Asymptotic Bode diagrams: exercises

- Trace asymptotic Bode diagrams of the following function $G(s)$:

$$G(s) = \frac{60\,(s^2 + 0.8\,s + 4)}{s(s-30)(1+\frac{s}{200})^2}.$$

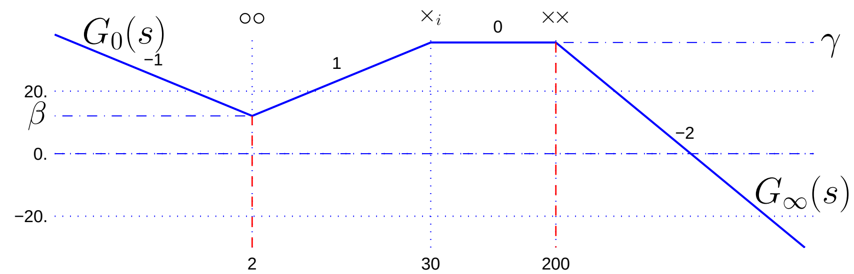
Initial slope: -20 db/dec. Critical pulsations: $\omega = 2$ (two complex conjugate zeros stable), $\omega = 30$ (an unstable pole) and ω =200 (two stable poles).

Function$G_0(s)$:

$$G_0(s) = \frac{60(4)}{s(-30)(1)^2} = -\frac{8}{s}$$

Initial phase $\varphi_0 = -\frac{3}{2}\pi$.

Diagramma asintotico dei moduli

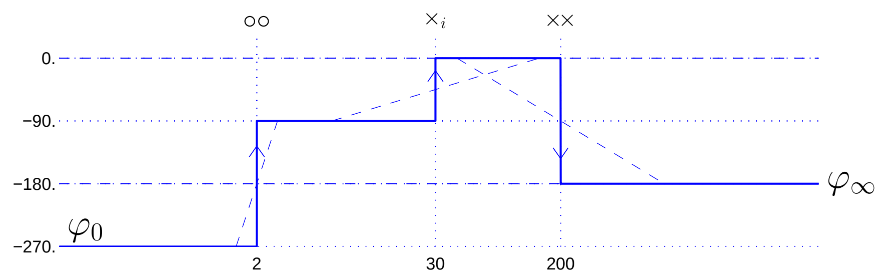


Function$G_\infty(s)$:

$$G_\infty(s) = \frac{60(s^2)}{s(s)(\frac{s}{200})^2} = \frac{2400000}{s^2}$$

Final phase $\varphi_\infty = -\pi$.

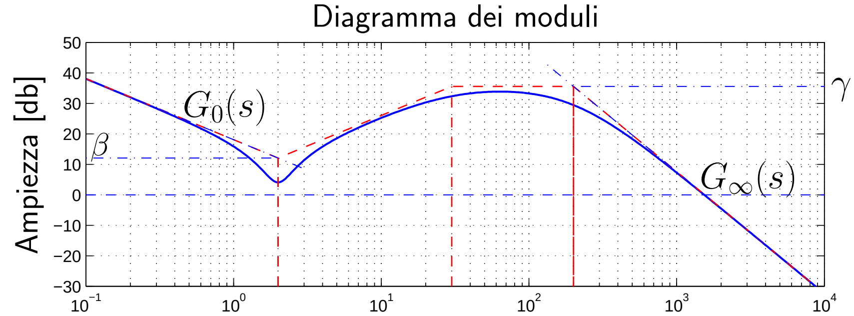
Diagramma a gradoni delle fasi



Gainβ:

$$\beta = |G_0(s)|_{s=2} = 4 = 12 \text{ db}.$$

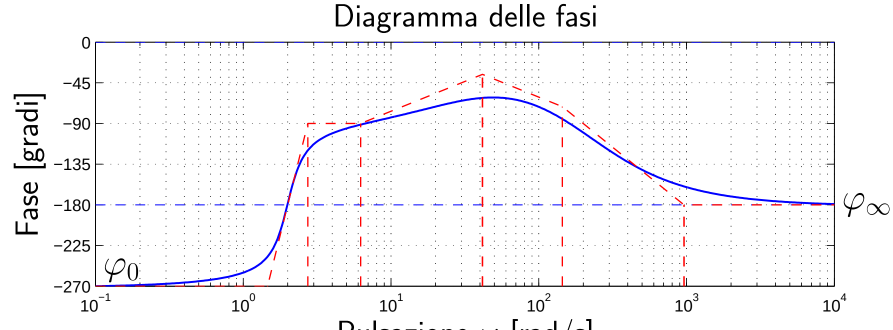
Diagramma dei moduli



Gainγ:

$$\gamma = |G_\infty(s)|_{s=200} = 60 = 35.56 \text{ db}.$$

Diagramma delle fasi

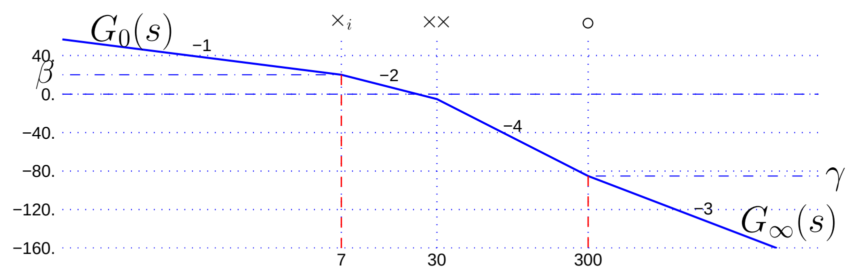


- Trace asymptotic Bode diagrams of the following function $G(s)$:

$$G(s) = \frac{1470\,(s+300)}{s\,(s-7)(s^2+15\,s+900)}.$$

Initial slope: -20 db/dec. Critical pulsations: $\omega = 7$ (unstable pole), $\omega = 30$ (two stable complex conjugate poles) and $\omega = 300$ (a stable zero).

Diagramma asintotico dei moduli

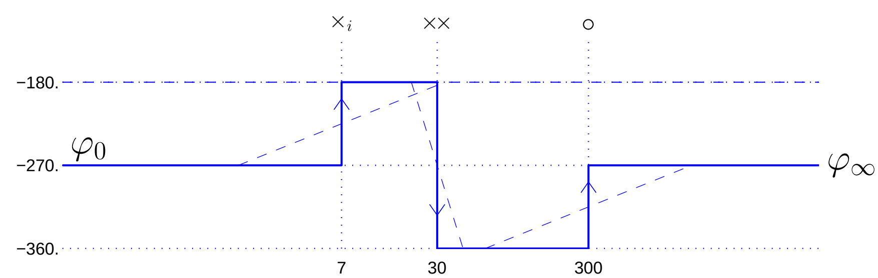


Function$G_0(s)$:

$$G_0(s) = \frac{1470\,(300)}{s\,(-7)(900)} = -\frac{70}{s}$$

Initial phase $\varphi_0 = -\frac{3}{2}\pi$.

Diagramma a gradoni delle fasi

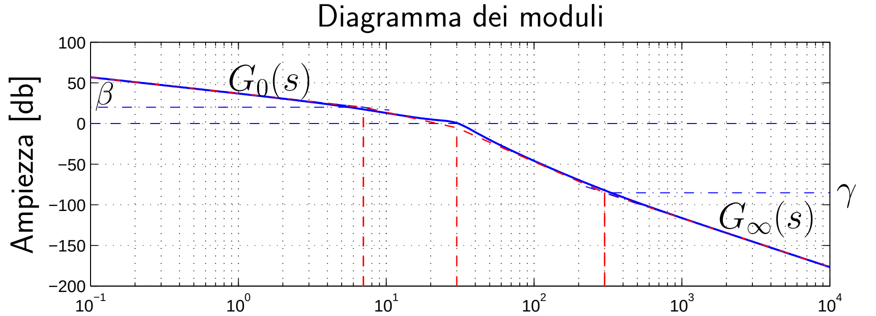


Function$G_\infty(s)$:

$$G_\infty(s) = \frac{1470\,(s)}{s\,(s)(s^2)} = \frac{1470}{s^3}$$

Final phase $\varphi_\infty = -\frac{3}{2}\pi$.

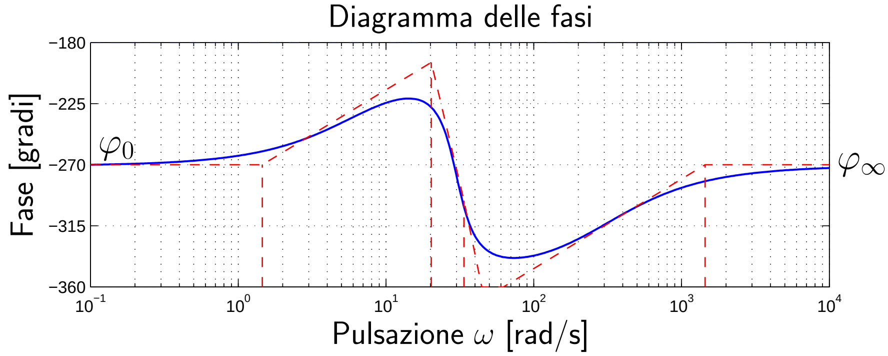
Diagramma dei moduli



Gainβ:

$$\beta = |G_0(s)|_{s=7}$$
$$= 10 = 20 \text{ db.}$$

Diagramma delle fasi



Gainγ:

$$\gamma = |G_\infty(s)|_{s=300}$$
$$= \frac{1470}{300^3} = -85.28 \text{ db.}$$

- Trace asymptotic Bode diagrams of the following function $G(s)$:

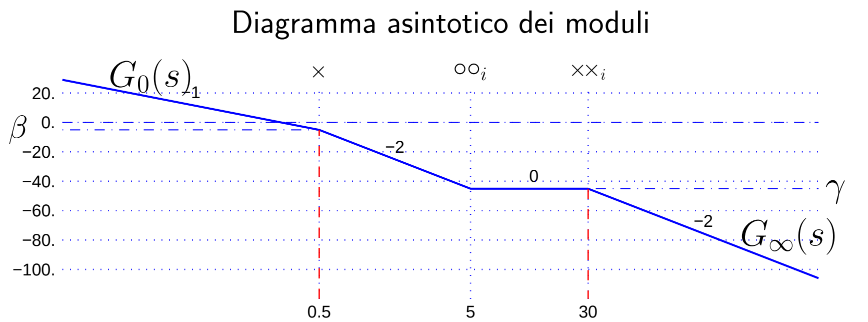
$$G(s) = \frac{50\,(5-s)^2}{s\,(s^2 - 18\,s + 900)(10\,s + 5)}.$$

Initial slope: -20 db/dec. Critical pulsations: $\omega = 0.5$ (a stable pole), $\omega = 5$ (two real unstable zeros) and $\omega = 30$ (two unstable conjugate complex poles).

Function$G_0(s)$:

$$G_0(s) = \frac{50\,(5)^2}{s\,(900)(5)} = \frac{5}{18\,s}$$

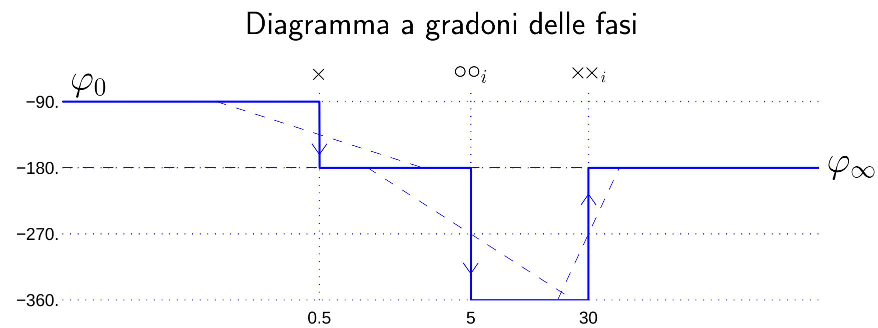
Initial phase $\varphi_0 = -\frac{\pi}{2}$.



Function$G_\infty(s)$:

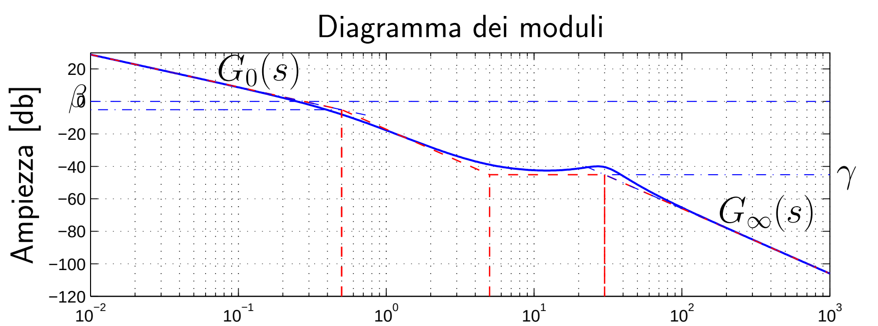
$$G_\infty(s) = \frac{50\,(-s)^2}{s\,(s^2)(10\,s)} = \frac{5}{s^2}$$

Final phase $\varphi_\infty = -\pi$.



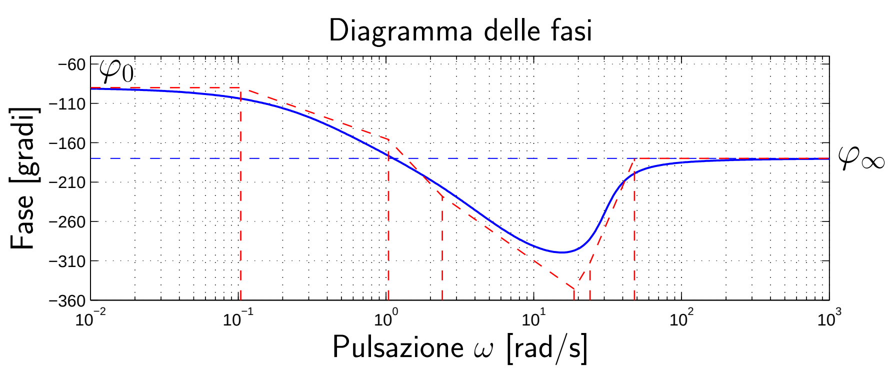
Gainβ:

$$\beta = |G_0(s)|_{s=0.5}$$
$$= \frac{5}{9} = -5.1 \text{ db}.$$



Gainγ:

$$\gamma = |G_\infty(s)|_{s=30}$$
$$= \frac{5}{30^2} = -45.1 \text{ db}.$$



- Trace asymptotic Bode diagrams of the following function $G(s)$:

$$G(s) = \frac{s\,(s+400)}{(1+3\,s)(s^2-1.5\,s+9)}.$$

Initial slope: +20 db/dec. Critical pulsations: $\omega = 0.333$ (a stable pole), $\omega = 3$ (two unstable conjugate complex poles) and $\omega = 400$ (a stable real zero).

Function$G_0(s)$:

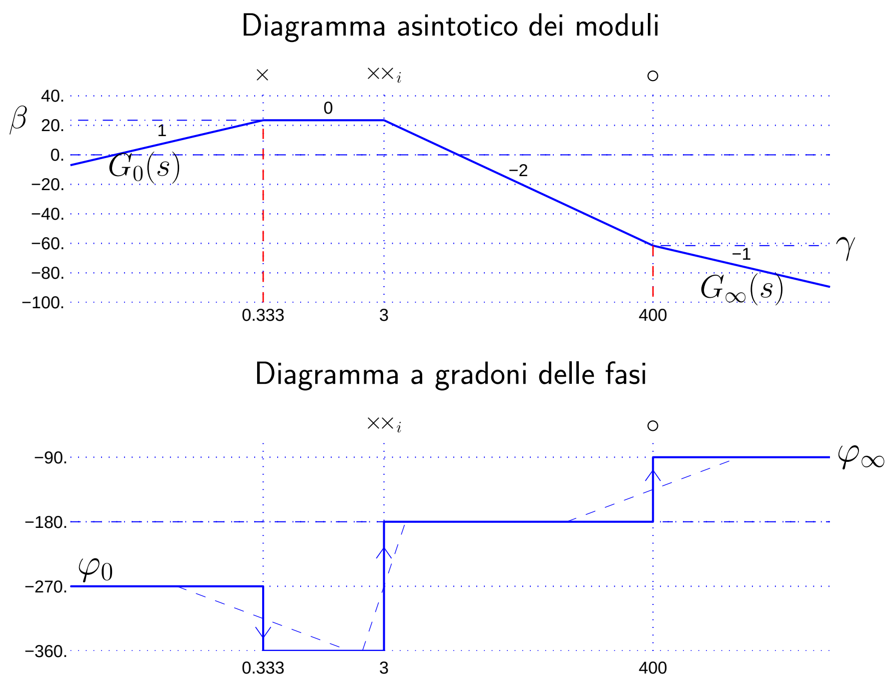
$$G_0(s) = \frac{s\,(400)}{(1)(9)} = \frac{400\,s}{9}$$

Initial phase $\varphi_0 = \frac{\pi}{2}$.

Function$G_\infty(s)$:

$$G_\infty(s) = \frac{s\,(s)}{(3\,s)(s^2)} = \frac{1}{3\,s}$$

Final phase $\varphi_\infty = -\frac{\pi}{2}$.

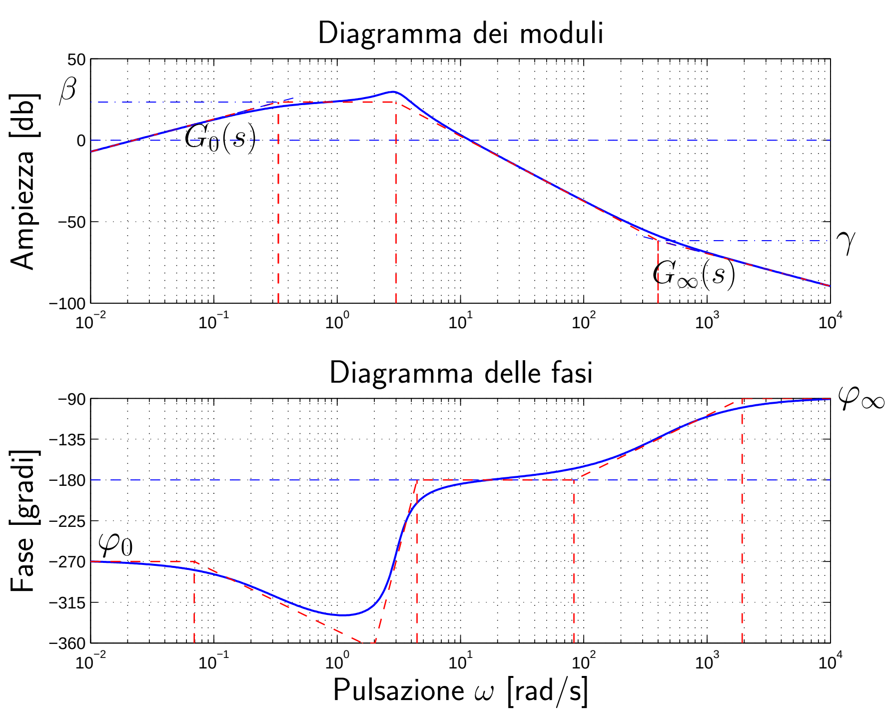


Gainβ:

$$\beta = |G_0(s)|_{s=0.333} = \frac{400}{27} = 23.4 \text{ db}.$$

Gainγ:

$$\gamma = |G_\infty(s)|_{s=400} = \frac{1}{1200} = -61.58 \text{ db}.$$



• Please refer to the Bode diagrams shown in the figure. Within the limits of the precision allowed by the graph calculate the expression of the function $G(s)$.

Gainβ for $\omega = 1$:

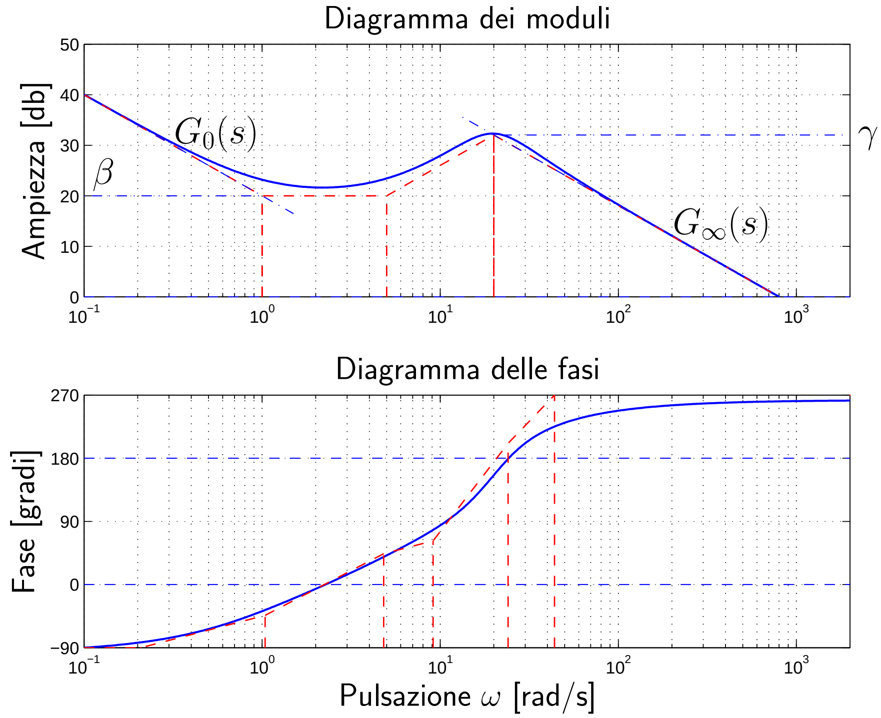
$$\beta = 20 \text{ db} = 10.$$

Critical frequenciesω:

0 $\rightarrow$ a pole
1 $\rightarrow$ uno zero stabile
5 $\rightarrow$ uno zero stabile
20 $\rightarrow$ due poli c.c instabili

Coefficient δ:

$$M_{\omega_n} = 1 \quad \rightarrow \quad \delta = 0.5.$$



The initial slope indicates the presence of a pole in the origin. The value of δ of the couple of complex conjugate poles is $\delta = 0.5$ because it is clear from the graph that for $\omega_n = 20$ the real diagram coincides with the asymptotic one.

The system transfer function is as follows:

$$G(s) \simeq \frac{\overbrace{800}^{K}(s+1)(s+5)}{s\,(s^2 - 20\,s + 400)} = \frac{10\,(1+s)(1+0.2\,s)}{s\,(1 - 0.05\,s + 0.025\,s^2)}.$$

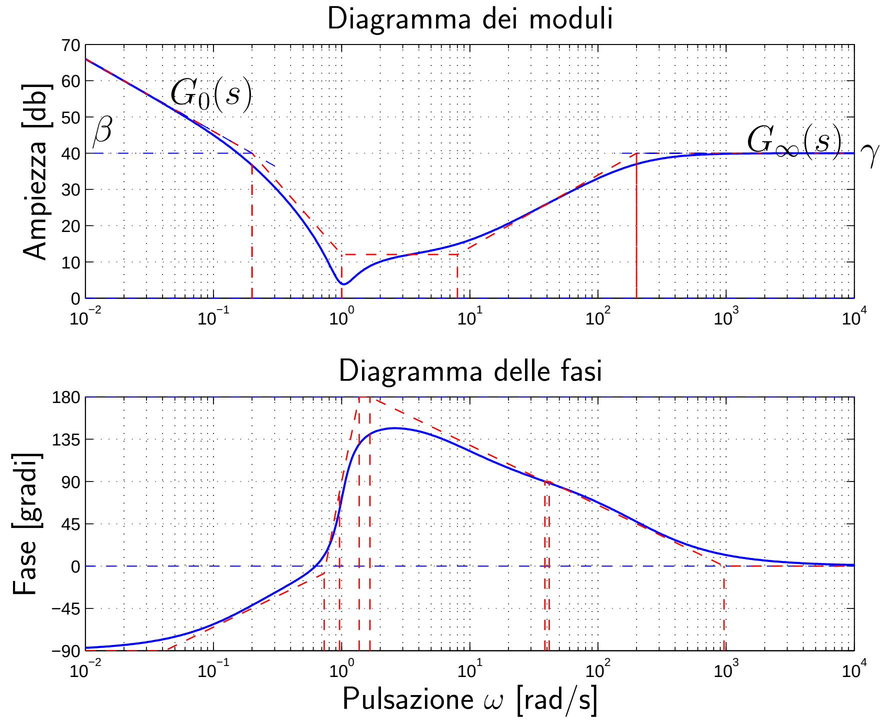
The value of the K gain of the $G(s)$ function is determined by imposing that the gain of the approximate $G_0(s)$ for $\omega = 1$ is equal to β:

$$|G_0(s)|_{s=j} = \left|\frac{5K}{400s}\right|_{s=j} = \frac{K}{80} = 10 \qquad \rightarrow \qquad K = 800.$$

The value of the gain K can also be determined by imposing that the gain of the approximate $GaInf(s)$ for $\omega = 20$ is equal to $\gamma = 32$ db:

$$|G_\infty(s)|_{s=j20} = \left|\frac{K}{s}\right|_{s=j20} = \frac{K}{20} = 32 \text{ db} = 40 \qquad \rightarrow \qquad K = 800.$$

• Please refer to the Bode diagrams shown in the figure. Within the limits of the precision allowed by the graph calculate the expression of the function $G(s)$.



Gainβ for $\omega = 0.2$:

$$\beta = 40 \text{ db} = 100.$$

Critical frequenciesω:

$0 \rightarrow$ a pole
$0.2 \rightarrow$ a stable pole
$1 \rightarrow$ two c.c. zeros
$8 \rightarrow$ an instable zero
$200 \rightarrow$ a stable pole

Coefficiente δ:

$$M_{\omega_n} = 2.5 \quad \rightarrow \quad \delta = 0.2.$$

The initial slope "-1" indicates the presence of a pole in the origin. The value of $\delta = 0.2$ of the couple of complex conjugate zeros is determined by the value $M_{\omega_n} = 8$ db at the pulsation $\omega_n = 1$.

The system transfer function is as follows:

$$G(s) \simeq \frac{\overbrace{100}^{K}(s^2 + 0.4\,s + 1)(s-8)}{s\,(s-0.2)(s+200)} = \frac{20\,(1+0.4\,s+s^2)(1-0.125\,s)}{s\,(1-5\,s)(1+0.005\,s)}.$$

The value of the K gain of the $G(s)$ function is determined by imposing that the gain of the approximate $G_0(s)$ for $\omega = 0.2$ is equal to β:

$$|G_0(s)|_{s=j0.2} = \left|\frac{8K}{40s}\right|_{s=j0.2} = \frac{8K}{8} = 100 \qquad \rightarrow \qquad K = 100.$$

The value of the gain K can also be determined by imposing that the gain of the approximate $GaInf(s)$ for $\omega = 200$ is equal to $\gamma = 40$ db:

$$|G_\infty(s)|_{s=j200} = K = 40 \text{ db} = 100 \qquad \rightarrow \qquad K = 100.$$

• Please refer to the Bode diagrams shown in the figure. To calculate: 1) the analytical expression of the function $G(s)$; 2) the steady-state response $y_\infty(t)$ of the system $G(s)$ when the signal is present: $x(t) = 5\,sin(0.02\,t) + 3\cos(400\,t)$.

Gainβ for $\omega = 0.2$:

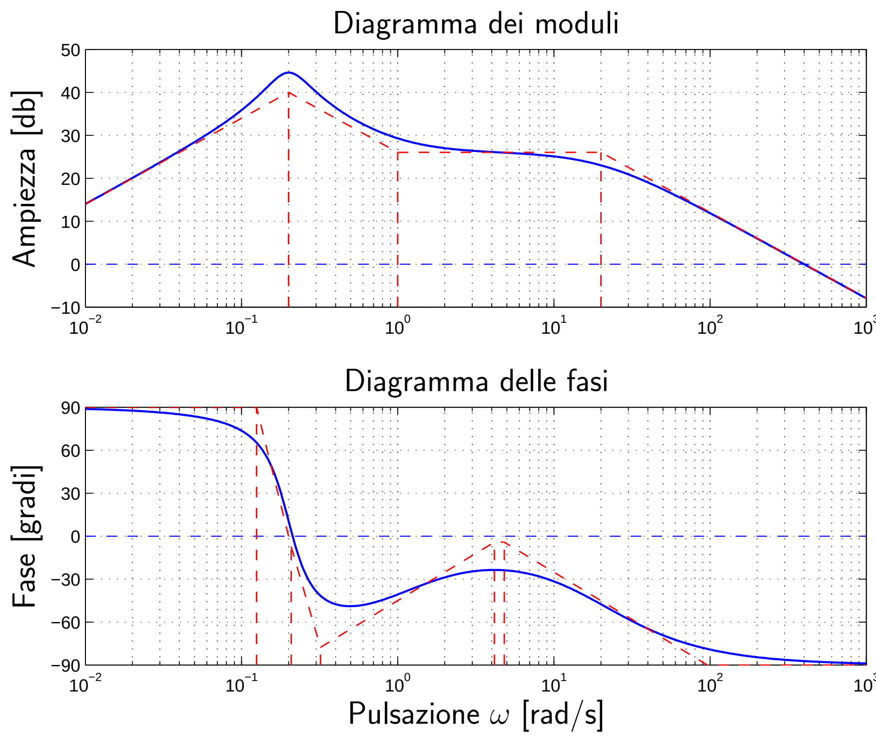
$$\beta = 40 \text{ db} = 100.$$

Critical frequenciesω:

- 0 → a zero
- 0.2 → two stable c.c. poles
- 1 → a stable zero
- 20 → a stable pole

Coefficient δ:

$$M_R = 1.66 \quad \to \quad \delta = 0.3.$$



1) The system transfer function is as follows:

$$G(s) = \frac{\overbrace{400}^{K}\; s\,(s+1)}{(s^2+0.12\,s+0.04)(s+20)} = \frac{500\,s\,(1+s)}{(1+3\,s+25\,s^2)(1+0.05\,s)}.$$

The value of the K gain of the $G(s)$ function is determined by imposing that the gain of the approximate $G_0(s)$ for $\omega = 0.2$ is equal to β:

$$|G_0(s)|_{s=j0.2} = \left|\frac{K\,s}{0.8}\right|_{s=j0.2} = \frac{K\,0.2}{0.8} = 100 \qquad \to \qquad K = 400.$$

2) The steady state response of the system $G(s)$ to the given signal is the following:

$$\begin{aligned} y_\infty(t) &= 5\,|G(0.02j)|\,\sin(0.02\,t + \arg G(0.02j)) \\ &\quad +3\,|G(400j)|\,\cos(400\,t + \arg G(400j)) \\ &= 50.4\,\sin(0.02\,t + 87.62°) + 2.996\,\cos(400\,t - 87.26°). \end{aligned}$$

The values of $|G(0.02j)|$, $\arg G(0.02j))$, $|G(400j)|$ and $\arg G(400j))$ are read directly on the diagrams of Bode of the modules and phases.

• Please refer to the Bode diagrams shown in the figure. Within the limits of the precision allowed by the graph: 1) obtain the analytical expression of the function $G(s)$; 2) draw the qualitative trend of the response to the unit step.

Static gain:

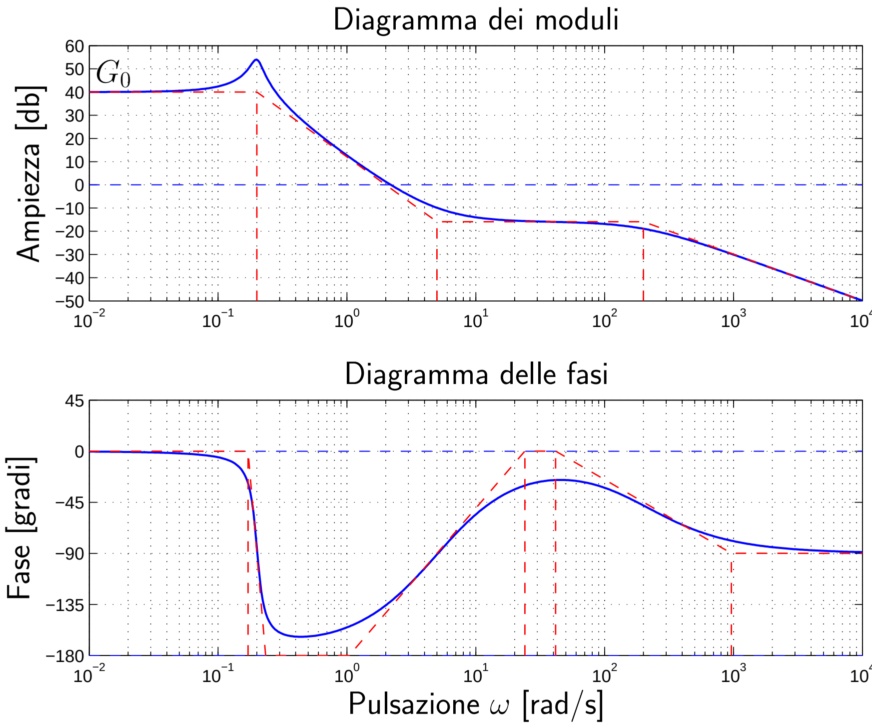
$$G(0) = 100.$$

Critical frequenciesω:

0.2 $\rightarrow$ due poli c.c. stabili
5 $\rightarrow$ due zeri stabili
200 $\rightarrow$ un polo stabile

Coefficient δ:

$$M_R = 5 \quad \rightarrow \quad \delta = 0.1.$$



1) The analytical expression of the $G(s)$ function is as follows:

$$G(s) = \frac{32\,(s+5)^2}{(s^2+0.04\,s+0.04)(s+200)} = \frac{100\,(1+0.2\,s)^2}{(1+s+25\,s^2)(1+0.005\,s)}.$$

2) The trend of the step response of the system $G(s)$ is shown in the figure.

Dominant poles:

$$p_{1,2} = -0.02 \pm j\,0.199.$$

Steady-state value:

$$y_\infty = G(0) = 100.$$

Settling Time:

$$T_a = \frac{3}{0.02} = 150 \text{ s.}$$

Period T_ω:

$$T_\omega = \frac{2\pi}{0.199} \simeq 31.57 \text{ s.}$$