

Contour Locus: examples

Let us refer to the following differential equation which describes the dynamics of a “mass, spring, damper” system:

$$M \ddot{x}(t) + b \dot{x}(t) + K x(t) = F(s)$$

The corresponding transfer function is:

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{M s^2 + b s + K}$$

The dynamic behavior of the given system depends on the position of the pole of function $G(s)$ and, therefore, on the value of parameters M , b and K .

Contour locus of $G(s)$ as a function of parameter $b > 0$. The poles of system $G(s)$ are the poles of the following equation:

$$M s^2 + b s + K = 0$$

which can be rewritten as follows:

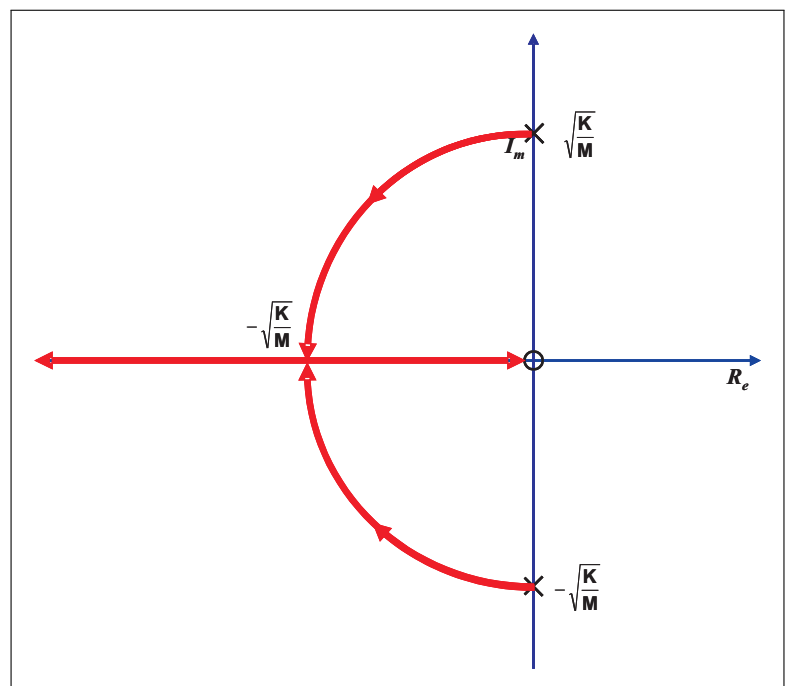
$$1 + \frac{b s}{M s^2 + K} = 0$$

The corresponding root locus when $b > 0$ (see the figure) shows that the minimum settling time is at the branching point σ_a :

$$\sigma_a = -\sqrt{\frac{K}{M}}$$

The corresponding “optimal value” of parameter b is:

$$b^* = -\left. \frac{1}{G(s)} \right|_{s=\sigma_a} = 2 \sqrt{K M}$$



Contour locus of $G(s)$ as a function of parameter $M > 0$. The equation which provides the poles of function $G(s)$ can be rewritten as follows:

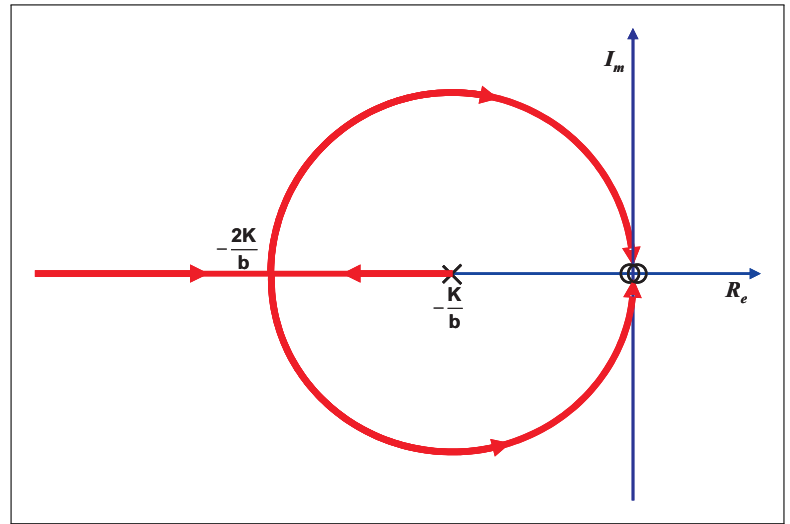
$$1 + \frac{M s^2}{b s + K} = 0$$

The corresponding root locus when $M > 0$ (see the figure) shows that the minimum settling time is obtained at the branching point:

$$\sigma_d = -\frac{2K}{b}$$

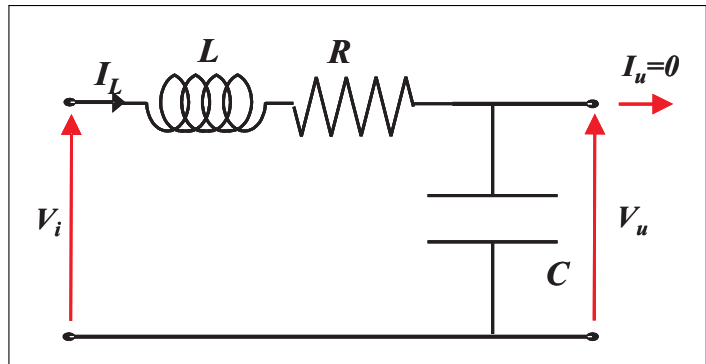
The corresponding “optimal value” of parameter M is:

$$M^* = -\frac{1}{G(s)} \Big|_{s=\sigma_d} = \frac{b^2}{4K}$$



Electric system R, L, C .

The transfer function $G(s)$ of the electrical system (see the figure) can be obtained as follows:



$$G(s) = \frac{V_u(s)}{V_i(s)} = \frac{\frac{1}{Cs}}{Ls + R + \frac{1}{Cs}} = \frac{\frac{1}{C}}{Ls^2 + Rs + \frac{1}{C}}$$

The parametric analysis of this electric system can be directly obtained from the parametric analysis of the previous mechanical system using the following analogies:

$$L \leftrightarrow M, \quad R \leftrightarrow b, \quad C \leftrightarrow \frac{1}{K}$$