The Power-Oriented Graphs for modeling mechanical systems with time-varying inertia

Roberto Zanasi
Department of Engineering "Enzo Ferrari",
University of Modena and Reggio Emilia, Modena, Italy
Email: roberto.zanasi@unimore.it

Federica Grossi
Department of Engineering "Enzo Ferrari",
University of Modena and Reggio Emilia, Modena, Italy
Email: federica.grossi@unimore.it

***Abstract*—The aim of this paper is to show how mechanical systems with time-varying inertia can be modeled by using the Power-Oriented Graphs technique. The reduced time-varying system is obtained starting from an extended Power-Oriented Graph model with masses and elasticities and applying a rectangular congruent transformation which eliminates the elasticities. The presented method is compared with the well established Bond Graphs modeling technique. The model of a crank-connecting rod system is considered as an example.**

I. Introduction

The main energy-based graphical modeling techniques developed for modeling dynamic physical systems are: the Bond-Graphs (BG) [1]-[2], the Power-Oriented Graphs (POG) [3], and the Energetic Macroscopic Representation (EMR) [4]. All these techniques exploit the concept of power flows within the system to build dynamic models. The lumped components of a physical system interact by energy ports through which the power flows. The energy-based techniques clearly show the power flows within the system. The BG and POG are mainly oriented toward dynamical system modeling while EMR is mainly oriented to the control of dynamic systems. Some comparison among these modeling techniques can be found in [5], [6] and [7].

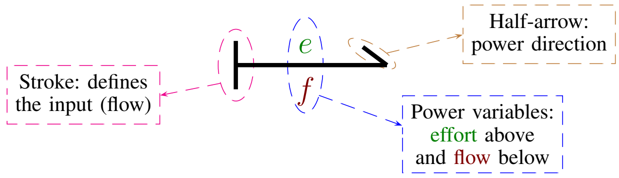
The focus of this paper is to show some aspects of POG giving a comparison with the well established BG. In particular the POG approach to reduce dynamical systems is shown and how to model a mechanical dynamic system with time-varying inertia is introduced. This particular topic, to the best of the author's knowledge, is not dealt with by the BG modeling approach. As an example for powertrain applications, the modeling of a crank-connecting rod system is addressed using the proposed approach.

II. Graphical energy-based modeling techniques

Both POG and BG techniques are graphical approaches to the energy-based modeling of dynamic systems: the components interact through energy ports and the power flowing through these ports is clearly shown in the graphical representation. The energy moves within a dissipative physical system from point to point only by means of two *power variables*. The product of the two power variables in each physical domain (electric, mechanical and hydraulic) has the physical meaning of instantaneous power flow.

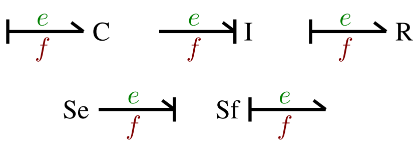
A. The Bond Graphs technique

This technique is well-known and therefore in this paper it will not be described in details. In the Bond Graphs representation a half arrow represents a "power bond" where the effort and flow variables are written respectively above and below the half arrow:



The direction of the half arrow defines the positive direction of power. The vertical stroke defines the input variable of the corresponding mathematical model: when the stroke is at the begin of the half-arrow the input variable is the *flow*, when the stroke is at the end of the half-arrow the input variable is the *effort*. The following *nine basic node-types* are distinguished in BG:

- Five 1-**port components**: C, I, R, Se, Sf

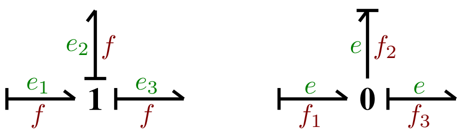


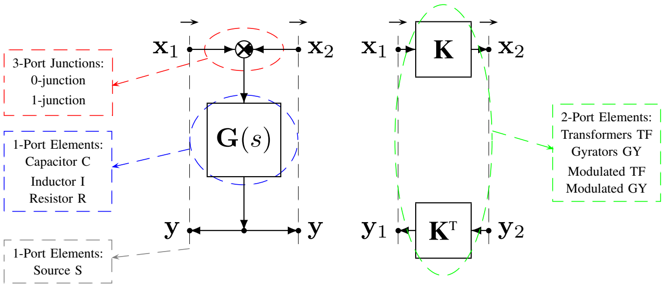
represent physical elements that store or dissipate energy (capacitor, inductor, resistor, etc.) or effort/flow sources.

- Two 2-**port components**: transformer TF and gyrator GY

represent physical elements that transform the power without storing nor dissipating energy (transformers, gear reduction, etc.). A letter M can also appear in the node symbol, MTF and MGY, standing for "modulated", thus expressing that the 2-port component depends on an external modulating signal.

- Two 3-**port components**: 1-junction and 0-junction





a) *Elaboration block* b) *Connection block*

Figure 1. POG basic blocks: *elaboration block* and *connection block* with the corresponding BG elements.

represent connections: the 1-junction connects elements sharing the same flow (i.e. the series connection in the electric domain), the 0-junction connects elements sharing the same effort (i.e. the parallel connection in the electric domain). Adding new bonds to the 3-port components one directly obtains the corresponding BG generalized n-port components.

B. The Power-Oriented Graphs technique

The main features of the POG modeling technique are stated in [3]. The POG technique uses only the two basic blocks shown in Fig. 1 for modeling physical systems:

a) the ***elaboration block*** (e.b.) is used for modeling all physical elements storing and/or dissipating energy (i.e. spring, mass, damper, capacitor, inductor, resistor, etc.). This block corresponds to the 1-port elements of types C, I, R used in BG. The summation element at the top of the block corresponds to all the 3-port connection elements (0-junction and 1-junction) of BG. The black spot within the summation element represents a minus sign that multiplies the entering variable. The e.b. can be scalar or vectorial and for linear systems matrix $\mathbf{G}(s)$ is always a square matrix of positive real transfer functions.

b) the ***connection block*** (c.b.) is used for modeling all physical elements transforming power without losses (i.e. gear reductions, transformers, etc.). This block corresponds to all the 2-port elements of BG (transformers TF and MTF, and gyrators GY and MGY). In the vectorial case, matrix $\mathbf{K}$ can also be rectangular, time-varying or a function of some state variables.

The vertical dashed lines in Fig. 1 represent the power sections which connect the two POG basic blocks to the external world. There are no restrictions on the choice of the vectors $\mathbf{x}$ and $\mathbf{y}$ involved in each dashed line except that their inner product $\langle \mathbf{x}, \mathbf{y} \rangle = \mathbf{x}^{\mathrm{T}}\mathbf{y}$ must have the physical meaning of instantaneous power flowing through the section.

III. Comparison between BG and POG

Both BG and POG have a direct correspondence between the power ports of the system and the power ports of their graphs. The power sections, represented by dashed lines in POG and by an half arrow in BG, always keep coupled the

	Variable	Electrical	Mec. Tra.	Mec. Rot.	Hydraulic
BG	**Effort:** e	V Voltage	F Force	τ Torque	P Pressure
	Flow: f	I Current	v Velocity	ω Angular Vel.	Q Flow Rate
POG	**Across:** v_e	V Voltage	v Velocity	ω Angular Vel.	P Pressure
	Through: v_f	I Current	F Force	τ Torque	Q Flow Rate

Figure 2. Bond Graphs definitions of the *effort* and *flow* variables and Power-Oriented Graphs definitions of *across* and *through* variables.

two power variables.

a) Power variables. The definitions of the power variables used in BG and POG schemes are shown in Fig. 2. In BG the power variables are defined as *effort* and *flow*, see for instance [9], while the POG uses the *across* and *through* variables, see [8]. An *across* variable is a variable defined between two points (i.e. voltage, speed, pressure, etc.), a *through* variable is a variable defined in each point of the space (i.e. current, force, flow rate, etc.). Note that in electrical and hydraulic domains the effort and flow variables correspond, respectively, to the across and through variables. On the contrary in the mechanical domain the opposite correspondences hold.

b) Series and parallel connections. Each physical element interacts with the external world, or with other physical elements, by means of the power ports associated to its terminals. The two possible ways of connecting a physical element are *in series* and *in parallel*: the connection is *in series* when the two terminals of the physical element share the same through-variable; the connection is *in parallel* when the two terminals share the same across-variable. These definitions are the ones used in the POG technique for all the energetic domains. The BG technique defines the connections between physical elements using a different approach: two elements are connected with a 0-junction if they share the same effort, while they are connected with a 1-junction if they share the same flow. The 0-junction and 1-junction correspond, respectively, to the parallel and series connections only for the electrical and hydraulic domains, while for the mechanical domain the opposite correspondences hold. Consider, for example, the spring and damper system shown in the left part of Fig. 3: the two physical elements are considered connected in parallel for the POG and connected with a 1-junction for the BG, see also [10] for a similar example.

c) Physical meaning of the graphical elements. It should be noted that the physical meaning of the nine graphical elements used in the BG technique is not absolute, but it depends on the particular definitions chosen for the effort and flow variables, see Fig. 2. The BG technique could be used exactly in the same way also choosing different definitions for the power variables, but in this case the physical meaning of the graphical elements would change too. In the mechanical domain, for example, the alternative definitions “velocity-effort” and “force-flow” would switch the meaning of the graphical elements used for describing masses, elasticities, transformers, gyrators, 0-junctions and 1-junctions. With these definitions, for example,

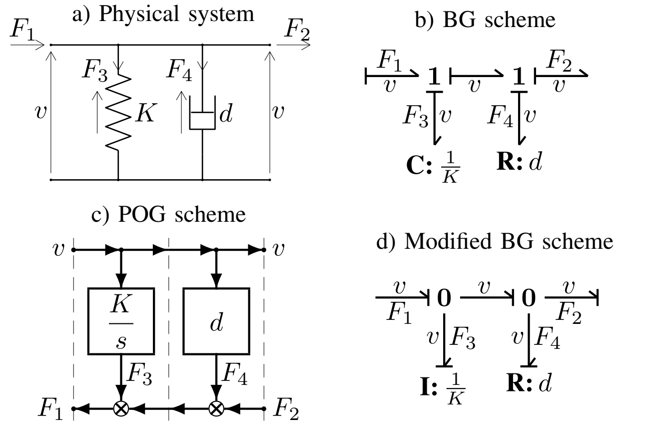


Figure 3. A spring and damper system: a) the physical system; b) the BG scheme; c) the POG scheme; d) the modified BG scheme.

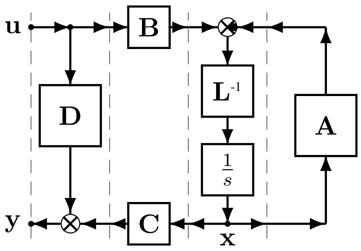


Figure 4. POG block scheme of a generic dynamic system.

the spring and damper system shown in Fig. 3.a can be represented using the modified BG scheme shown in Fig. 3.d. In this case the $\mathbf{C}$-element has been replaced by an $\mathbf{I}$-element and the 1-junctions have been replaced by 0-junctions.

d) Vectorial notation and causality. Usually BG use a scalar notation, but also vectorial Bond Graphs, called multi Bond Graphs, exist in the literature, see [11] and [5], with the restriction that all the vectors must be homogeneous, i.e. the vector components must be all efforts or all flows. In order to have a more compact scheme, POG often use a vectorial notation where the variable vectors can also contain mixed across or through variables. Like in BG, a POG can be written with derivative or integral causality, but obviously integral causality is mandatory in order to have POG models suitable for simulations.

e) State space equations. The state space equations of a physical system can be deduced from both BG and POG. For the BG a procedure to obtain such equations is introduced in [12], while for POG a procedure is given in [6]. The POG schemes can always be written in the following POG compact state space form:

$$\begin{cases} \mathbf{L}\,\dot{\mathbf{x}} = -\mathbf{A}\,\mathbf{x} + \mathbf{B}\,\mathbf{u} \\ \mathbf{y} = \mathbf{C}\,\mathbf{x} + \mathbf{D}\,\mathbf{u} \end{cases} \tag{1}$$

where $\mathbf{x} \in \mathbb{R}^n$ is the state vector, $\mathbf{u} \in \mathbb{R}^m$ is the input, $\mathbf{y} \in \mathbb{R}^m$ is the output, $\mathbf{L}$ is the *energy matrix*, $\mathbf{A}$ is the *power matrix*, $\mathbf{B}$ is the *input matrix*, $\mathbf{C}$ is the *output matrix* and $\mathbf{D}$ is the *input-output matrix*. The POG system (1) can be graphically represented by using the POG scheme shown in Fig. 4. For the POG system (1) the following properties hold. The energy matrix $\mathbf{L}$ is always symmetric and positive definite $\mathbf{L} = \mathbf{L}^{\mathrm{T}} > 0$. The *energy E_s stored in the system* can be expressed as

$$E_s = \frac{1}{2}\mathbf{x}^{\mathrm{T}}\mathbf{L}\,\mathbf{x}. \tag{2}$$

The power matrix $\mathbf{A}$ can be decomposed in symmetric and skew-symmetric part

$$\mathbf{A} = \mathbf{A}_s + \mathbf{A}_w = \frac{(\mathbf{A} + \mathbf{A}^{\mathrm{T}})}{2} + \frac{(\mathbf{A} - \mathbf{A}^{\mathrm{T}})}{2} \tag{3}$$

where $\mathbf{A}_s$ contains only dissipation terms and $\mathbf{A}_w$ only connection terms. The *dissipating power P_d in the system* can be expressed as

$$P_d = \mathbf{x}^{\mathrm{T}}\mathbf{A}\,\mathbf{x} = \mathbf{x}^{\mathrm{T}}\mathbf{A}_s\,\mathbf{x}. \tag{4}$$

The POG form (1) is similar to the notations used for the port-Hamiltonian equations in co-energy variables [13] and in generalized state-space equations [14].

IV. State space transformations

A. Constant state space transformations.

Using a constant "congruent" transformation $\mathbf{x} = \mathbf{T}\mathbf{z}$, where $\mathbf{z} \in \mathbb{R}^p$, $p \le n$ and $\mathbf{T} \in \mathbb{R}^{n\times p}$, the dynamic model (1) can be transformed as follows

$$\begin{cases} \overline{\mathbf{L}}\,\dot{\mathbf{z}} = -\overline{\mathbf{A}}\,\mathbf{z} + \overline{\mathbf{B}}\,\mathbf{u} \\ \mathbf{y} = \overline{\mathbf{C}}\,\mathbf{z} + \overline{\mathbf{D}}\,\mathbf{u} \end{cases} \tag{5}$$

where $\overline{\mathbf{L}} = \mathbf{T}^{\mathrm{T}}\mathbf{L}\mathbf{T}$, $\overline{\mathbf{A}} = \mathbf{T}^{\mathrm{T}}\mathbf{A}\mathbf{T}$, $\overline{\mathbf{B}} = \mathbf{T}^{\mathrm{T}}\mathbf{B}$, $\overline{\mathbf{C}} = \mathbf{C}\mathbf{T}$ and $\overline{\mathbf{D}} = \mathbf{D}$. One can easily show that the transformed system (5) maintains all the power and energy properties of system (1).

When an eigenvalue of matrix $\mathbf{L}$ tends to zero (or to infinity), system (1) degenerates towards a lower dimension dynamic system. In this case the reduced and transformed system (5) can be obtained using a "rectangular" matrix $\mathbf{T}$, see [6].

B. Time-varying state space transformations.

When the transformation matrix $\mathbf{T}(t)$ is time-varying, the transformed system is

$$\begin{cases} \underbrace{\mathbf{T}^{\mathrm{T}}\mathbf{L}\mathbf{T}}_{\bar{\mathbf{L}}(t)}\,\dot{\mathbf{z}} + \underbrace{\mathbf{T}^{\mathrm{T}}\mathbf{L}\dot{\mathbf{T}}}_{\mathbf{N}_2(t)}\,\mathbf{z} = -\underbrace{\mathbf{T}^{\mathrm{T}}\mathbf{A}\mathbf{T}}_{\bar{\mathbf{A}}}\,\mathbf{z} + \underbrace{\mathbf{T}^{\mathrm{T}}\mathbf{B}}_{\bar{\mathbf{B}}}\,\mathbf{u} \\ \mathbf{y} = \underbrace{\mathbf{C}\mathbf{T}}_{\bar{\mathbf{C}}}\,\mathbf{z} + \underbrace{\mathbf{D}}_{\bar{\mathbf{D}}}\,\mathbf{u} \end{cases} \tag{6}$$

where matrices $\bar{\mathbf{L}}$, $\bar{\mathbf{A}}$, $\bar{\mathbf{B}}$ and $\bar{\mathbf{C}}$ are time-varying and $\mathbf{N}_2(t) = \mathbf{T}^{\mathrm{T}}\mathbf{L}\dot{\mathbf{T}}$. System (6) can be graphically represented by the POG scheme of Fig. 5: note that block $\mathbf{N}_2(t)$ is structurally connected with the dynamic block $\bar{\mathbf{L}}(t)$, therefore only the two blocks considered together properly describe a time-varying dynamic element. When $\mathbf{L}$ is constant the transformed system (6) can also be written in the following equivalent POG form:

$$\begin{cases} \dfrac{d(\mathbf{T}^{\mathrm{T}}\mathbf{L}\mathbf{T}\mathbf{z})}{dt} - \dot{\mathbf{T}}^{\mathrm{T}}\mathbf{L}\mathbf{T}\,\mathbf{z} = -\mathbf{T}^{\mathrm{T}}\mathbf{A}\mathbf{T}\,\mathbf{z} + \mathbf{T}^{\mathrm{T}}\mathbf{B}\,\mathbf{u} \\ \mathbf{y} = \mathbf{C}\mathbf{T}\,\mathbf{z} + \mathbf{D}\,\mathbf{u} \end{cases} \tag{7}$$

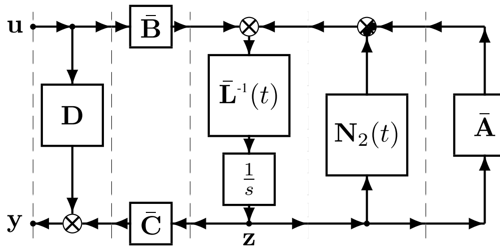


Figure 5. POG block scheme of the transformed dynamic system (6).

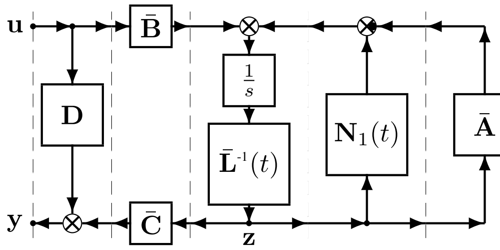


Figure 6. POG block scheme of the transformed dynamic system (7).

that is $\frac{d}{dt}(\bar{\mathbf{L}}\mathbf{z}) - \mathbf{N}_1(t)\mathbf{z} = -\bar{\mathbf{A}}\mathbf{z} + \bar{\mathbf{B}}\mathbf{u}$ and $\mathbf{y} = \bar{\mathbf{C}}\mathbf{z} + \bar{\mathbf{D}}\mathbf{u}$, where $\mathbf{N}_1(t) = \dot{\mathbf{T}}^{\mathrm{T}}\mathbf{L}\mathbf{T}$. It can be proved that the following relations hold:

$$\dot{\bar{\mathbf{L}}} = \mathbf{N}_1 + \mathbf{N}_2, \qquad \mathbf{N}_1^{\mathrm{T}} = \mathbf{N}_2, \qquad \mathbf{N}_2^{\mathrm{T}} = \mathbf{N}_1,$$

from which it follows that

$$\mathbf{N}_{1s} = \frac{\mathbf{N}_1 + \mathbf{N}_1^{\mathrm{T}}}{2} = \frac{\dot{\bar{\mathbf{L}}}}{2} = \frac{\mathbf{N}_2 + \mathbf{N}_2^{\mathrm{T}}}{2} = \mathbf{N}_{2s} \tag{8}$$

When $p = 1$ (i.e. $\mathbf{T}$ is a column matrix) matrices $\bar{\mathbf{L}}$, $\bar{\mathbf{A}}$, $\bar{\mathbf{B}}$, $\bar{\mathbf{C}}$, $\mathbf{N}_1$ and $\mathbf{N}_2$ degenerate to scalars and from (8) it follows

$$N_1 = N_2 = \frac{\dot{\bar{L}}}{2} \tag{9}$$

C. Mechanical systems with time-varying inertia.

The mechanical systems with time-varying inertia can always be obtained as the limit case of an extended POG model with masses and elasticities, see the POG model given in (5), when the elasticities tend to zero. The time-varying reduced model (6) can always be obtained from the extended POG model using a time-varying rectangular and congruent POG transformation $\mathbf{z} = \mathbf{T}(t)\,\mathbf{x}$ which eliminates the elasticities. An example of this method is given in the following section.

V. Model of a crank-connecting rod system

Consider the elastic crank-connecting rod system shown in Fig. 7. The meaning of the system parameters is shown in Fig. 8. The relation which links the angular position $\theta(t)$ of the crank with the translational position x_k of point P_k is the following:

$$x_k(\theta) = R\cos\theta + \sqrt{L^2 - (R\sin\theta - d)^2}.$$

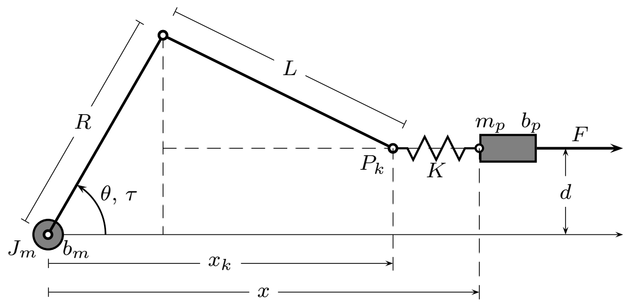


Figure 7. Basic structure of the elastic crank-connecting rod system.

J_m	shaft moment of inertia
b_m	shaft linear friction coefficient
K	stiffness of the fictitious spring
m_p	mass of the piston
b_p	linear friction coefficient of the piston
θ	shaft angular position
ω	shaft angular velocity ($\omega = \dot{\theta}$)
τ	torque acting on the shaft
$\dot{x}$	translational velocity of the piston
F	external force acting on the piston

Figure 8. Parameters of the crank-connecting rod system.

The velocity $\dot{x}_k$ can be expressed as a function of the velocity $\omega = \dot{\theta}$ as follows:

$$\begin{aligned}\dot{x}_k(t) &= R\left[-\sin\theta - \frac{(R\sin\theta - d)\cos\theta}{\sqrt{L^2 - (R\sin\theta - d)^2}}\right]\dot{\theta}\\ &= \underbrace{R\left[-\sin\theta - \frac{(\sin\theta - \beta)\cos\theta}{\sqrt{\alpha^2 - (\sin\theta - \beta)^2}}\right]}_{H(\theta)}\omega \;=\; H(\theta)\,\omega\end{aligned}$$

where function $H(\theta)$ and the auxiliary parameters α and β are defined as follows:

$$H(\theta) = \frac{\partial x(\theta)}{\partial\theta}, \qquad \alpha = \frac{L}{R} > 1, \qquad \beta = \frac{d}{R} < 1.$$

The time derivative of function $H(\theta)$ is:

$$\dot{H}(\theta) = -R\left[\cos\theta + \frac{\alpha^2(\cos(2\theta) + \beta\sin\theta) + \sin\theta(\sin\theta - \beta)^3}{(\alpha^2 - (\sin\theta - \beta)^2)^{\frac{3}{2}}}\right]\omega$$

The POG dynamic model of the considered dynamic system is shown in Fig. 9 and the corresponding POG state space equations are expressed by (1) where:

$$\begin{aligned}\mathbf{L} &= \begin{bmatrix} J_m & & \\ & K^{-1} & \\ & & m_p \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} b_m & H(\theta) & 0 \\ -H(\theta) & 0 & 1 \\ 0 & -1 & b_p \end{bmatrix},\\ \mathbf{B} &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} \omega \\ F_k \\ \dot{x} \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} \tau \\ F \end{bmatrix}\end{aligned} \tag{10}$$

The fictitious stiffness K between the connecting rod and the piston has been inserted in the system as a good way to obtain

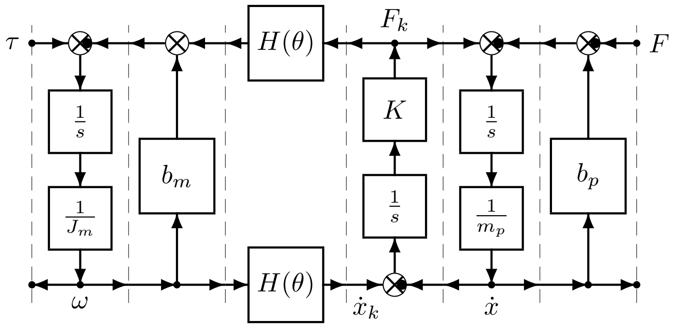


Figure 9. POG dynamic model of the extended dynamic system: the motor, the crank, the connecting rod, the fictitious stiffness and the piston.

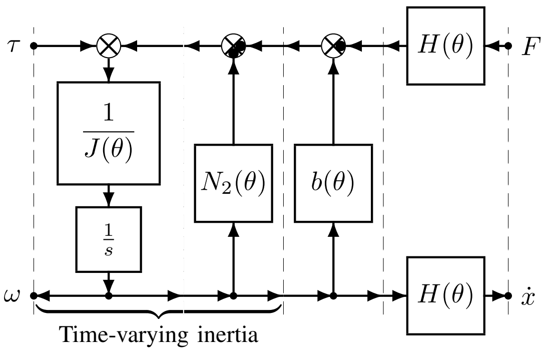


Figure 10. Equivalent POG scheme of the reduced rigid system (11).

the time-varying dynamic model of the corresponding rigid system when stiffness K tends to infinity. When $K \to \infty$, the state space variables $\dot{x}$ and ω are related by the following constraint:

$$\dot{x} = H(\theta)\,\omega.$$

Applying to system (1)-(10) the following rectangular congruent state space transformation

$$\mathbf{x} = \mathbf{T}(\theta)\,\omega, \qquad \Leftrightarrow \qquad \underbrace{\begin{bmatrix} \omega \\ F_k \\ \dot{x} \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} 1 \\ 0 \\ H(\theta) \end{bmatrix}}_{\mathbf{T}(\theta)} \omega$$

one obtains the following transformed and reduced system:

$$J(\theta)\,\dot{\omega} + N_2(\theta)\omega = -b(\theta)\,\omega + \overline{\mathbf{B}}(\theta)\,\mathbf{u} \qquad (11)$$

where:

$$\begin{aligned} J(\theta) &= \mathbf{T}^{\mathrm{T}}\mathbf{L}\mathbf{T} = J_m + m_p\,H^2(\theta) \\ N_2(\theta) &= \mathbf{T}^{\mathrm{T}}\mathbf{L}\dot{\mathbf{T}} = m_p\,H(\theta)\dot{H}(\theta) = \dot{J}(\theta)/2 \\ b(\theta) &= \mathbf{T}^{\mathrm{T}}\mathbf{A}\mathbf{T} = b_m + H^2(\theta)\,b_p \\ \overline{\mathbf{B}}(\theta) &= \mathbf{T}^{\mathrm{T}}\mathbf{B} = \begin{bmatrix} 1 & -H(\theta) \end{bmatrix} \end{aligned}$$

The POG scheme of the dynamic equations (11) is shown in Fig. 10.

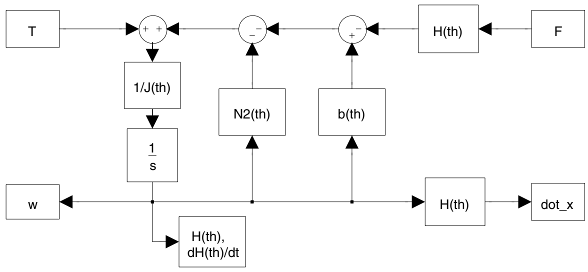


Figure 11. Simulink model of the reduced POG scheme shown in Fig. 10.

Table I
MAIN PARAMETERS OF THE SYSTEM

R	10	cm
L	40	cm
d	5	cm
J_m	12.5	kg cm^2
b_m	0.0955	Nm/(rad/s)
m_p	0.8	kg
b_p	2	N/(m/s)

VI. SIMULATIONS

The Simulink model corresponding to the POG scheme of Fig. 10 is shown in Fig. 11. The main parameters used in simulation are shown in Table I. The simulation results have been obtained using the input torque $\tau = 10$ Nm, the input force $F = 0$ N, the initial speed $\omega_0 = 200$ rpm and the initial position $\theta_0 = 0$ rad. Function $H(\theta)$ and its time derivative $\dot{H}(\theta)$ are shown in Fig. 12. The shaft angular velocity ω and piston speed $\dot{x}$ are shown in Fig. 13. The time-varying inertia $J(\theta)$ and the energy E_m stored within the system are shown in Fig. 14. The motor torque τ_m, the dynamic torque $-N_2(\theta)\omega$ and the friction torque $-b(\theta)\omega$ are shown in Fig. 15. This figure clearly shows that for the considered system the total torque $\tau_t = \tau_m - N_2(\theta)\omega - b(\theta)\omega$ is very close to the dynamic torque $\tau_t \simeq -N_2(\theta)\omega$, that is the considered mechanical system is dominated by the dynamic torque $-N_2(\theta)\omega$.

VII. CONCLUSION

The Power-Oriented Graphs modeling technique is a useful tool for modeling physical dynamic systems. In the paper the POG modeling of physical systems with time-varying dynamical elements has been presented and applied to mechanical systems with time-varying inertia. In particular the proposed modeling approach has been applied to a crank-connecting rod system.

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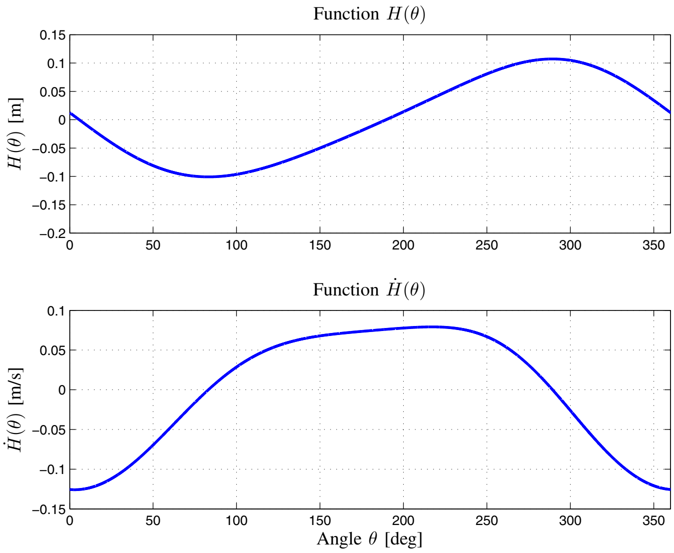

Figure 12. Functions $H(\theta)$ and $\dot{H}(\theta)$.

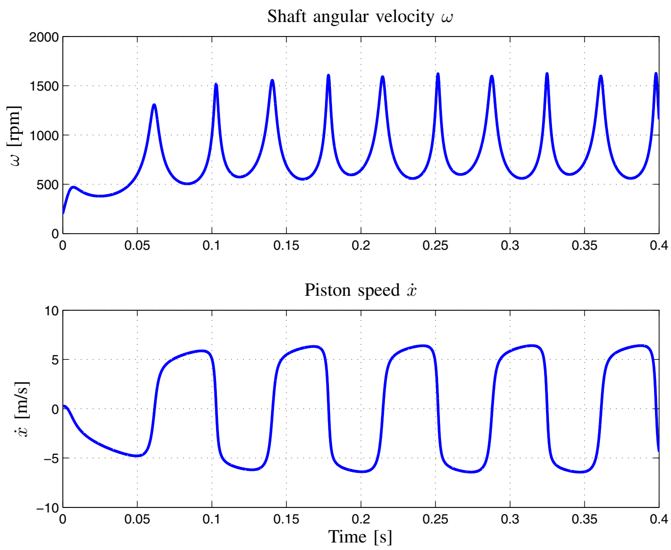

Figure 13. Shaft angular velocity ω and piston speed $\dot{x}$.

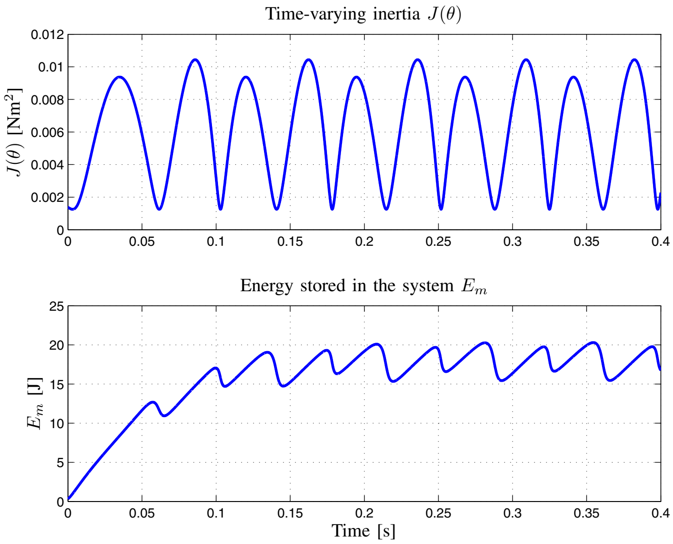

Figure 14. Time-varying inertia $J(\theta)$ and energy E_m stored in the system.

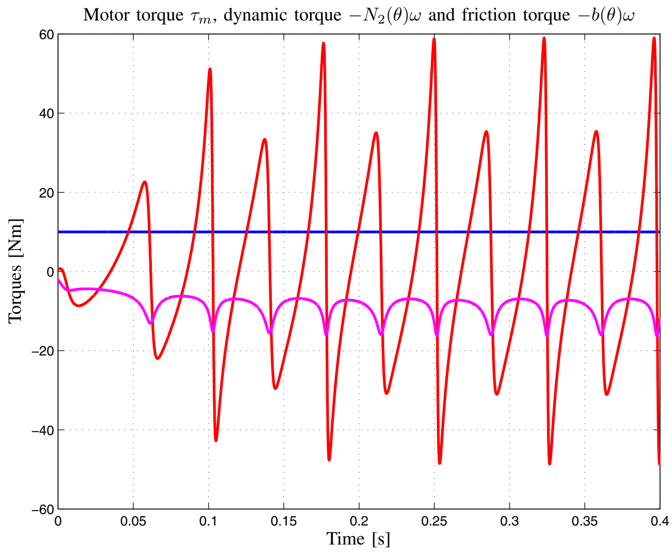

Figure 15. Motor torque τ_m (blue), dynamic torque $-N_2(\theta)\omega$ (red) and friction torque $-b(\theta)\omega$ (magenta).

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