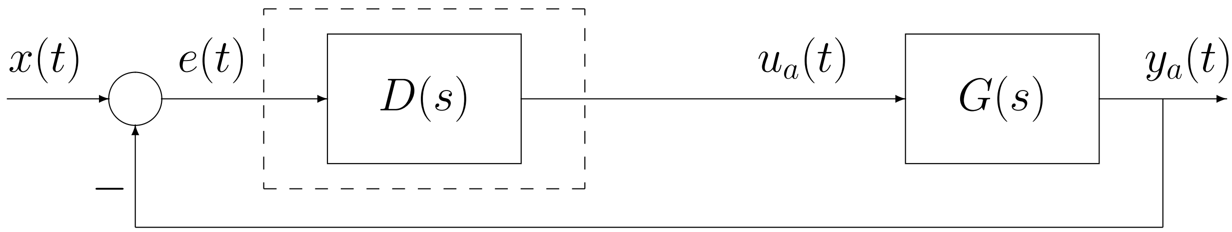
$$G(z) = k\frac{z + 0.6}{z^2 - z + 0.61}$$

The corresponding root locus is the following:

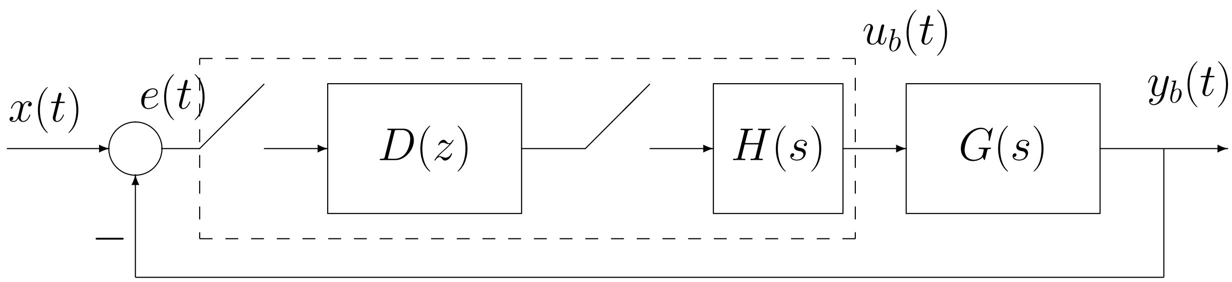


DISCRETIZATION OF CONTINUOUS TIME CONTROLLERS

- Let $D(s)$ be a continuous time controller designed for the continuous system $G(s)$, see figure (a). The corresponding "discretized" controller $D(z)$ can be inserted into the control loop as it is shown in figure (b):



(a)



(b)

- THREE OF THE POSSIBLE DISCRETIZATION TECHNIQUES:
  1. Backward difference method
  2. Forward difference method
  3. Bilinear transformation

- All these techniques are "approximated", that is, they provide a discrete controller $D(z)$ that reproduces well, but not exactly, the dynamic behavior of the $D(s)$ controller.

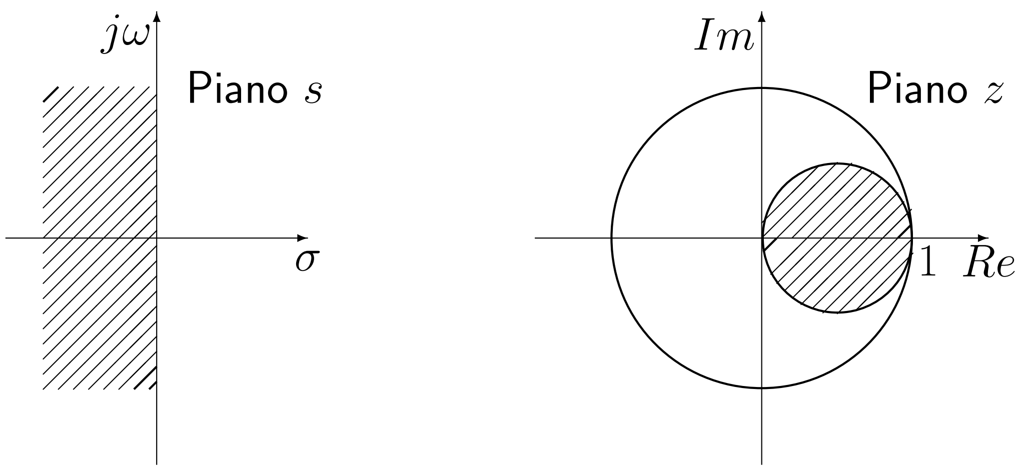
- The smaller is the sampling period T, the more the $D(z)$ controller has a dynamic behavior similar to that of the $D(s)$ controller.

1. BACKWARD DIFFERENCE METHOD

- The method consists in the following replacement:

$$\boxed{D(z) = D(s)\Big|_{s = \frac{1-z^{-1}}{T}}}$$

- Correspondence between the s and z planes:



Example. Using the backward difference method, discretize the following controller using the sampling period $T = 0.1$:

$$D(s) = \frac{M(s)}{E(s)} = \frac{1+s}{1+0.2s}$$

.

[Solution.] Using the backward difference method one obtains:

$$D(s) = \frac{1+s}{1+0.2s} = 5\frac{s+1}{s+5} \quad \rightarrow \quad D(z) = D(s)\big|_{s=\frac{1-z^{-1}}{T}} = \frac{5(1+T-z^{-1})}{1+5T-z^{-1}}$$

The corresponding difference equation can be obtained starting from the following relation:

$$D(z) = \frac{M(z)}{E(z)} = \frac{5(1+T-z^{-1})}{1+5T-z^{-1}}$$

and applying the inverse $\mathcal{Z}$-transform:

$$M(z)(1+5T-z^{-1}) = 5E(z)(1+T-z^{-1}) \quad \leftrightarrow \quad M(z)(1.5-z^{-1}) = E(z)(5.5-5z^{-1})$$

Finally one obtains:

$$m(k) = \frac{1}{1.5}[m(k-1) + 5.5e(k) - 5e(k-1)]$$

that is:

$$m(k) = 0.666\, m(k-1) + 3.666\, e(k) - 3.333\, e(k-1)]$$

Example. Using the backward difference method, discretize the following function:

$$D(s) = \frac{M(s)}{E(s)} = 2\,\frac{s+2}{s+5}$$

using the sampling period $T = 0.1$.

[Solution.] Using the backward difference method one obtains

$$D(z) = D(s)|_{s=\frac{1-z^{-1}}{T}} = 2\,\frac{1+2T-z^{-1}}{1+5T-z^{-1}} = 2\,\frac{1.2-z^{-1}}{1.5-z^{-1}}$$

From this equation one can directly obtains the the corresponding difference equation:

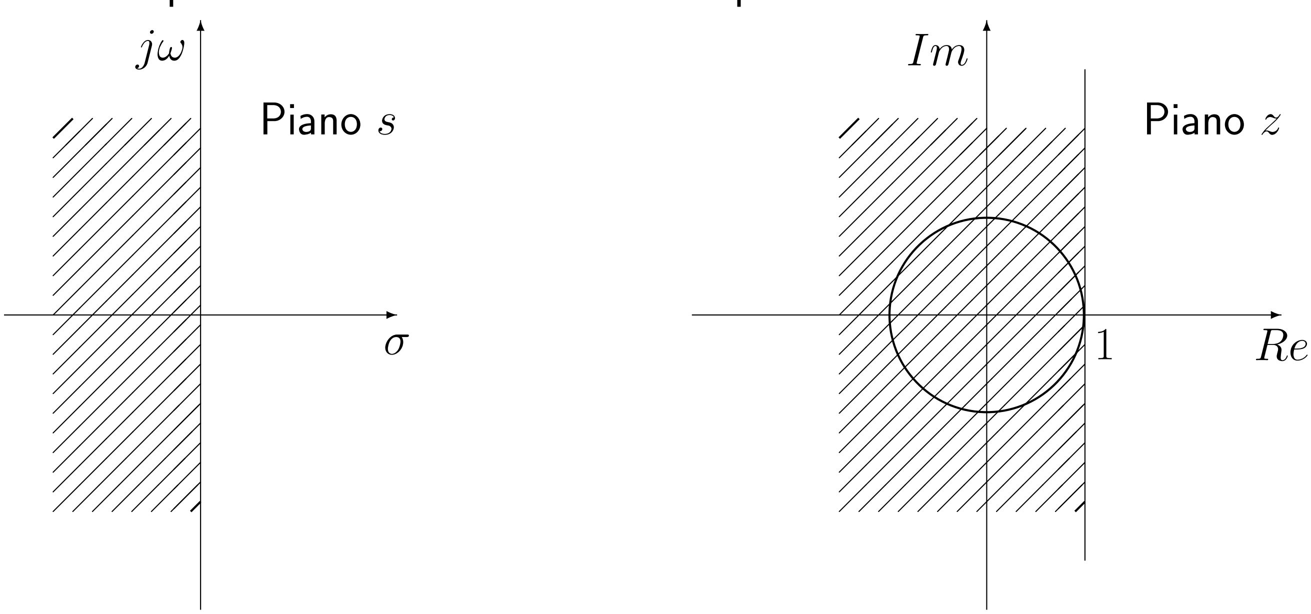
$$m(n) = \frac{1}{1.5}\left[m(n-1) + 2.4e(n) - 2e(n-1)\right]$$

2. FORWARD DIFFERENCE METHOD

- The method consists in the following replacement:

$$\boxed{D(z) = D(s)|_{s = \frac{z-1}{T}}}$$

- Correspondence between the s and z planes:



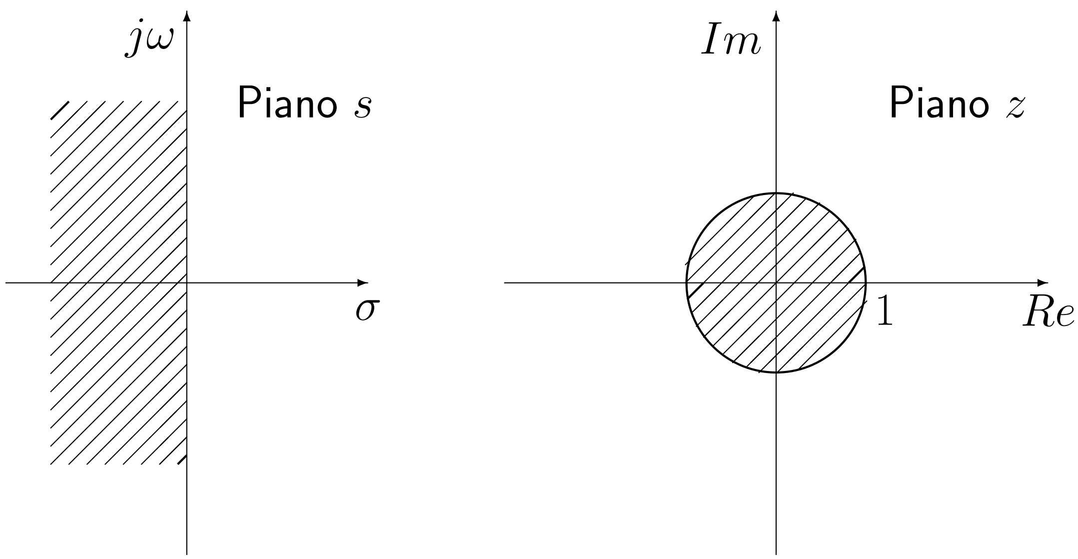
- This method can be used only if the poles of the $D(s)$ controller are mapped by relation $z = e^{sT}$ into the unitary circle of the plane z.

3. BILINARY TRANSFORMATION (or TUSTIN TRANSFORMATION)

- The method consists in the following replacement:

$$D(z) = D(s)|_{s=\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}}$$

- Biunivocal correspondence between the s and z planes:



Example. Using the bilinear transformation method, discretize the following PI regulator:

$$D(s) = \frac{M(s)}{E(s)} = \frac{s+1}{s}$$

using the sampling period $T = 0.2$.

[Solution.] Using the bilinear transformation method one obtains:

$$D(z) = D(s)|_{s=\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}} = \frac{10(1-z^{-1}) + (1+z^{-1})}{10(1-z^{-1})} = \frac{11 - 9\,z^{-1}}{10(1-z^{-1})}$$

The corresponding difference equation can be obtained from the following relation:

$$M(z)(1-z^{-1}) = \frac{E(z)}{10}(11 - 9\,z^{-1})$$

and applying the inverse $\mathcal{Z}$-transform method:

$$m(k) = m(k-1) + 1.1\,e(k) - 0.9\,e(k-1)$$

Example. Using the bilinear transformation method, discretize the following controller:

$$D(s) = \frac{M(s)}{E(s)} = 2\frac{(1+0.25s)}{(1+0.1s)}$$

using the sampling period $T = 0.05$.

[Solution] Function $D(s)$ can also be rewritten as follows:

$$D(s) = 2\frac{(1+0.25\,s)}{(1+0.1\,s)} = 5\frac{(s+4)}{(s+10)}$$

Using the bilinear transformation method and the sampling period $T = 0.05$ one obtains:

$$D(z) = 5\frac{(s+4)}{(s+10)}\bigg|_{s=\frac{2}{T}\frac{z-1}{z+1}} = 4.4\frac{z-\frac{9}{11}}{z-\frac{3}{5}} = 4.4\,\frac{1-0.8182\,z^{-1}}{1-0.6\,z^{-1}}$$

The corresponding difference equation is obtained writing the following relation

$$(1-0.6\,z^{-1})M(z) = 4.4(1-0.8182\,z^{-1})E(z)$$

and then applying the inverse $\mathcal{Z}$-transform method:

$$m(k) = 0.6\,m(k-1) + 4.4\,e(k) - 3.6\,e(k-1).$$

Ideal Sampling

- Consider the following continuous time system:

$$G(s) = \frac{25}{s^2 + 2\,s + 5}$$

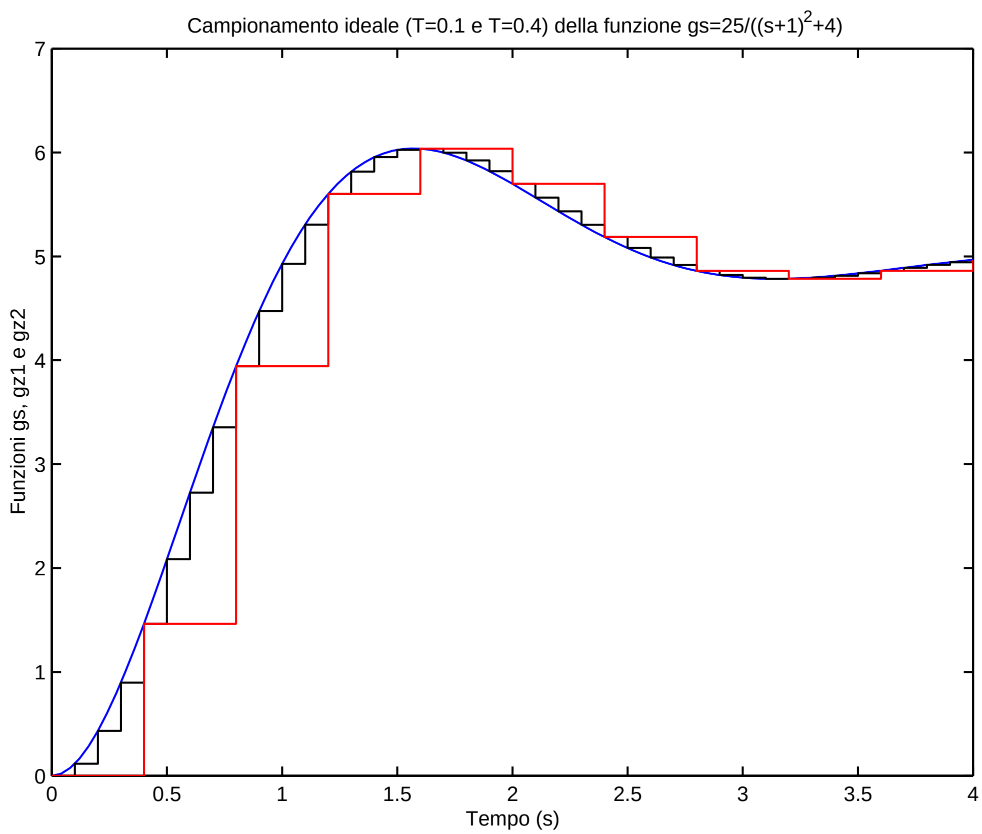
- Discretizing this function using the sampling period $T = 0.1$ one obtains:

$$G_1(z) = \mathcal{Z}[H_0(s)G(s)]_{T=0.1} = \frac{0.1166z + 0.1091}{z^2 - 1.774z + 0.8187}$$

- Using the sampling period $T = 0.4$, one obtains:

$$G_2(z) = \mathcal{Z}[H_0(s)G(s)]_{T=0.4} = \frac{1.463\,z + 1.114}{z^2 - 0.934\,z + 0.4493}$$

- Step response of the 3 functions $G(s)$, $G_1(z)$ and $G_2(z)$:



- Note the exact coincidence of the 3 step responses in the sampling instants.

- In Matlab, a continuous time function "gs" can be discretized using the following command: "`gz=c2d(gs,T)`".

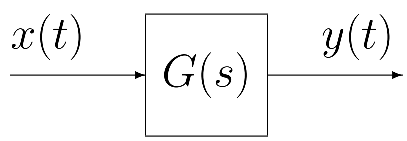
Proportional discrete controller

- Let us consider the following system:

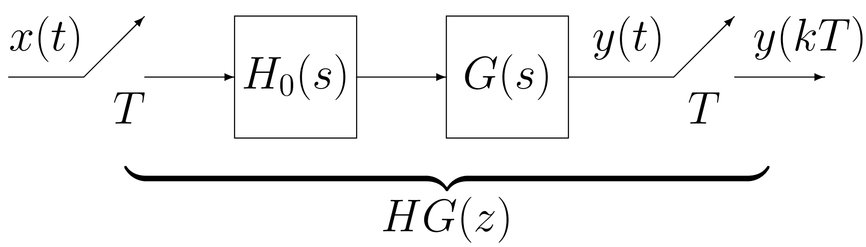
$$G(s) = \frac{25}{s(s+1)(s+10)}$$

and let us compare the frequency responses of the following 3 systems:

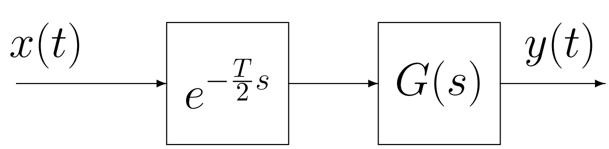
1) The system $G(s)$:



2) The system $HG(z) = \mathcal{Z}[H_0(s)G(s)]$ obtained by inserting a sampler and a zero order hold in cascade to the $G(s)$ system:

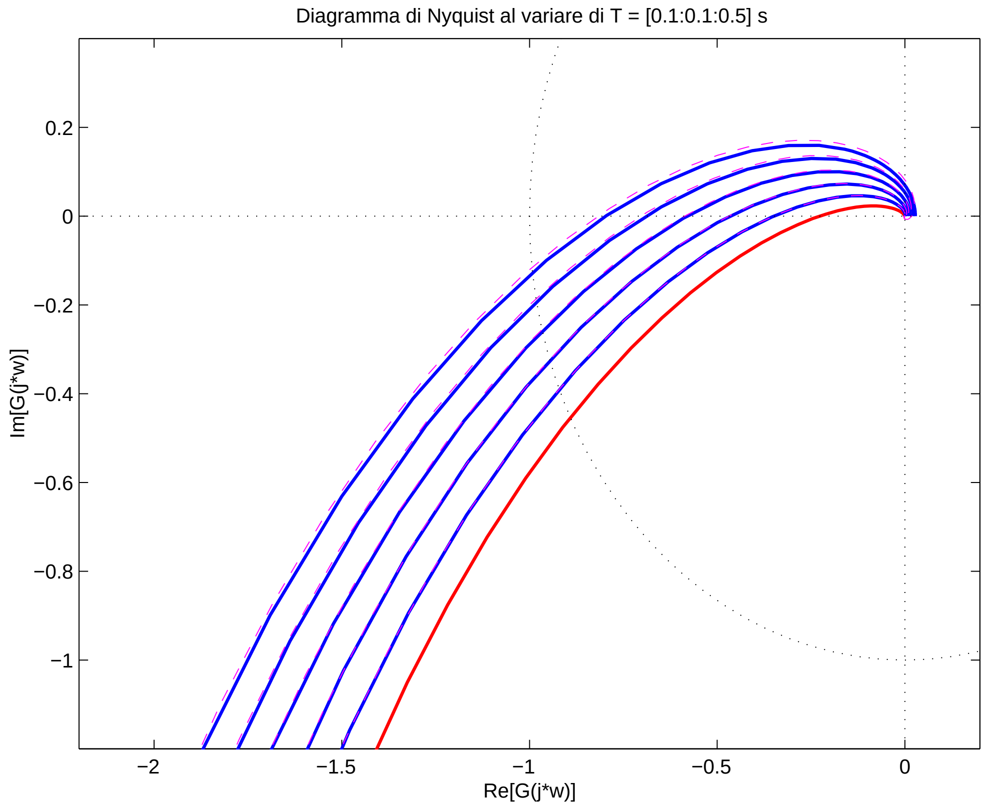


3) The system $G(s)\,e^{-\frac{T}{2}s}$ obtained by replacing the sampler and the zero-order hold with a pure delay $e^{-\frac{T}{2}s}$:



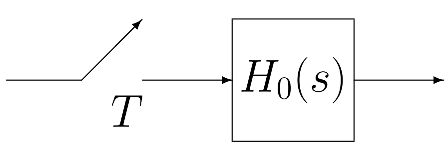
- Function $HG(z, T) = \mathcal{Z}[H_0(s)G(s)]$ has been obtained by discretizing the system $G(s)$ in cascade with a zero order hold $H_0(s)$.

- Nyquist diagrams of systems $G(s)$, $HG(z,T)$ and $G(s)\,e^{-\frac{T}{2}s}$ for $T \in [0.1,\, 0.2,\, 0.3,\, 0.4,\, 0.5]$:



- Note: when the sampling period T increases, also phase shift present within the system increases.

- From the above Nyquist diagrams it is evident that the cascade of the sampler and the zero order hold can be well approximated by a pure delay:



$$\simeq \qquad e^{-\frac{T}{2}s}$$

Comparison between different discretization methods

- Let us consider the following system:

$$G(s) = \frac{25}{s(s+1)(s+10)}$$

and the following lead network properly designed for system $G(s)$:

$$D(s) = \frac{1 + \tau_1\, s}{1 + \tau_2\, s} = \frac{1 + 0.806\, s}{1 + 0.117\, s}$$

- Let discretize controller $D(s)$ using, for example, the following methods:

1) backward difference method:

$$D_1(z) = D(s)\big|_{s=\frac{1-z^{-1}}{T}} = \frac{T + \tau_1 - \tau_1\, z^{-1}}{T + \tau_2 - \tau_2\, z^{-1}}$$

2) forward difference method:

$$D_2(z) = D(s)\big|_{s=\frac{z-1}{T}} = \frac{\tau_1 + (T - \tau_1)\, z^{-1}}{\tau_2 + (T - \tau_2)\, z^{-1}}$$

3) bilinear transformation:

$$D_3(z) = D(s)\big|_{s=\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}} = \frac{T + 2\,\tau_1 + (T - 2\,\tau_1)\, z^{-1}}{T + 2\,\tau_2 + (T - 2\,\tau_2)\, z^{-1}}$$

4) poles-zeros correspondence:

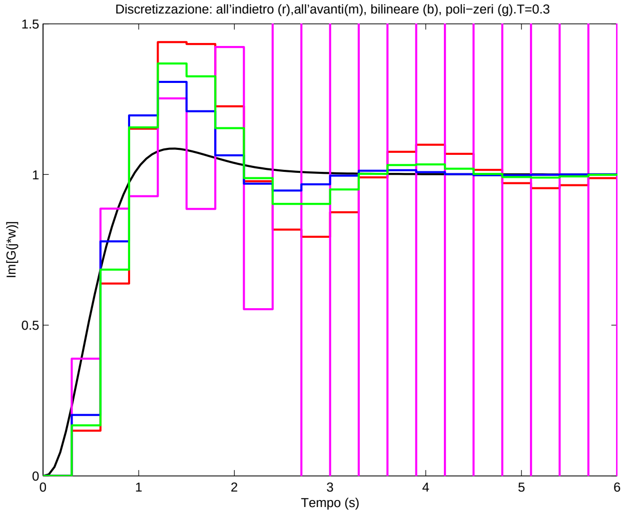
$$D(s) \qquad \rightarrow \qquad D_4(z) = \frac{(1-\beta) - \alpha\,(1-\beta)\, z^{-1}}{(1-\alpha) - \beta\,(1-\alpha)\, z^{-1}}$$

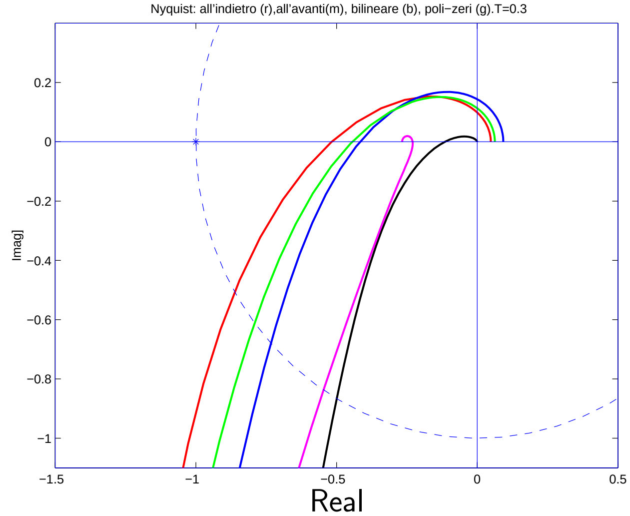
where

$$\alpha = e^{-\frac{T}{\tau_1}}, \qquad \beta = e^{-\frac{T}{\tau_2}}$$

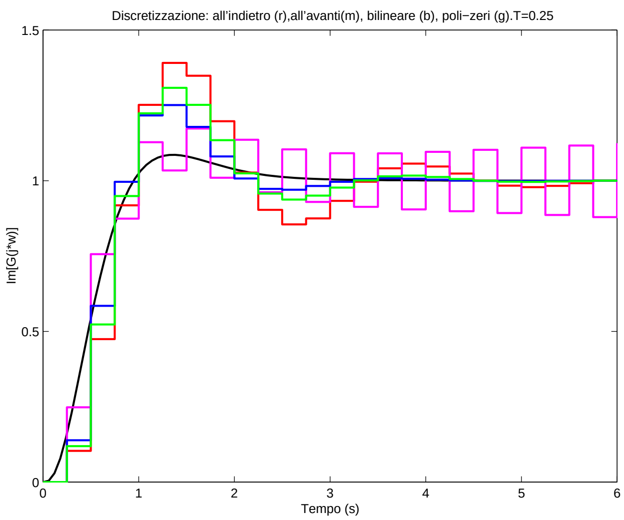
- Note: controller $D_2(z)$ is stable only if $T < 2\,\tau_2$ while the other regulators are stable for $T > 0$.

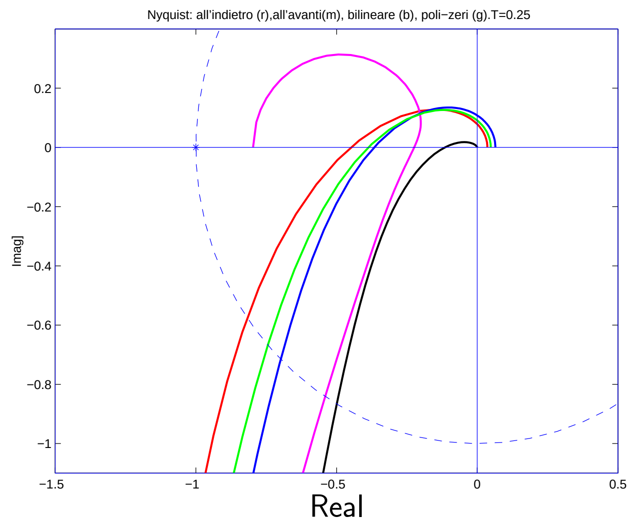
- Step responses and Nyquist diagrams when $T = 0.3$:



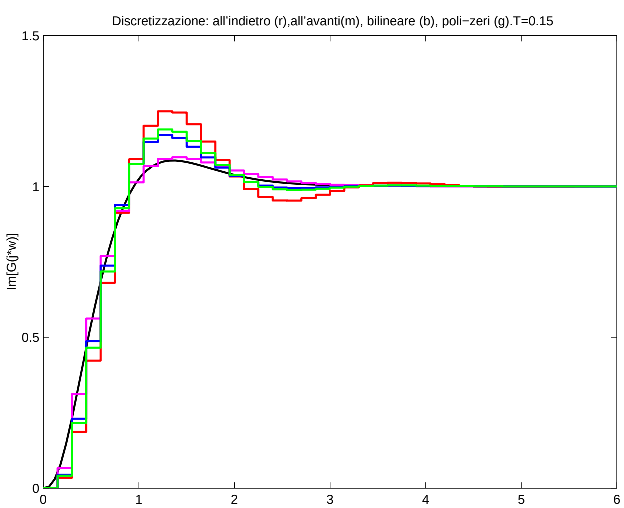


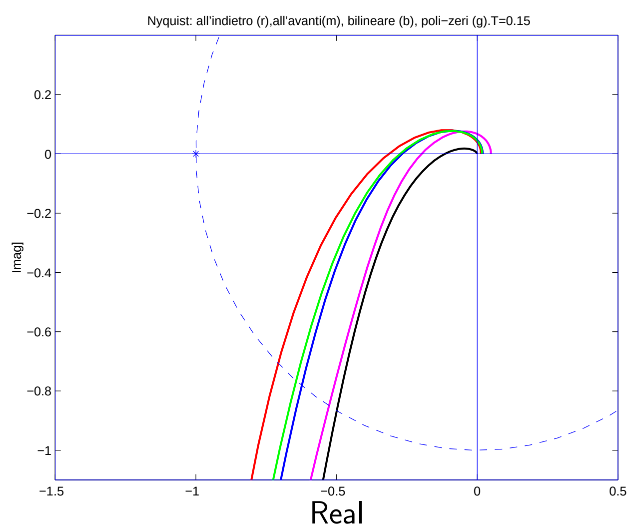
- Step responses and Nyquist diagrams when $T = 0.25$:



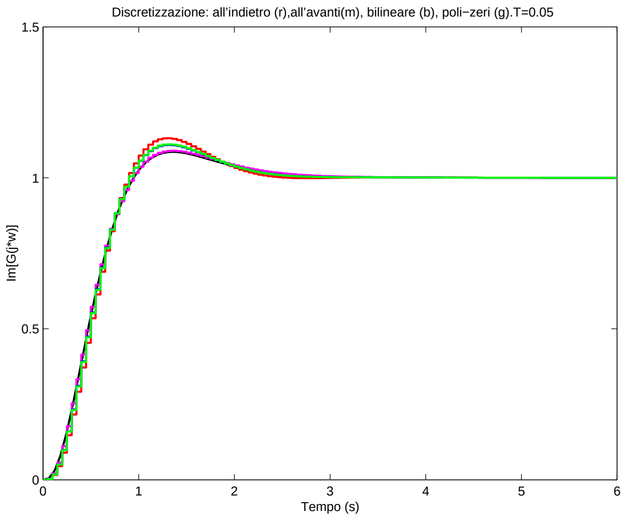


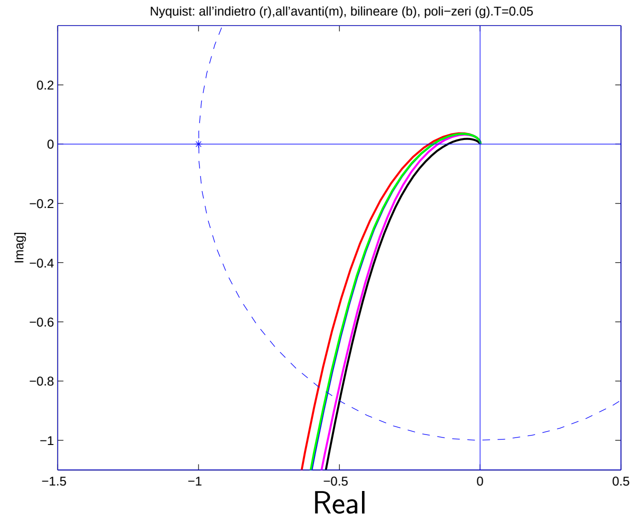
- Step responses and Nyquist diagrams when $T = 0.15$:



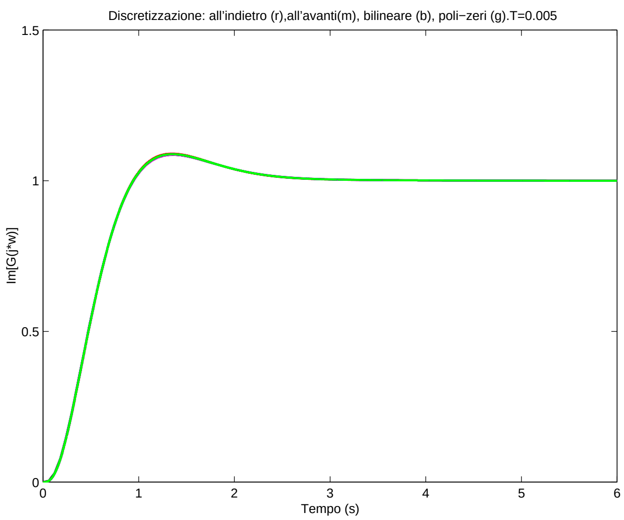


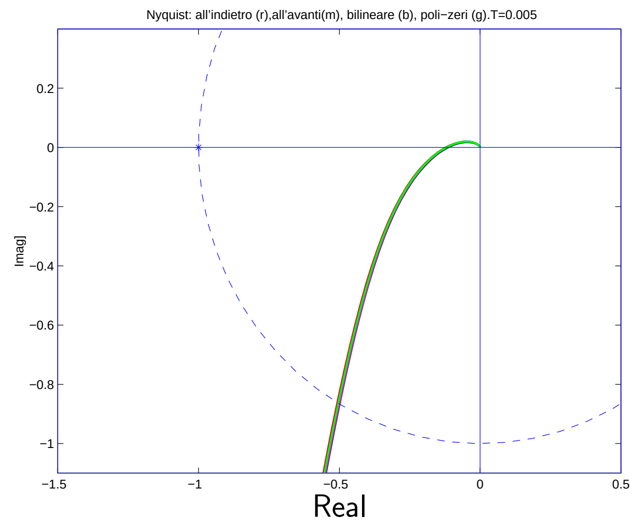
- Step responses and Nyquist diagrams when $T = 0.05$:





- Step responses and Nyquist diagrams when $T = 0.005$:





- For small sampling periods T all the considered discrete controllers provides the same dynamics for the feedback system.