Dynamic Models of Thermal Systems Using an Energy-Based Modeling Approach

Federica Grossi[1]
Department of Engineering "Enzo Ferrari,"
University of Modena and Reggio Emilia,
Modena 41121, Italy
e-mail: federica.grossi@unimore.it

Roberto Zanasi
Professor
Department of Engineering "Enzo Ferrari,"
University of Modena and Reggio Emilia,
Modena 41121, Italy
e-mail: roberto.zanasi@unimore.it

The aim of this work is to give a new approach to obtain compact dynamic thermal models suitable for a variety of systems where the heat transfer can be caused by conduction, internal convection (not at the boundary), and evaporation/condensation of water. The structural properties of the proposed dynamic model are presented and discussed in this paper. These properties guarantee conservation of energy and mass within the system, thus giving a good confidence in the correctness of the model. This paper shows that the proposed model has a simple structure, can be easily implemented in SIMULINK*, and provides simulation times much shorter compared with those usually obtained using CFD programs. The proposed model proves to be suitable for real-time simulations and for control design purposes.* [DOI: 10.1115/1.4033543]

1 Introduction

In today's industrial applications, the use of models and simulations is becoming increasingly important for many reasons: models can be used as a design tool, they allow to reduce time and costs related to real tests, and they help in the control design.

Thermal systems can involve a very large variety of phenomena. A large model able to catch the thermal behavior in full details becomes soon very complex and likely to be completely useless. A good model must offer a trade-off between accuracy and simulation time. It must be able to catch all the main phenomena and neglect the aspects of secondary importance. Moreover, a model supported by structural properties based on thermodynamic laws guarantees a certain level of confidence in the model. The main approaches used in this field are computational fluid dynamics (CFD) and reduced order models, both distributed and lumped parameters, see Refs. [1] and [2] for discussions and comparison among them.

Compact thermal models (CTM) have been proposed in Refs. [3] and [4] for electronic systems. In CTMs, the system is seen as a number of nodes connected by a resistive network. In Ref. [3], some conditions are stated in order to find a complete set of experiments to construct the compact model. In order to predict temperature profiles, the so-called flexible profile approach is introduced in Ref. [5]. A generalization of CTMs toward convective heat transfer is given in Ref. [6]. These compact models are static, thus transients are not considered and this is reasonable when thinking of systems in which transients can be disregarded. However, there are many systems where dynamic thermal effects may be important or settling times may be very large, for example, in drying machines or in air-conditioning/heating plants for buildings. Transient effects are considered in Refs. [7] and [8] obtaining the dynamic compact thermal model (DCTM) that considers distributive spatial and temporal effects. A new topology for DCTM is introduced in Ref. [9] to generalize the approach for transient problems.

In this paper, a modeling approach coming from energy-based modeling techniques is exploited. The so-called energy-based modeling techniques are based on the concept of power flows within a physical system. In the past years, many graphical energy-based modeling techniques have been introduced and developed: the bond graphs (BG) [10,11], the power-oriented graphs (POG) [12], and the energetic macroscopic representation [13]. The use of analogies among power variables allows to deal with the modeling of multidomain systems with a common approach. In Refs. [14] and [15], the analogies for variables in different energy domains are presented concerning the BG approach. An example of CTM exploiting BG can be found in Ref. [16].

The thermal modeling approach proposed in this paper is general and suitable for thermal systems involving heat transfer by conduction, moist air flows dynamics, and air–water interaction with evaporation and condensation. The model is lumped parameter and dynamic, essentially based on the POG modeling technique. Once the system topology has been defined, the model can be written in a straightforward way, and therefore, it can be also built automatically. The used POG modeling technique guarantees the conservation of energy and mass within the system, thus giving a certain confidence in the model. The mathematical properties of the POG proposed model are described and proved in Sec. 2.1 of this paper. The number of parameters of the model is limited. The particular structure of the model allows a very easy implementation in MATLAB/SIMULINK environment and it ensures small simulation times. Moreover, adding new elements to the model does not require to rewrite the model but only to increase the size of the matrices of the model, which keeps the same structure. The simulation results reported at the end of this paper show that the simulation times obtained with the proposed lumped parameters model are quite short compared with those usually obtained using CFD programs which can provide more precise results but also require a higher number of system and boundary layer parameters. The CFD models and simulations can be useful in the product development process, but they are not particularly useful for the design of system control strategies or real-time simulations. The proposed lumped parameters model is characterized by a limited number of parameters that can be easily identified from experimental data, and for this reason, the model is suitable for real-time simulations and for control design purposes: it can be used to analyze the system dynamics, to predict nonmeasured variables, to study the effect of parameters variations, to test different control strategies, and as a model-based design tool.

2 Thermal Model

2.1 Model C: Thermal Model With Heat Transfer Only Due to Conduction. The following assumptions are made: (1) the considered system model is composed of n elements (nodes) and a set of thermal conductivities that may go between all pairs of

[1]Corresponding author.

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nodes; (2) a model node is a portion of the model that can be represented by a thermal capacity C_i at temperature T_i and that can exchange heat with the other nodes through the thermal conductivities g_{ij}; (3) the system interacts with the external world only through m surfaces at constant temperature T_i^u for $i \in \{1, 2, \ldots, m\}$ and through p heat sources supplying the thermal power $\dot{Q}_i^u$ for $i \in \{1, 2, \ldots, p\}$; (4) no mass flows are present in the system, i.e., the heat fluxes are only due to conduction; and (5) evaporation and condensation phenomena are not considered. The dynamic model of the considered thermal system can be written in the following state-space form:

$$\begin{cases} \mathbf{L}\dot{\mathbf{x}} = \mathbf{A}_g\mathbf{x} + \mathbf{B}\mathbf{u} \\ \mathbf{y} = \mathbf{B}^{\mathrm{T}}\mathbf{x} + \mathbf{D}\mathbf{u} \end{cases} \tag{1}$$

where $\mathbf{u}$ is the input vector, $\mathbf{x}$ is the state vector, $\mathbf{y}$ is the output vector, $\mathbf{L}$ is the energy matrix, $\mathbf{A}_g$ is the power matrix, $\mathbf{B}$ is the input matrix, and $\mathbf{D}$ is the input–output matrix. Matrix $\mathbf{L}$ and vectors $\mathbf{x}$, $\mathbf{u}$, and $\mathbf{y}$ have the following structure:

$$\mathbf{L} = \begin{bmatrix} C_1 & & & \\ & C_2 & & \\ & & \ddots & \\ & & & C_n \end{bmatrix}, \mathbf{x} = \begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_n \end{bmatrix}, \mathbf{u} = \begin{bmatrix} \mathbf{T}_u \\ \hline \dot{\mathbf{Q}}_u \end{bmatrix} = \begin{bmatrix} T_1^u \\ \vdots \\ T_m^u \\ \hline \dot{Q}_1^u \\ \vdots \\ \dot{Q}_p^u \end{bmatrix},$$

$$\mathbf{y} = \begin{bmatrix} \dot{\mathbf{Q}}_y \\ \hline \mathbf{T}_y \end{bmatrix} = \begin{bmatrix} \dot{Q}_1^y \\ \vdots \\ \dot{Q}_m^y \\ \hline T_1^y \\ \vdots \\ T_p^y \end{bmatrix} \tag{2}$$

Model (1) is characterized by the fact that the input and output vectors $\mathbf{u}$ and $\mathbf{y}$ are conjugated with respect to the variation of entropy, i.e., the ratios $\dot{S}_i^y = \dot{Q}_i^y / T_i^u$ for $i \in \{1, 2, \ldots, m\}$ and $\dot{S}_i^u = \dot{Q}_i^u / T_i^y$ for $i \in \{1, 2, \ldots, p\}$ have the physical meaning of variation of entropy. Matrix $\mathbf{L}$ is diagonal and contains all the thermal capacities C_i of the system. The thermal capacities are constant (the time-variant case is considered later). The state vector $\mathbf{x}$ is the vector of the temperatures T_i of the nodes, i.e., the average temperature of the ith element. Matrices $\mathbf{A}_g$, $\mathbf{B} = \begin{bmatrix} \mathbf{B}_g & \mathbf{B}_f \end{bmatrix}$ and $\mathbf{D}$ of system (1) are characterized only by the thermal conductivities g_{ij}, for $i, j \in \{1, 2, \ldots, n\}$, which go between nodes i and j and the thermal conductivities g_{ij}^u, for $i \in \{1, 2, \ldots, n\}$ and $j \in \{1, 2, \ldots, m\}$, which go between node i and input j. Matrices $\mathbf{A}_g = \begin{bmatrix} a_{ij}^g \end{bmatrix} \in \mathbb{R}^{n \times n}$, $\mathbf{B}_g = \begin{bmatrix} b_{ij}^g \end{bmatrix} \in \mathbb{R}^{n \times m}$, $\mathbf{B}_f = \begin{bmatrix} b_{ij}^f \end{bmatrix} \in \mathbb{R}^{n \times p}$, and $\mathbf{D} = \begin{bmatrix} d_{ij} \end{bmatrix} \in \mathbb{R}^{(m+p) \times (m+p)}$ are defined as follows:

$$a_{ij}^g = \begin{cases} -\sum_{k=1}^{n} g_{ik} - \sum_{k=1}^{m} g_{ik}^u & \text{if} \quad i = j \\ g_{ij} & \text{if} \quad i \neq j \end{cases}, \qquad b_{ij}^g = g_{ij}^u \tag{3}$$

$$b_{ij}^f = \begin{cases} 1 & \text{if } \dot{Q}_j^u \neq 0 \text{ enters element } i \\ 0 & \text{otherwise} \end{cases}, \quad d_{ij} = \begin{cases} -\sum_{k=1}^{n} g_{kj}^u & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$$

where $g_{ij} = g_{ji}$ and $g_{ii} = 0$ for $i, j \in \{1, 2, \ldots, n\}$, and $g_{ij}^u = 0$ for $j > m$. It should be noted that coefficients g_{ij} always appear four times in matrix $\mathbf{A}_g$ in symmetric positions. Considering g_{ij}, a heat flux given by $g_{ij}(T_i - T_j)$ goes from the node at temperature T_i to the node at temperature T_j. The term $g_{ij}(T_i - T_j)$ appears with a negative sign in the energy balance of node i and with a positive sign in the energy balance of node j. This particular structure of matrix $\mathbf{A}_g$ guarantees that all the heat fluxes within the system occur without generation nor dissipation of energy. System (1) can also be written as

$$\begin{bmatrix} \mathbf{L}\dot{\mathbf{x}} \\ \mathbf{y} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B}^{\mathrm{T}} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{u} \end{bmatrix} = \mathbf{H} \begin{bmatrix} \mathbf{x} \\ \mathbf{u} \end{bmatrix} \tag{4}$$

where matrix $\mathbf{H}$ is symmetric.

Numerical Example. Consider, for example, the thermal system shown in Fig. 1. This system has nine node elements associated to thermal capacities. Matrix $\mathbf{L}$ and vector $\mathbf{x}$ are

$$\begin{aligned} \mathbf{L} &= \mathrm{diag}[\mathbf{C}_0], \\ \mathbf{C}_0 &= \begin{bmatrix} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 & C_9 \end{bmatrix}^{\mathrm{T}}, \\ \mathbf{x} &= \begin{bmatrix} T_1 & T_2 & T_3 & T_4 & T_5 & T_6 & T_7 & T_8 & T_9 \end{bmatrix}^{\mathrm{T}} \end{aligned} \tag{5}$$

where diag $[\mathbf{x}]$ is the diagonal matrix having on its diagonal the elements of vector $\mathbf{x}$. Matrix $\mathbf{A}_g$ is

$$\mathbf{A}_g = \begin{bmatrix} -g_{15} - g_{18} & 0 & 0 & 0 & g_{15} & 0 & 0 & g_{18} & 0 \\ 0 & -g_{25} - g_{29} & 0 & 0 & g_{25} & 0 & 0 & 0 & g_{29} \\ 0 & 0 & -g_{34} & g_{34} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & g_{34} & -g_{34} - g_{45} & g_{45} & 0 & 0 & 0 & 0 \\ g_{15} & g_{25} & 0 & g_{45} & -g_{15} - g_{25} - g_{45} - g_{56} & g_{56} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & g_{56} & -g_{56} - g_{67} & g_{67} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & g_{67} & -g_{67} - g_{71}^u & 0 & 0 \\ g_{18} & 0 & 0 & 0 & 0 & 0 & 0 & -g_{18} & 0 \\ 0 & g_{29} & 0 & 0 & 0 & 0 & 0 & 0 & -g_{29} \end{bmatrix} \tag{6}$$

The considered system interacts with the external world through a surface at constant temperature T_1^u and a heat source supplying the thermal power $\dot{Q}_1^u$ to element 1. The input vector $\mathbf{u}$, the output vector $\mathbf{y}$, and the matrices $\mathbf{B}$ and $\mathbf{D}$ are

$$\mathbf{u} = \begin{bmatrix} T_1^u \\ \dot{Q}_1^u \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} \dot{Q}_1^y \\ T_1^y \end{bmatrix}, \quad \mathbf{B} = \left[\begin{array}{c} \mathbf{B}_g^{\mathrm{T}} \\ \hline \mathbf{B}_f^{\mathrm{T}} \end{array}\right]^{\mathrm{T}} = \left[\begin{array}{cccccccc|c} 0 & 0 & 0 & 0 & 0 & 0 & g_{71}^u & 0 & 0 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]^{\mathrm{T}}, \quad \mathbf{D} = \begin{bmatrix} -g_{71}^u & 0 \\ 0 & 0 \end{bmatrix}$$

Properties of the Thermal Model With Heat Transfer Due to Conduction. For the thermal model (1), the following properties hold:

(1) Matrix $\mathbf{L}$ is always symmetric and positive definite, and therefore, all its eigenvalues are real and positive.

(2) The thermal energy E_t stored in the system is given by

$$E_t = \sum_{i=1}^{n} C_i T_i = 1_n^{\mathrm{T}} \mathbf{L}\, \mathbf{x} = \mathbf{C}_0^{\mathrm{T}} \mathbf{x}, \quad 1_n = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \tag{7}$$

where 1_n is the all-ones column vector of dimension n. Relation (7) means that the thermal energy E_t is equal to the sum of all thermal energies $C_i T_i$ stored in the thermal capacities C_i of the system.

(3) Matrix $\mathbf{A}_g$ describes the power redistribution within the system. The components of vector $\mathbf{A}_g\mathbf{x} + \mathbf{B}\mathbf{u}$ represent the thermal power fluxes entering the thermal capacities C_i: $\mathbf{A}_g\mathbf{x}$ represents the thermal fluxes inside the system, and $\mathbf{B}\mathbf{u}$ represents the thermal fluxes entering the system.

(4) Matrix $\mathbf{A}_g$ is symmetric, as stated also in Refs. [3] and [4]. When $g_{ij}^u = 0$, matrix $\mathbf{A}_g$ can be written as follows:

$$\mathbf{A}_g = -\sum_{i=1}^{n}\sum_{j=1}^{n} (\mathbf{e}_i - \mathbf{e}_j) g_{ij} (\mathbf{e}_i - \mathbf{e}_j)^{\mathrm{T}} \tag{8}$$

where $\mathbf{e}_i$ is the ith vector of the standard basis in the n-dimensional space. Matrix $\mathbf{A}_g$, see Eq. (8), is the sum of symmetric and negative semidefinite matrices, and therefore, it is negative semidefinite and all its eigenvalues are zero or negative real. From this property, it follows that the considered thermal system is always stable in every operating condition and for any matrix $\mathbf{L}$, and therefore, when inputs are constant, the system always reaches a stable final operating point.

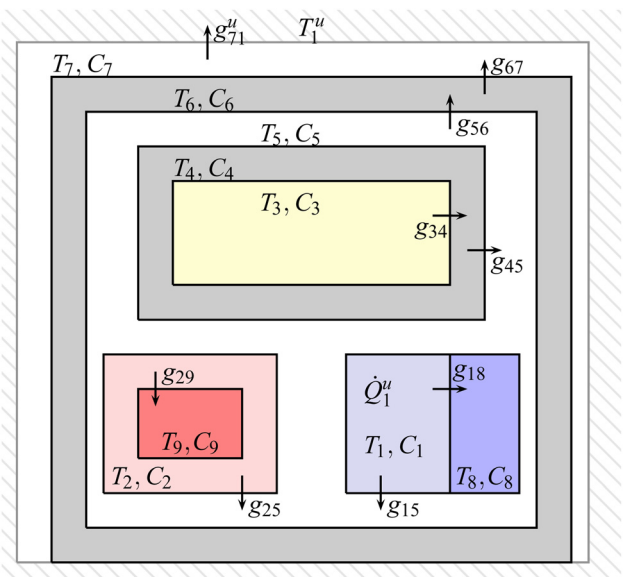

Fig. 1 Thermal system with heat transfer only due to conduction

(5) When $g_{ij}^u = 0$, there is no thermal conduction interaction of the system with the external environment. In this case, it holds that $\mathbf{A}_g 1_n = 0$, i.e., the sum of all the columns of matrix $\mathbf{A}_g$ is zero, see Refs. [3,4], and [8]. This is equivalent to say that matrix $\mathbf{A}_g$ is not full rank and vector 1_n is the eigenvector associated to the null eigenvalue of matrix $\mathbf{A}_g$.

(6) It can be proved that the number of zero eigenvalues of matrix $\mathbf{A}_g$ is equal to the number of isolated subsystems within the considered system. The elements C_i belonging to each isolated subsystem can be determined computing the eigenvectors associated to the zero eigenvalue.

(7) When $g_{ij}^u = 0$ matrix $\mathbf{A}_g$ satisfies the relation: $1_n^{\mathrm{T}} \mathbf{A}_g = 0$, i.e., the sum of all the rows of matrix $\mathbf{A}_g$ is equal to the null vector and this means that the sum of all the thermal fluxes within the system is always zero: generation or loss of energy can never occur within the system. This property can be found also in Refs. [3,4], and [8].

(8) When $\mathbf{u} = 0$, the free response of the system with initial condition $\mathbf{x}_0$ is

$$\mathbf{x}_l(t) = e^{\mathbf{L}^{-1}\mathbf{A}_g t}\mathbf{x}_0$$

Matrix $e^{\mathbf{L}^{-1}\mathbf{A}_g t}$ satisfies the relations $e^{\mathbf{L}^{-1}\mathbf{A}_g t}\, 1_n = 1_n$ and $\mathbf{C}_0^{\mathrm{T}}\, e^{\mathbf{L}^{-1}\mathbf{A}_g t} = \mathbf{C}_0^{\mathrm{T}}$, i.e., vectors 1_n and $\mathbf{C}_0$ are, respectively, the right and the left eigenvectors of matrix $e^{\mathbf{L}^{-1}\mathbf{A}_g t}$ associated to the eigenvalue $\lambda = 1$. Whatever the initial condition $\mathbf{x}_0$ is, the total energy E_t stored in the system remains constant along the free response

$$E_t = \mathbf{C}_0^{\mathrm{T}} \mathbf{x}_l(t) = \mathbf{C}_0^{\mathrm{T}} \mathbf{x}_0 = \mathbf{C}_0^{\mathrm{T}} \mathbf{x}_f = \text{constant}$$

where $\mathbf{x}_f$ is the final state of the system

$$\mathbf{x}_f = \lim_{t\to\infty} \mathbf{x}_l(t) = 1_n T_f, \quad T_f = \frac{\mathbf{C}_0^{\mathrm{T}} \mathbf{x}_0}{\mathbf{C}_0^{\mathrm{T}} 1_n}$$

where T_f is the final constant temperature of the considered system.

(9) When the input vector $\mathbf{u} = \bar{\mathbf{u}}$ is constant and matrix $\mathbf{A}_g$ is invertible, i.e., no isolated subsystems are present within the system (see Property 6), matrix $\mathbf{B}_g \neq 0$ and the state vector $\bar{\mathbf{x}}$ in steady-state condition is

$$\bar{\mathbf{x}} = -\mathbf{A}_g^{-1}\, \mathbf{B}\, \bar{\mathbf{u}}$$

(10) If the input is $\mathbf{u} = T_1^u \neq 0$, after a transient the system reaches the final steady-state condition

$$\bar{\mathbf{x}} = -\mathbf{A}_g^{-1}\mathbf{B} T_1^u = 1_n T_1^u$$

i.e., all the final temperatures within the system are equal to the external temperature T_1^u.

(11) The temperatures T_i in the state vector $\mathbf{x}$ can be expressed both in Kelvin and degree Celsius without modifying the dynamics (1) of the thermal system. Consider, for example, the case $\mathbf{T}_u \neq 0$ and all the temperatures in Eq. (1) are expressed in Kelvin. In this case, the first equation of system (1) can be rewritten as

$$\mathbf{L}\dot{\mathbf{x}} = \mathbf{A}_g\mathbf{x} + \mathbf{B}_g\, \mathbf{T}_u + \mathbf{B}_f\, \dot{\mathbf{Q}}_u \tag{9}$$

Let T_0 denote the temperature in Kelvin corresponding to 0 °C and let $\bar{\mathbf{x}}$ and $\bar{\mathbf{T}}^u$ denote the vectors $\mathbf{x}$ and $\mathbf{T}_u$ expressed in degree Celsius. The following relations hold:

$$\mathbf{x} = \bar{\mathbf{x}} + 1_n T_0, \quad \mathbf{T}_u = \bar{\mathbf{T}}^u + 1_m T_0 \tag{10}$$

Substituting these relations in Eq. (9), one obtains

$$\mathbf{L}\dot{\bar{\mathbf{x}}} = \mathbf{A}_g \bar{\mathbf{x}} + \mathbf{B}_g \bar{\mathbf{T}}^u + \mathbf{B}_f \dot{\mathbf{Q}}_u + \underbrace{\mathbf{A}_g\, 1_n T_0 + \mathbf{B}_g\, 1_m T_0}_{\mathbf{w}=0} \tag{11}$$

The term $\mathbf{w}$ is zero because, due to definition (3), it satisfies the following relation:

$$\mathbf{w} = \mathbf{A}_g\, 1_n T_0 + \mathbf{B}_g\, 1_m T_0 = \begin{bmatrix} \mathbf{A}_g & \mathbf{B}_g \end{bmatrix} \begin{bmatrix} 1_n \\ 1_m \end{bmatrix} T_0 = 0$$

i.e., the sum of the columns of matrix $\begin{bmatrix} \mathbf{A}_g & \mathbf{B}_g \end{bmatrix}$ is always zero. It follows that the system dynamics (11) expressed in degree Celsius is equal to the system dynamics (9) expressed in Kelvin. What is different is the value of the energy E_t stored within the system. Indicating with $\bar{E}_t$ the stored energy calculated with temperatures in degree Celsius, from Eqs. (7) and (10) it follows that:

$$E_t = 1_n^{\mathrm{T}} \mathbf{L}\mathbf{x} = \underbrace{1_n^{\mathrm{T}} \mathbf{L}\bar{\mathbf{x}}}_{\bar{E}_t} + \underbrace{1_n^{\mathrm{T}} \mathbf{L} 1_n T_0}_{E_0} = \bar{E}_t + E_0 \quad \rightarrow \quad \bar{E}_t = E_t - E_0$$

where E_0 is the energy stored in the system when all the thermal capacities are at temperature T_0.

For the sake of simplicity, the thermal model considered in this section, i.e., heat transfer due only to conduction, will be referred to as "model C" in the following.

2.2 Model CC: Thermal Model With Heat Transfer and Mass Flows. The following assumptions are made: (1) the physical elements of the system can be solid, liquid, or gaseous; if all the elements are isolated, no mass can flow among them and their thermal capacities are constant; if some of the gaseous elements of the system are not isolated, mass can flow among them by means of air ducts and their thermal capacities C_i are no longer constant; (2) the elements of type "gas" of the considered system are supposed to be constituted by moist air and are characterized both by a temperature T_i and a pressure P_i; (3) the internal convection mass flows are due to the pressure drop $P_i - P_j$ at the terminals of the air duct connecting the ith and jth elements, or due to the pressure jump $\Delta\bar{\mathbf{P}}_{ij}$ forced by a fan inserted in the air duct, as it happens in building heating/cooling systems or in drying machines; and (4) the model parameters do not depend on the boundary layer thermal phenomena associated with the system local geometry. Let $\mathcal{V}$ denote the set of q indices of the moist air elements of the system. The mass $\mathbf{m}_{vi}$ and the mass density ρ_i of the ith moist air element, for $i \in \mathcal{V}$, can be expressed as follows:

$$\mathbf{m}_{vi} = [m_{ia} \;\; m_{iv}]^{\mathrm{T}}, \quad \rho_i = [\rho_{ia} \;\; \rho_{iv}]^{\mathrm{T}} = \frac{\mathbf{m}_{vi}}{V_i} \tag{12}$$

where m_{ia}, m_{iv}, ρ_{ia}, and ρ_{iv} are the masses and the mass densities of the dry air and the vapor of the ith element, and V_i is the volume of the element. The specific heat at constant pressure $\mathbf{c}_\phi$ and the thermal capacity C_i of the ith moist air element can be expressed as follows:

$$\mathbf{c}_\phi = [c_a \;\; c_v]^{\mathrm{T}}, \quad C_i = \mathbf{m}_{vi}^{\mathrm{T}} \mathbf{c}_\phi$$

Table 1 Values of parameters of the thermal system

ρ_{air}	1.1842	kg/m^3	c_a	1005	J/(kg K)
R_{gas}	8.314	J/(mol K)	c_w	4186	J/(kg K)
M_w	18.015	g/mol	c_v	1900	J/(kg K)
M_a	28.84	g/mol	c_l	2272	kJ/kg

where c_a and c_v are the specific heats of the dry air and the vapor, respectively, see Table 1. The dynamic model of the thermal system with heat transfer due to internal convection can be written as follows:

$$\begin{cases} \mathbf{L}\dot{\mathbf{x}} = (\mathbf{A}_g + \mathbf{A}_f)\mathbf{x} + \mathbf{B}\mathbf{u} \\ \mathbf{y} = \mathbf{B}^{\mathrm{T}}\mathbf{x} + \mathbf{D}\mathbf{u} \end{cases}, \quad \dot{\mathbf{m}}_v = \mathbf{A}_m \mathbf{m}_v \tag{13}$$

where matrices $\mathbf{L}$, $\mathbf{A}_g$, $\mathbf{B}$, and $\mathbf{D}$ and vectors $\mathbf{x}$, $\mathbf{u}$, and $\mathbf{y}$ are the same as in system (1). Coefficients a_{ij}^f of matrix $\mathbf{A}_f = \left[a_{ij}^f \right] \in \mathbb{R}^{n \times n}$ are defined as follows:

$$a_{ij}^f = \begin{cases} -\sum_{k=1}^{n} f_{ik} & \text{if} \quad i = j \\ f_{ji} & \text{if} \quad i \neq j \end{cases} \tag{14}$$

where f_{ij} is the specific power coefficient which defines the thermal power $f_{ij} T_i$ subtracted from element i and added to element j. The specific power coefficients f_{ij} are defined as follows:

$$f_{ij} = \begin{cases} \rho_i^{\mathrm{T}} \phi_{V_{ij}}^{+} \mathbf{c}_\phi, & \text{if } i,j \in \mathcal{V} \\ 0 & \text{otherwise} \end{cases}, \quad \phi_{V_{ij}}^{+} = \begin{cases} \phi_{V_{ij}} & \text{if} \quad \phi_{V_{ij}} > 0 \\ 0 & \text{if} \quad \phi_{V_{ij}} \leq 0 \end{cases} \tag{15}$$

where $\phi_{V_{ij}}$ is the volume flow rate from element i toward element j. From Eq. (15), it is evident that $f_{ij} \geq 0$, for $i,j \in \mathcal{V}$, and if $f_{ij} \neq 0$ then $f_{ji} = 0$. Moreover, the coefficients f_{ij} always appear with opposite signs in two different positions of the ith column of matrix $\mathbf{A}_f$ and this fact guarantees the energy conservation within the system. The coefficients f_{ij} of the proposed lumped parameters model do not depend on the boundary layer thermal phenomena associated with the local geometry of the system. Vector $\mathbf{m}_v \in \mathbb{R}^{2q}$ and coefficients $\mathbf{a}_{ij}^m$ of matrix $\mathbf{A}_m = \left[\mathbf{a}_{ij}^m \right] \in \mathbb{R}^{2q \times 2q}$, for $i,j \in \mathcal{V}$, see Eq. (13), are defined as follows:

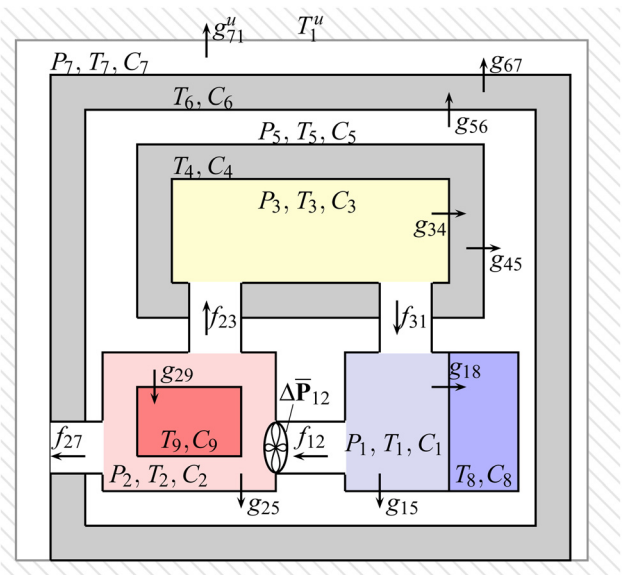

Fig. 2 Thermal system with conduction and internal convection

$$\mathbf{m}_v = \begin{bmatrix} \mathbf{m}_{v1} \\ \mathbf{m}_{v2} \\ \vdots \\ \mathbf{m}_{vq} \end{bmatrix} = \begin{bmatrix} \mathbf{m}_{\mathcal{V}_1} \\ \mathbf{m}_{\mathcal{V}_2} \\ \vdots \\ \mathbf{m}_{\mathcal{V}_q} \end{bmatrix}, \quad \mathbf{a}_{ij}^m = \begin{cases} -\dfrac{1}{V_j}\displaystyle\sum_{k=1}^{n} \phi_{V_{ij}}^+ \mathbf{I}_2 & \text{if} \quad i = j \\ \dfrac{1}{V_j}\, \phi_{V_{ij}}^+ \mathbf{I}_2 & \text{if} \quad i \neq j \end{cases}$$

where $\mathbf{I}_2$ is the identity matrix of order 2, and $\mathcal{V}_k \in \mathcal{V}$ for $k \in \{1, 2, \ldots, q\}$. The coefficients $\phi_{V_{ij}}^+$ always appear with opposite signs in two different positions of the same column of matrix $\mathbf{A}_m$ and this fact guarantees the conservation of mass within the system. The volume flow rates $\phi_{V_{ij}}$, for $i,j \in \mathcal{V}$, are defined as follows:

$$\phi_{V_{ij}} = \frac{1}{\mathcal{R}_{ij}} \left(P_i - P_j + \Delta \bar{\mathbf{P}}_{ij} \right) \tag{16}$$

where $\mathcal{R}_{ij}$ is the hydraulic resistance of the air duct between elements i and j, and P_i, for $i \in \mathcal{V}$, is the pressure of the ith moist air element obtained as follows using the ideal gas law:

$$P_i = T_i R_{\text{gas}} \begin{bmatrix} \dfrac{1}{M_a V_i} & \dfrac{1}{M_w V_i} \end{bmatrix} \mathbf{m}_{vi} \tag{17}$$

The values of parameters R_{gas}, M_a, and M_w in Eq. (17) are reported in Table 1. The pressure drop $\Delta \bar{\mathbf{P}}_{ij}$ in Eq. (16) satisfies relation $\Delta \bar{\mathbf{P}}_{ii} = 0$, for $i \in \mathcal{V}$.

Numerical Example. Referring to the thermal system of Fig. 2, suppose that the moist air elements are 1, 2, 3, 5, and 7. In this case, it is $\mathcal{V} = \{1, 2, 3, 5, 7\}$ and $q = 5$. Matrices $\mathbf{L}$, $\mathbf{A}_g$, and vector $\mathbf{x}$ are the same as in Eqs. (5) and (6). Matrix $\mathbf{A}_f$ defined in Eq. (14) has the following structure:

$$\mathbf{A}_f = \begin{bmatrix} -f_{12} - f_{13} & f_{21} & f_{31} & 0 & 0 & 0 & 0 & 0 & 0 \\ f_{12} & -f_{21} - f_{23} - f_{27} & f_{32} & 0 & 0 & 0 & f_{72} & 0 & 0 \\ f_{13} & f_{23} & -f_{31} - f_{32} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & f_{27} & 0 & 0 & 0 & 0 & -f_{72} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{18}$$

The structures of matrix $\mathbf{A}_m$ and vector $\mathbf{m}_v$ are

$$\mathbf{A}_m = \begin{bmatrix} \dfrac{-(\phi_{V12}^+ + \phi_{V13}^+)\mathbf{I}_2}{V_1} & \dfrac{\phi_{V21}^+ \mathbf{I}_2}{V_2} & \dfrac{\phi_{V31}^+ \mathbf{I}_2}{V_3} & 0 & 0 \\ \dfrac{\phi_{V12}^+ \mathbf{I}_2}{V_1} & \dfrac{-(\phi_{V21}^+ + \phi_{V23}^+ + \phi_{V27}^+)\mathbf{I}_2}{V_2} & \dfrac{\phi_{V32}^+ \mathbf{I}_2}{V_3} & 0 & \dfrac{\phi_{V72}^+ \mathbf{I}_2}{V_7} \\ \dfrac{\phi_{V13}^+ \mathbf{I}_2}{V_1} & \dfrac{\phi_{V23}^+ \mathbf{I}_2}{V_2} & \dfrac{-(\phi_{V31}^+ + \phi_{V32}^+)\mathbf{I}_2}{V_3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & \dfrac{\phi_{V27}^+ \mathbf{I}_2}{V_2} & 0 & 0 & \dfrac{-\phi_{V72}^+ \mathbf{I}_2}{V_7} \end{bmatrix}, \quad \mathbf{m}_v = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \\ \mathbf{m}_5 \\ \mathbf{m}_7 \end{bmatrix} \tag{19}$$

Note that, for the considered case, the fourth row and the fourth column of matrix $\mathbf{A}_m$ are zero because the moist air element 5 of the system does not have any connections with the other four moist air elements 1, 2, 3, and 7.

For the sake of simplicity, the thermal model of this type, i.e., heat transfer due to both conduction and internal convection, will be referred to as "model CC" in the following.

2.3 Model CCE: Thermal Model With Evaporation and Condensation. Let us now consider the case of a thermal system with conduction, internal convection, and condensation/evaporation of water. The following assumptions are made: (1) a few elements of the system are supposed to be made of liquid water. Let $\mathcal{W}$ denote the set of the r indices of the water elements of the system; (2) the evaporation and condensation phenomena can happen only between the liquid water and the moist air elements of the system; (3) each liquid water element can interact with only one moist air element, while each moist air element can interact with one or more liquid water elements. The pressure of each liquid water element is equal to the pressure of the moist air element with which it interacts; (4) no mass flows between the liquid water elements will be considered in the proposed dynamic model; and (5) the fluid saturation pressure of the moist air elements is supposed to be independent from the small pressure drop which is present between adjacent moist air elements. The dynamic model of the thermal system with evaporation and condensation can be written as follows:

$$\begin{cases} \mathbf{L}\dot{\mathbf{x}} = (\mathbf{A}_g + \mathbf{A}_f + \mathbf{A}_w)\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{l}(\mathbf{x}, \mathbf{m}) \\ \mathbf{y} = \mathbf{B}^{\mathrm{T}}\mathbf{x} + \mathbf{D}\mathbf{u} \end{cases},$$
$$\dot{\mathbf{m}} = \begin{bmatrix} \dot{\mathbf{m}}_v \\ \dot{\mathbf{m}}_w \end{bmatrix} = \begin{bmatrix} \mathbf{A}_m \mathbf{m}_v + \bar{\varphi}_v \\ \bar{\varphi}_w \end{bmatrix} \tag{20}$$

where matrices $\mathbf{L}$, $\mathbf{A}_g$, $\mathbf{A}_f$, $\mathbf{A}_m$, $\mathbf{B}$, and $\mathbf{D}$ and vectors $\mathbf{x}$, $\mathbf{u}$, and $\mathbf{y}$ are the same as in Eq. (13). Vector $\mathbf{m}_w \in \mathbb{R}^r$ and coefficients a_{ij}^w of matrix $\mathbf{A}_w = \left[a_{ij}^w \right] \in \mathbb{R}^{n \times n}$ are defined as follows:

$$\mathbf{m}_w = \begin{bmatrix} m_{w1} \\ m_{w2} \\ \vdots \\ m_{wr} \end{bmatrix} = \begin{bmatrix} m_{\mathcal{W}_1} \\ m_{\mathcal{W}_2} \\ \vdots \\ m_{\mathcal{W}_r} \end{bmatrix}, \qquad a_{ij}^w = \begin{cases} -\sum_{k=1}^{n} h_{ik} & \text{if} \quad i = j \\ h_{ji} & \text{if} \quad i \neq j \end{cases}$$

where $\mathcal{W}_i \in \mathcal{W}$, for $i \in \{1, 2, \ldots, r\}$, and h_{ij} are the specific power coefficients which define the thermal power $a_{ij}^w T_j$ flowing from element i to element j due to evaporation/condensation. The coefficients h_{ij} are defined as follows:

$$h_{ij} = \begin{cases} c_w \, \varphi_{ij}^+, & \text{if } (i,j) \in \mathcal{W} \times \mathcal{V} \\ c_v \, \varphi_{ij}^+, & \text{if } (i,j) \in \mathcal{V} \times \mathcal{W}, \\ 0 & \text{otherwise} \end{cases}$$

$$\varphi_{ij}^+ = \begin{cases} \varphi_{ij} & \text{if } \varphi_{ij} > 0 \text{ and } (i,j) \in (\mathcal{V} \times \mathcal{W}) \cup (\mathcal{W} \times \mathcal{V}) \\ 0 & \text{otherwise} \end{cases} \tag{21}$$

where φ_{ij} is the mass flow rate from element i toward element j due to evaporation/condensation. When the mass flow rate φ_{ij} is positive, i.e., $\varphi_{ij} = \varphi_{ij}^+ > 0$, a real mass flux flows from ith element toward the jth element. If $i \in \mathcal{W}$ is a liquid water element and $j \in \mathcal{V}$ is a moist air element, the mass flow represents an "evaporation" phenomenon, and therefore, to calculate the heat flux, the water-specific heat c_w must be used. Vice versa, if $i \in \mathcal{V}$ is a moist air element and $j \in \mathcal{W}$ is a liquid water element, the mass flow represents a "condensation" phenomenon, and therefore, to calculate the heat flux, the vapour-specific heat c_v must be used. The values of coefficients c_v and c_w are reported in Table 1. Coefficients l_i of vector $\mathbf{l} = \begin{bmatrix} l_i \end{bmatrix} \in \mathbb{R}^n$ in Eq. (20) are defined as follows:

$$l_i = c_l \left(\sum_{k=1}^{n} \varphi_{ki}^+ - \sum_{k=1}^{n} \varphi_{ik}^+ \right)$$

where c_l is the latent heat of evaporation/condensation of the water, see Table 1. The term $c_l \varphi_{ij}^+$ represents the latent heat flux associated to evaporation/condensation phenomena when a positive mass flow rate φ_{ij}^+ moves from element i toward element j, and therefore, the latent heat flux $c_l \varphi_{ij}^+$ is subtracted from element i and added to element j. Coefficients φ_{vi} of vector $\bar{\varphi}_v = \begin{bmatrix} \varphi_{vi} \end{bmatrix} \in \mathbb{R}^q$ and coefficients φ_{wj} of vector $\bar{\varphi}_w = \begin{bmatrix} \varphi_{wj} \end{bmatrix} \in \mathbb{R}^r$ in Eq. (20) are defined as follows:

$$\bar{\varphi}_{vi} = \begin{bmatrix} 0 \\ \sum_{k=1}^{n} \varphi_{k\mathcal{V}_i}^+ - \sum_{k=1}^{n} \varphi_{\mathcal{V}_i k}^+ \end{bmatrix}, \qquad \bar{\varphi}_{wj} = \sum_{k=1}^{n} \varphi_{k\mathcal{W}_j}^+ - \sum_{k=1}^{n} \varphi_{\mathcal{W}_j k}^+$$

where $\mathcal{V}_i \in \mathcal{V}$ for $i \in \{1, 2, \ldots, q\}$ and $\mathcal{W}_j \in \mathcal{W}$ for $j \in \{1, 2, \ldots, r\}$. Note that when evaporation/condensation occurs, the mass and the volume of the liquid water elements vary. The volume variation of each liquid water element causes a volume variation of the corresponding moist air element with which it interacts.

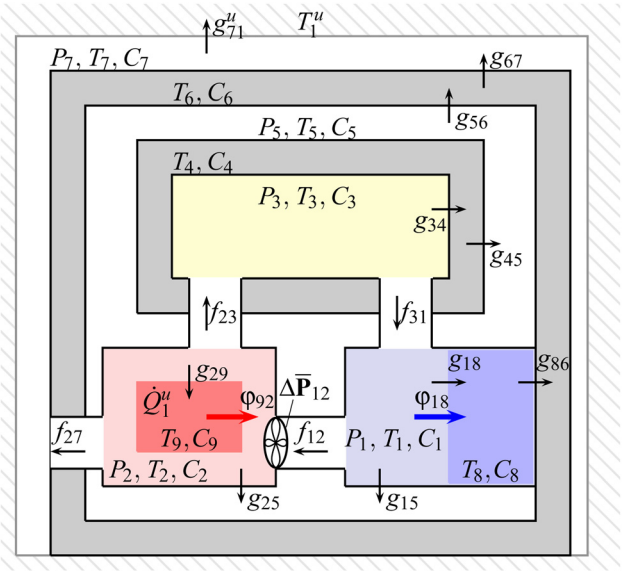

Fig. 4 Thermal system with conduction, internal convection, and evaporation/condensation of water

Evaporation/Condensation Mass Flow Rate φ_{ij}. Some equations for the calculation of evaporation mass flow rate have been proposed in Ref. [17]. In Ref. [18], a new function $\psi_0(T)$ for the evaporation mass flow rate has been introduced, see Fig. 3(*a*): $\psi_0(T)$ is the mass flow rate of water for unit surface that evaporates in dry air at ambient pressure when the water and the dry air are at the same temperature T expressed in degree Celsius

$$\psi_0(T) = f_0 \, e^{hT}, \quad h = \frac{1}{100} \ln\left(\frac{f_{100}}{f_0}\right)$$

The parameters f_0 and f_{100} represent the water evaporation mass flow rates at temperatures 0 °C and 100 °C, respectively. The evaporation/condensation mass flow rate φ_{ij} from element i toward element j, used in Eq. (21), is defined as follows:

$$\varphi_{ij} = \begin{cases} \Psi_{ij} & \text{if } (i,j) \in (\mathcal{W} \times \mathcal{V}) \text{ (evaporation)} \\ -\Psi_{ji} & \text{if } (i,j) \in (\mathcal{V} \times \mathcal{W}) \text{ (condensation)} \end{cases} \tag{22}$$

The function Ψ_{ij} describing the water mass flow rate between elements i and j is defined as

$$\Psi_{ij} = \Psi(T_i, T_j, m_i, u_j) = \alpha(m_i) \left(\psi_0(T_i) - \psi_0(\max(T_i, T_j)) \frac{u_j}{u_{sat}(T_j)} \right), \quad (i,j) \in (\mathcal{W} \times \mathcal{V}) \tag{23}$$

where T_i and m_i are the temperature and the mass of the liquid water element i, and T_j and u_j are the temperature and the specific

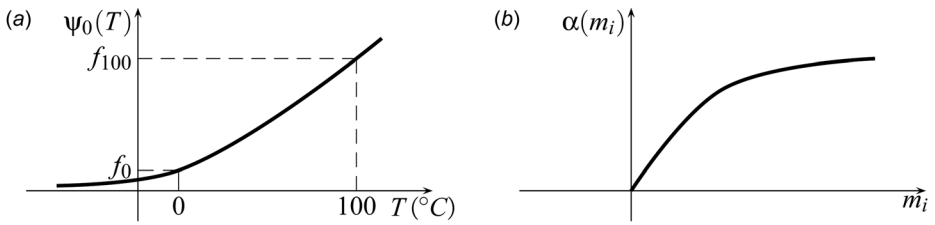

Fig. 3 (*a*) Shape of the evaporation function $\psi_0(T)$ and (*b*) qualitative shape of function $\alpha(m_i)$

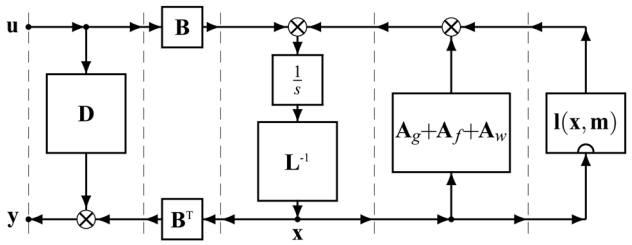

Fig. 5 POG block scheme of the thermodynamic system (20)

humidity of the moist air element j. The specific humidity u_j and the humidity ratio $\tilde{u}_j$ of the moist air element j are defined as $u_j = m_{jv}/m_{ja}$ and $\tilde{u}_j = 100\left[u_j/u_{\mathrm{sat}}(T_j)\right]$. The specific humidity u_j is always positive and must be lower than the specific humidity at saturation $u_{\mathrm{sat}}(T_j)$ corresponding to temperature T_j, i.e., $0 \le u_j \le u_{\mathrm{sat}}(T_j)$. Coefficient $\alpha(m_i)$ is a positive function of the water mass m_i, see Fig. 3(*b*), which describes the area of the contact surface between the liquid water and the moist air elements. The condition $\Psi_{ij} = 0$, corresponding to the absence of evaporation/condensation, is obtained when $\alpha(m_i) = 0$, i.e., when $m_i = 0$ see Fig. 3, or when air-specific humidity u_j is equal to the particular value $u_{j\mathrm{eq}} = u_{\mathrm{sat}}(T_j)\psi_0(T_i)/\psi_0(\max(T_i, T_j))$. If the temperature T_i of the water is equal to the temperature T_j of the moist air, i.e., if $T_i = T_j$, the condition of no evaporation/condensation arises when $u_j = u_{\mathrm{sat}}(T_j)$, that is, when the air-specific humidity u_j is equal to the saturation humidity $u_{\mathrm{sat}}(T_j)$. Consider, for example, the case $T_i > T_j$ with $(i,j) \in (\mathcal{W} \times \mathcal{V})$: if $u_j < u_{\mathrm{sat}}(T_j)$, then $\Psi_{ij} > 0$ and evaporation occurs; if $u_j > u_{\mathrm{sat}}(T_j)$, then $\Psi_{ij} < 0$ and condensation occurs.

Numerical Example. Consider the thermal system shown in Fig. 4. For this system: (a) the moist air elements are $\mathcal{V} = \{1,2,3,5,7\}$ with $q = 5$; (b) the liquid water elements are $\mathcal{W} = \{8,9\}$ with $r = 2$; (c) evaporation/condensation mass flow rates are present between the liquid water elements 8 and 9 and the moist air elements 1 and 2, respectively; (d) the liquid water element 9 is heated by the heat flux $\dot{Q}_1^u$ which improves the evaporation; and (e) the liquid water element 8 is in contact with the solid element 6. Matrices $\mathbf{L}$, $\mathbf{A}_f$, and $\mathbf{A}_m$ and vectors $\mathbf{x}$ and $\mathbf{m}_v$ are the same as in Eqs. (5), (18), and (19). Vectors $\mathbf{u}$ and $\mathbf{y}$ and matrices $\mathbf{B}$ and $\mathbf{D}$ are

$$\mathbf{u} = \begin{bmatrix} T_1^u \\ \dot{Q}_1^u \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} \dot{Q}_1^y \\ T_1^y \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & g_{71}^u & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^{\mathrm{T}}, \quad \mathbf{D} = \begin{bmatrix} -g_{71}^u & 0 \\ 0 & 0 \end{bmatrix}$$

The structure of matrix $\mathbf{A}_g$ is the same as in Eq. (6) with the addition of thermal conductivity g_{86} in the proper symmetrical positions. The structure of matrix $\mathbf{A}_w$ is

$$\mathbf{A}_w = \begin{bmatrix} -c_v\varphi_{18}^+ & 0 & 0 & 0 & 0 & 0 & c_w\varphi_{81}^+ & 0 \\ 0 & -c_v\varphi_{29}^+ & 0 & 0 & 0 & 0 & 0 & c_w\varphi_{92}^+ \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ c_v\varphi_{18}^+ & 0 & 0 & 0 & 0 & 0 & -c_w\varphi_{81}^+ & 0 \\ 0 & c_v\varphi_{29}^+ & 0 & 0 & 0 & 0 & 0 & -c_w\varphi_{92}^+ \end{bmatrix} \tag{24}$$

The structures of vectors $\mathbf{l}$, $\bar{\varphi}_v$, $\bar{\varphi}_w$, and $\mathbf{m}_w$ are

$$\mathbf{l} = \begin{bmatrix} c_l(\varphi_{81}^+ - \varphi_{18}^+) \\ c_l(\varphi_{92}^+ - \varphi_{29}^+) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ c_l(\varphi_{18}^+ - \varphi_{81}^+) \\ c_l(\varphi_{29}^+ - \varphi_{92}^+) \end{bmatrix}, \quad \bar{\varphi}_v = \begin{bmatrix} \bar{\varphi}_{v1} \\ \bar{\varphi}_{v2} \\ \bar{\varphi}_{v3} \\ \bar{\varphi}_{v4} \\ \bar{\varphi}_{v5} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 0 \\ \varphi_{81}^+ - \varphi_{18}^+ \end{bmatrix} \\ \begin{bmatrix} 0 \\ \varphi_{92}^+ - \varphi_{29}^+ \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{bmatrix},$$

$$\bar{\varphi}_w = \begin{bmatrix} \bar{\varphi}_{w1} \\ \bar{\varphi}_{w2} \end{bmatrix} = \begin{bmatrix} \varphi_{18}^+ - \varphi_{81}^+ \\ \varphi_{29}^+ - \varphi_{92}^+ \end{bmatrix}, \quad \mathbf{m}_w = \begin{bmatrix} m_{w1} \\ m_{w2} \end{bmatrix} = \begin{bmatrix} m_8 \\ m_9 \end{bmatrix} \tag{25}$$

For the sake of simplicity, the thermal model of this type, i.e., heat transfer due to both conduction and internal convection and presence of water evaporation/condensation, will be referred to as "model CCE" in the following.

3 Simulation

The thermodynamic system (20) can be graphically represented by the block scheme shown in Fig. 5, which has the typical structure of the POG schemes, see Ref. [18]. Usually in the POG schemes, the variables involved in each vertical dashed line are

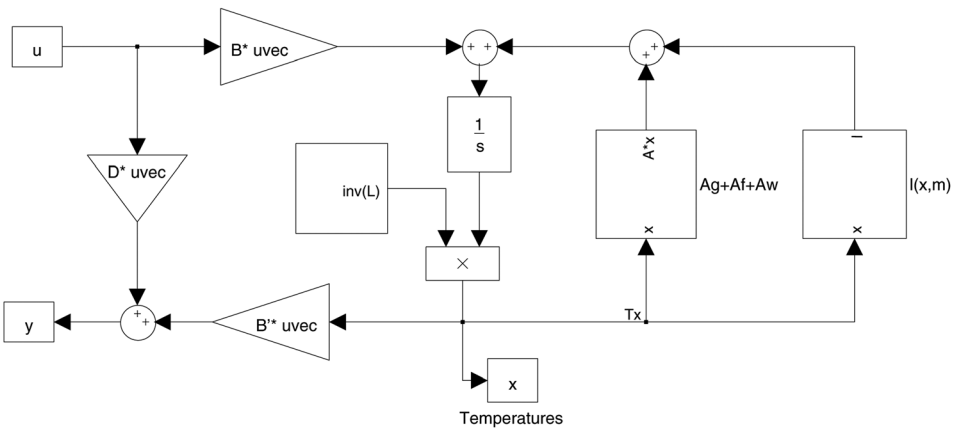

Fig. 6 SIMULINK block scheme of the thermodynamic system (20)

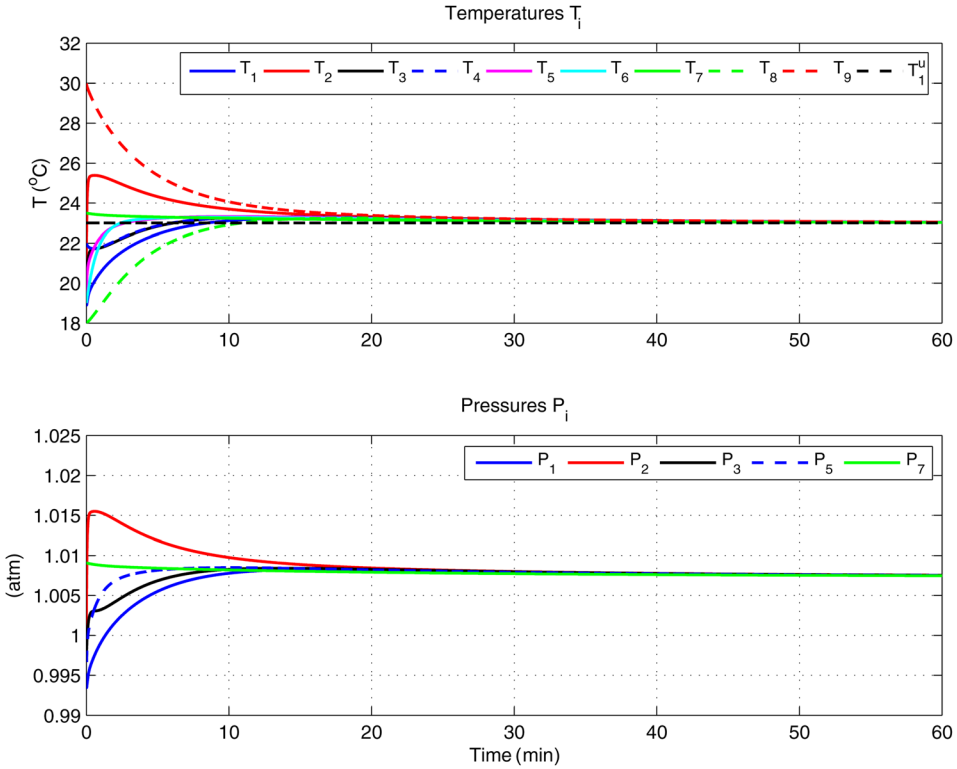

Fig. 7 Temperatures T_i and pressures P_i of model C

conjugated with respect to power. Otherwise, in the particular case of the thermodynamical system shown in Fig. 5, the variables T_i and $\dot{Q}_i$ involved in each dashed line are conjugated with respect to the variation of entropy as stated in Sec. 2.1.

The block scheme in Fig. 5 has been implemented in MATLAB using the SIMULINK model shown in Fig. 6. The simulations have been performed on a standard personal computer equipped with an Intel Core2 Duo CPU E8500 at 3.16 GHz and 3.46 GB of ram using a variable step solver ODE23TB. The same SIMULINK model with different parameters has been used to simulate all the three dynamic models (1), (13), and (20): the conduction model C, the internal convection model CC, and the evaporation/

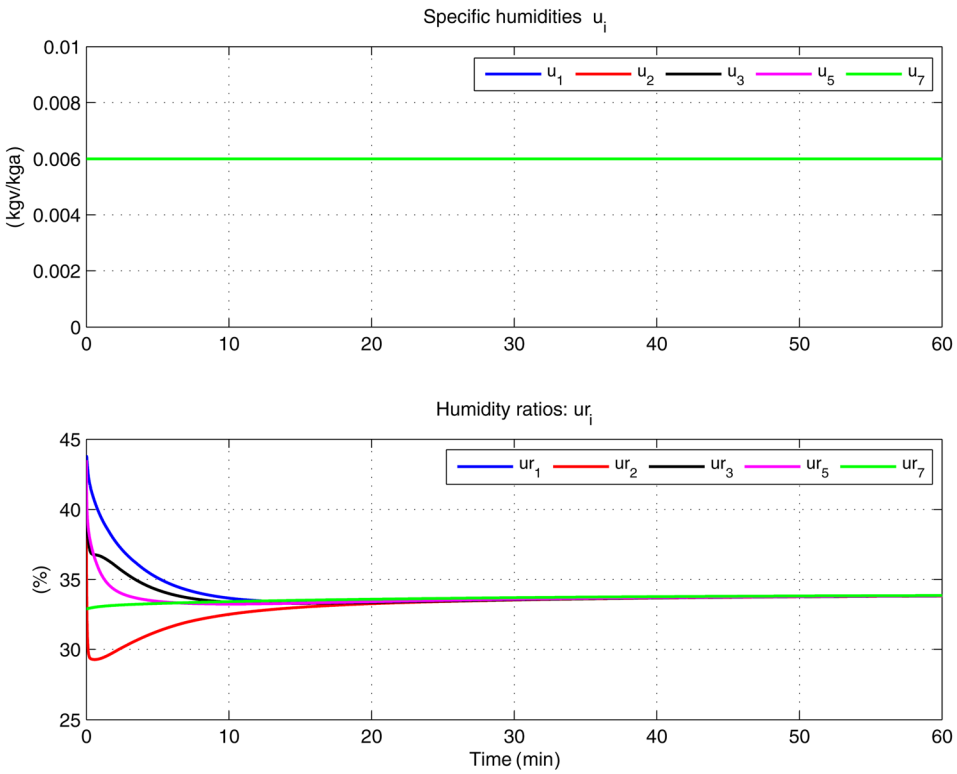

Fig. 8 Specific humidities u_i and humidity ratios $\tilde{u}_i$ of model C

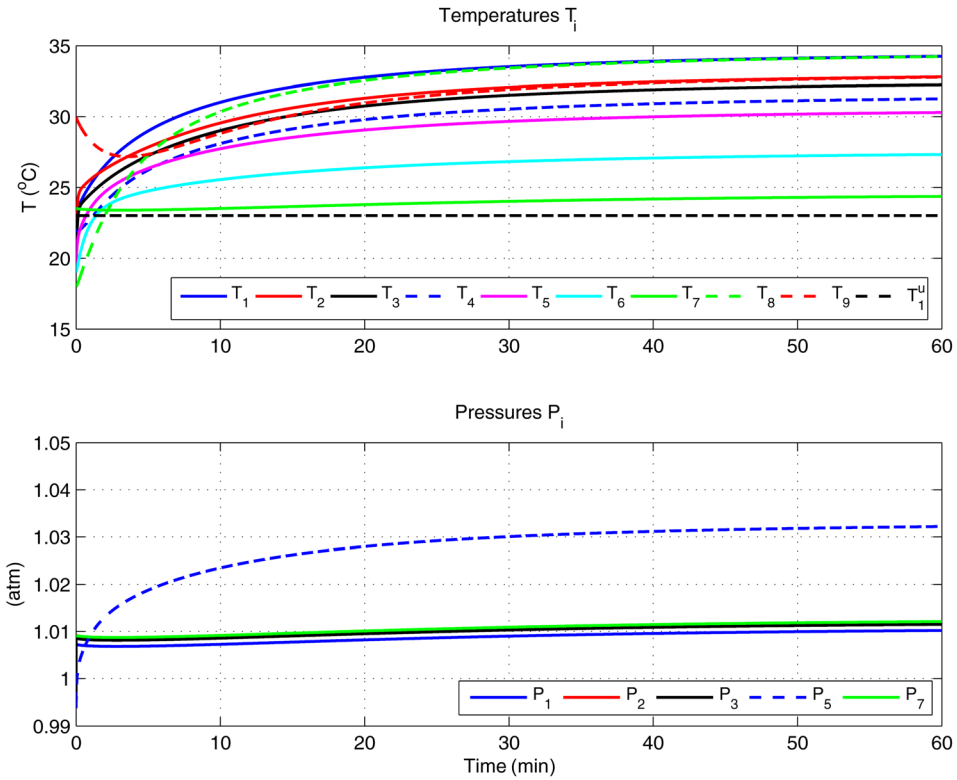

Fig. 9 Temperatures T_i and pressures P_i of model CC

condensation model CCE. In simulation, the following initial conditions have been used: $u_1 = u_2 = u_3 = u_5 = u_7 = 6$ g/kg, $P_1 = 0.9938$ atm, $\mathbf{x}^{\mathrm{T}} = \begin{bmatrix} T_1 & T_2 & T_3 & T_4 & T_5 & T_6 & T_7 & T_8 & T_9 \end{bmatrix} = \begin{bmatrix} 19 & 21 & 20 & 22 & 19 & 19 & 23.5 & 18 & 30 \end{bmatrix}^{\circ}$C, $P_2 = 1.0006$ atm, $P_3 = 0.9972$ atm, $P_5 = 0.9938$ atm, and $P_7 = 1.0091$ atm.

3.1 Simulation of Model C With Conduction. The simulation of model C has been performed considering no heat sources within the system and with the external temperature $T_1^u = 23\,^{\circ}$C, see Fig. 1. The obtained simulation time was 9.82 s. The temperatures T_i of the system elements and the pressures P_i of the moist

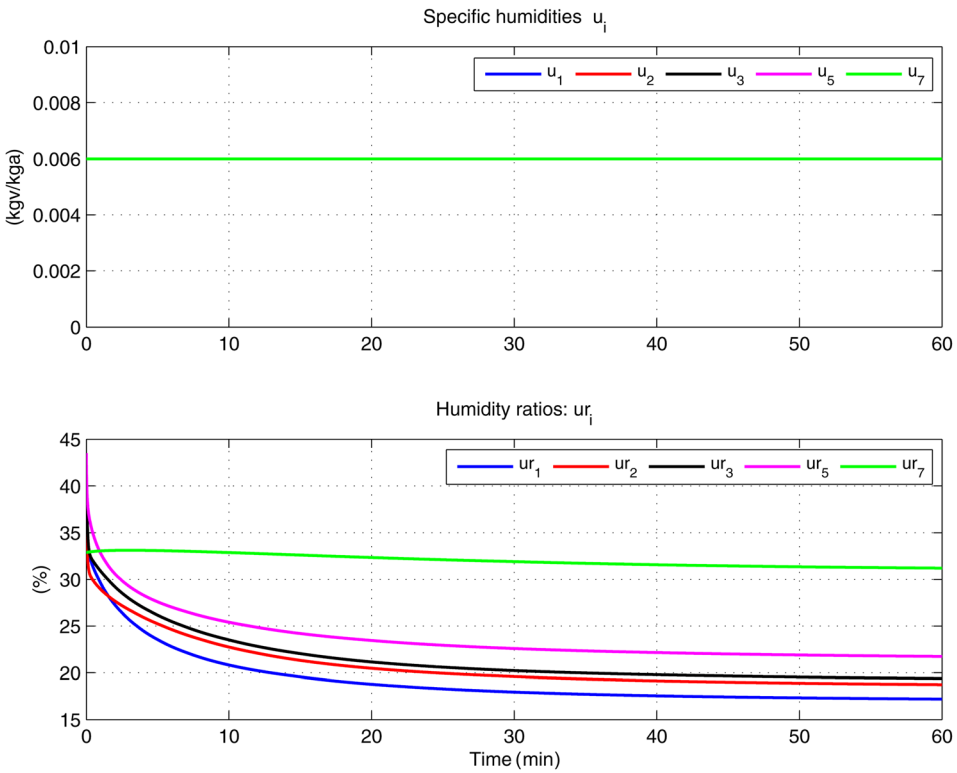

Fig. 10 Specific humidities u_i and humidity ratios $\tilde{u}_i$ of model CC

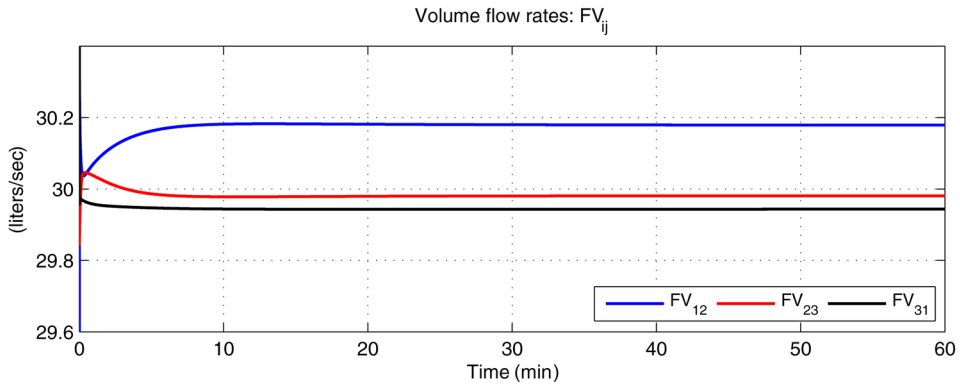

Fig. 11 Volume flow rates $\phi_{V_{12}}$, $\phi_{V_{23}}$, and $\phi_{V_{31}}$ of model CC

air elements 1, 2, 3, 5, and 7 are shown in Fig. 7. Note that, after a transient, all the temperatures T_i tend to the constant temperature T_1^u and all the pressures tend to the same value $P_i = 1.0075$ atm. The specific humidities and the humidity ratios of the moist air elements are shown in Fig. 8. Since no mass flows are present within the system, the specific humidities u_i remain unchanged, while the humidity ratios $\tilde{u}_i$ change with the temperature.

3.2 Simulation of Model CC With Conduction and Internal Convection. The simulation of model CC has been performed considering the external temperature $T_1^u = 23\,^{\circ}\mathrm{C}$ and the heat flux $\dot{Q}_1^u = 150$ W in element 1, see Fig. 2. The obtained simulation time was 10.41 s. In this case, the air can circulate within the system flowing through the four air ducts characterized by the hydraulic resistances $\mathcal{R}_{12}$, $\mathcal{R}_{23}$, $\mathcal{R}_{31}$, and $\mathcal{R}_{27}$. Moreover, a fan located between elements 1 and 2 produces a pressure jump $\Delta\bar{\mathbf{P}}_{12} = 240$ Pa. The temperatures T_i of the system elements and the pressures P_i of the moist air elements are shown in Fig. 9. Note that, after a transient, the temperatures T_i settle to different values. The pressure P_5 increases because temperature T_5 increases and the air of element 5 is isolated, i.e., it cannot expand. The specific humidities and the humidity ratios of the moist air elements are shown in Fig. 10. Also, in this case, the specific humidities remain unchanged, while the humidity ratios change with the temperature. The volume flow rates $\phi_{V_{12}}$, $\phi_{V_{23}}$, and $\phi_{V_{31}}$ are shown in Fig. 11. The volume flow rate $\phi_{V_{27}}$ is almost zero.

3.3 Simulation of Model CCE With Conduction, Internal Convection, and Evaporation/Condensation of Water. The simulation of model CCE has been performed considering the external surface temperature $T_1^u = 23\,^{\circ}\mathrm{C}$ and the heat flux $\dot{Q}_1^u = 50$ W entering the liquid water element 9, see Fig. 4. The obtained simulation time was 13.38 s. Like in the previous case, the air can circulate between the moist air elements 1, 2, 3, and 7, and the pressure jump $\Delta\bar{\mathbf{P}}_{12} = 240$ Pa has been inserted between elements 1 and 2. The initial masses of the liquid water elements are: $m_8(0) = 0.5$ kg and $m_9(0) = 0.7$ kg. The temperatures T_i of the

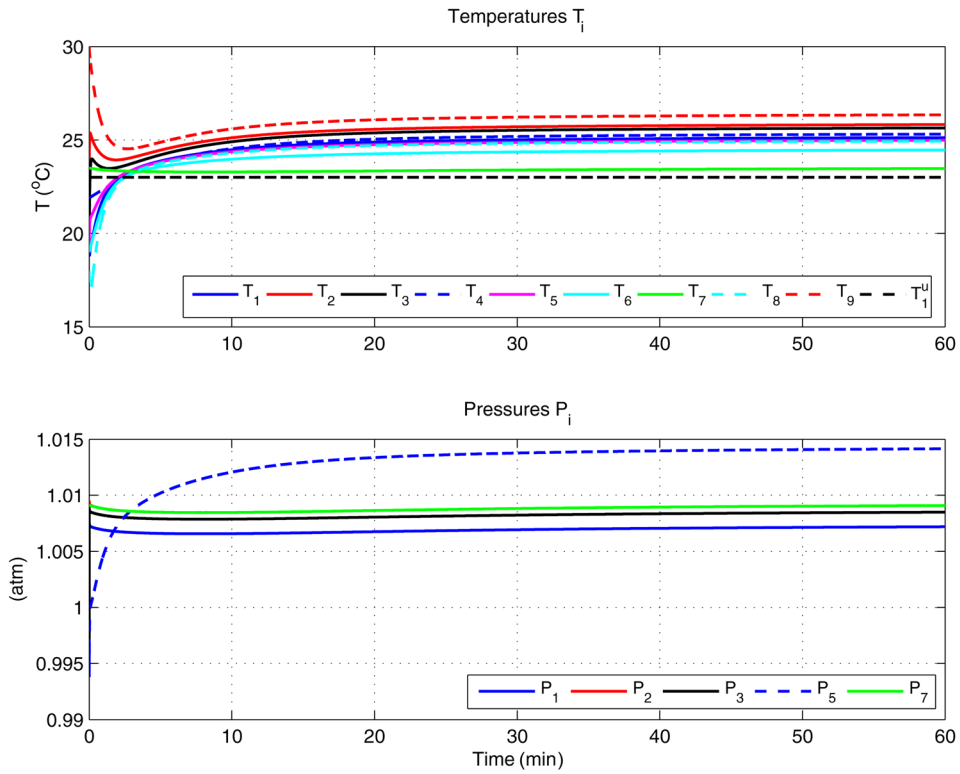

Fig. 12 Temperatures T_i and pressures P_i of model CCE

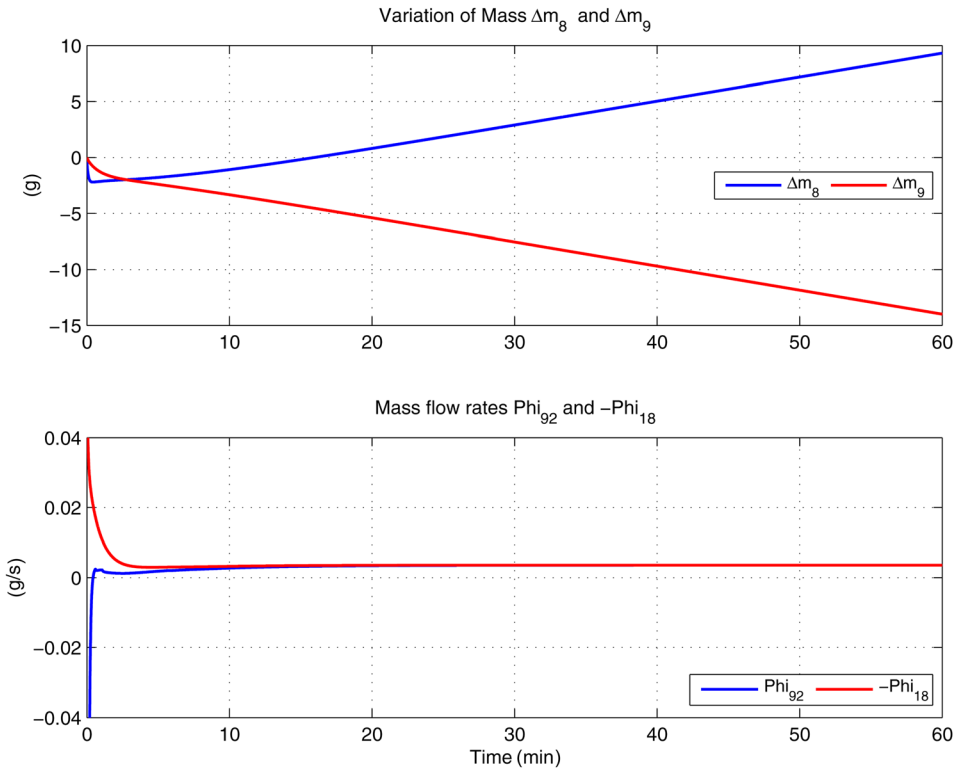

Fig. 13 Mass variations Δm_8, Δm_9, and evaporation/condensation mass flow rates $-\varphi_{18}$ and φ_{92} of model CCE

system elements and the pressures P_i of the moist air elements are shown in Fig. 12. Note that, after a transient, the temperatures settle to different values. The pressure P_5 increases because T_5 increases and the air of element 5 is isolated. The mass variations $\Delta m_8 = m_8(t) - m_8(0)$ and $\Delta m_9 = m_9(t) - m_9(0)$ of the liquid water elements and the evaporation/condensation mass flow rates $-\varphi_{18}$ and φ_{92} are shown in Fig. 13. It should be noticed that mass m_9 decreases, while mass m_8 increases according to the fact that element 9 is heated and the water tends to evaporate, while element 8 is at a lower temperature and vapor tends to condensate.

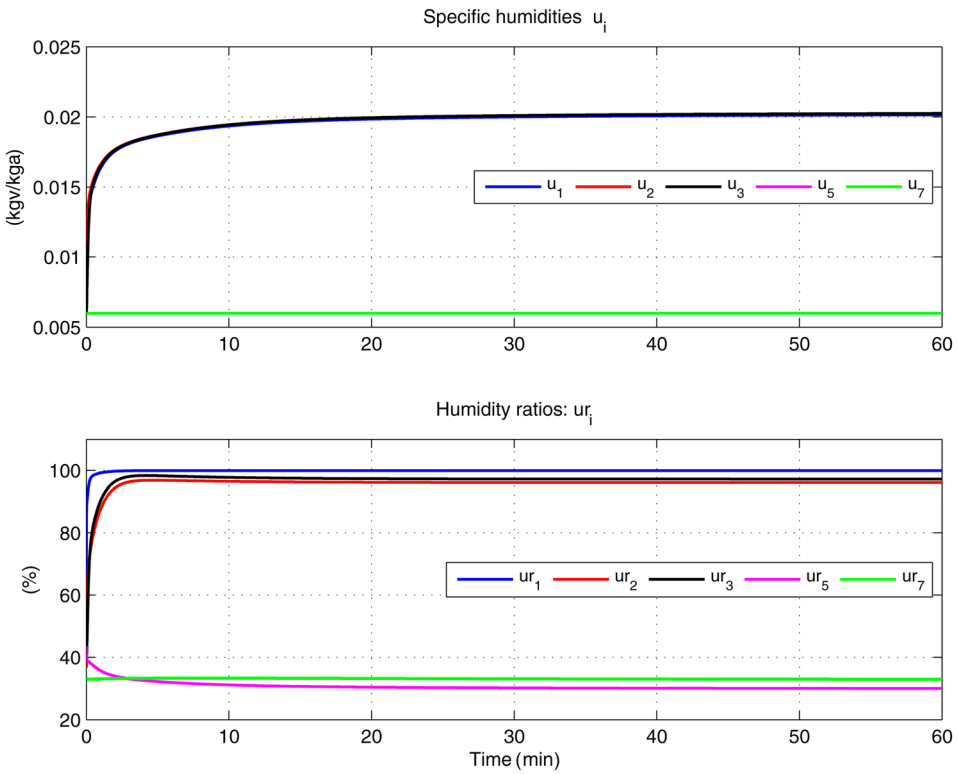

Fig. 14 Specific humidities u_i and humidity ratios $\tilde{u}_i$ of model CCE

The specific humidities and humidity ratios of the moist air elements are shown in Fig. 14. Note that, in this case, the specific humidities in the moist air elements 1, 2, and 3 increase because the evaporation of water tends to saturate the air humidity.

4 Conclusion

A new method to obtain compact dynamic thermal models is provided in the paper, based on an energy-based modeling approach. The proposed lumped parameters model is able to describe different types of heat transfer within the physical system: conduction, internal convection, and evaporation/condensation of water. The structure of the presented model is simple because it tries to describe only the main dynamics of the considered physical system. In particular, it does not take into account the boundary layer phenomena and the local geometry dependence of the system parameters. The structure of the proposed model guarantees the conservation of energy and mass within the system, thus providing a certain level of confidence in the model. The proposed model can be easily implemented in SIMULINK and the obtained simulation times are much shorter compared with those usually obtained using CFD programs. The proposed model does not guarantee very precise modeling, but it is suitable for real-time simulations and for control design purposes: it can be used to analyze the system dynamics, to predict nonmeasured variables, to study the effect of parameters variations, and to test different control strategies. The proposed model can be fruitfully used for designing the control laws of dryers, drying rooms, building conditioning systems, engine cooling systems, and industrial cooling systems. The next work on this topic will address the development of algorithms for fast and adaptive identification of the system parameters, based on experimental data, in order to obtain a precise and robust control of the considered thermal system.

Nomenclature

c_a = specific heat of dry air at constant pressure (J/(kg K))
c_l = latent heat of evaporation/condensation of water (J/kg)
c_v = specific heat of water vapor at constant pressure (J/(kg K))
c_w = specific heat of water (J/(kg K))
C_i = thermal capacity of element i (J/K)
f_{ij} = specific power coefficient between elements i and j (J/(s K))
g_{ij} = thermal conductivity between elements i and j (J/(s K))
M_a = molar mass of dry air (kg/mol)
M_w = molar mass of water (kg/mol)
m_{ia} = mass of dry air in element i (kg)
m_{iv} = mass of vapor in element i (kg)
P_i = pressure of element i (Pa)
$\dot{Q}_i$ = thermal power (entering or exiting) of element i (J/s)
R_{gas} = ideal gas constant (J/(mol K))
$\mathcal{R}_{ij}$ = hydraulic resistance between elements i and j (Pa·s/m^3)
T_i = temperature of element i (K)
u_i = specific humidity of moist air element i (kg/kg)
$\tilde{u}_i$ = humidity ratio of moist air element i (%)
V_i = volume of element i (m^3)
ρ_{air} = air mass density at $T = 25\ ^\circ$C and $P = 1$ atm (kg/m^3)
ρ_{ia} = mass density of dry air in element i (kg/m^3)
ρ_{iv} = mass density of vapor in element i (kg/m^3)
φ_{ij} = mass flow rate between elements i and j (kg/s)
$\phi_{V_{ij}}$ = volume flow rate between elements i and j (m^3/s)

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