Open and Closed Logarithmic Nyquist Plots

Roberto Zanasi, Federica Grossi

***Abstract*— The Nyquist plot is a fundamental tool in the investigation of the stability of control systems. Usually Nyquist polar diagrams are plotted in a linear scale and often, in particular when poles at the origin are present, the diagrams need different levels of magnification in order to inspect the behavior of the frequency response in the areas very close to or very far from the origin of the complex plane. In this paper a new logarithmic Nyquist plot is proposed where the amplitude is in a logarithmic scale and the diagram is entirely contained and shown in a circle of finite radius. This method does not need to zoom in or zoom out the plot. All the considerations made by Nyquist stability criterion can be done with this plot which maintains all the properties of polar plots such as gain and phase margins, intersection points with the real axis, encirclements of the critical point. The design of first order lead and lag compensators can be done on this diagram in a simple way. The proposed new Nyquist plot is implemented in a Matlab function available to users.**

I. INTRODUCTION

The Nyquist stability criterion plays a central role in classical control theory for discussing the stability of closed-loop systems, see control textbooks [1] and [2] among many others. One of the most frequent inconvenients in studying the stability of closed-loop systems by means of Nyquist plots is that often the open-loop frequency response has a large span in amplitudes, thus resulting, in a linear scale, in the need for the user to zoom in and zoom out the polar plot to inspect the intersections with the real axis.

A way to face this problem has been given in [3] where the idea is to draw the polar diagram with a logarithmic scale for amplitudes instead of the linear scale used for Nyquist plot. However the log-polar diagram proposed in [3] does not show the whole curve for the entire range of frequencies. Another alternative to Nyquist plot is the projection of Nyquist plot onto the Riemann sphere, see [4], but here some difficulties can arise in handling a three-dimensional object.

In this paper some new functions are introduced to deform the amplitude of the frequency response function in logarithmic way making the polar diagram to be all contained in a finite area of the plane. In particular the function showing the best performances in terms of simplicity and preservation of basic properties of polar diagrams is the so called “Closed Logarithmic Nyquist plot”.

The paper is organized as follows. Sec. II reminds the features of linear and logarithmic Nyquist diagrams. Sec. III proposes a first formulation for a logarithmic conversion of the frequency response amplitude. Sec. IV introduces the new “Closed Logarithmic Nyquist plot” giving its main features and a comparison with [3]. Sec. V provides a procedure to manually draw the Closed Logarithmic Nyquist plot. Sec. VI explains how the design of first order lead and lag compensators can also be addressed on this new Nyquist diagram. The Matlab function implemented for plotting the logarithmic diagrams is available at the link given in Sec. VII. Finally conclusions are given.

Roberto Zanasi and Federica Grossi are with the Department of Engineering ”Enzo Ferrari”, University of Modena and Reggio Emilia, Modena, Italy roberto.zanasi@unimore.it, federica.grossi@unimore.it

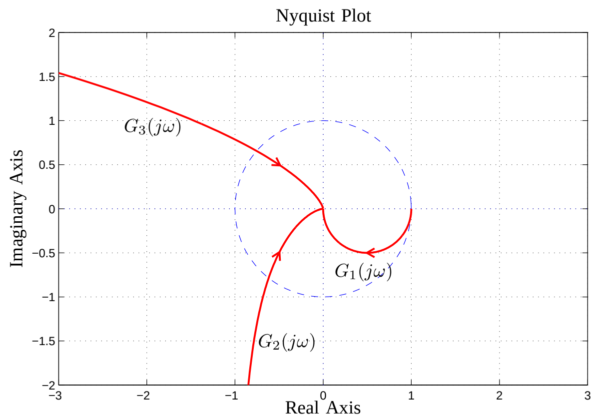

Fig. 1. Linear Nyquist plot of $G_1(j\omega)$, $G_2(j\omega)$ and $G_3(j\omega)$.

II. LINEAR AND LOGARITHMIC NYQUIST PLOT

Consider the Nyquist plots of the following functions shown in red in Fig. 1:

$$G_1(s)=\frac{1}{(s+1)},\quad G_2(s)=\frac{1}{s(s+1)},\quad G_3(s)=\frac{1}{s^2(s+1)}$$

The Nyquist diagram plots the amplitude of the frequency response $|G(j\omega)|$ in a linear scale: this can be a strong drawback because in many cases, also with the help of numerical tools, the obtained polar plots may result unreadable because of the large span in the magnitude over the entire frequency range, that hides the local behavior of the curve, in particular in the region near the unit circle. This situation is quite common when the considered systems have poles on the imaginary axis. In order to cope with this problem, [3] proposes to scale the magnitude of polar plots by adopting a dB scale, that allows to magnify the parts of the polar plot close to the origin without losing the diagram overview. The author of [3] provides the MATLAB function `nyqlog(sys)` to obtain the "log-polar" plot of a transfer function. In this case the modulus $M = |G(j\omega)|$ is transformed using the following function $LP(M, n)$:

$$LP(M,n) = \begin{cases} 1 + \dfrac{1}{n}\log_{10}(M) & \text{if } \; M > 10^{-n} \\ 0 & \text{if } \; M \leq 10^{-n} \end{cases} \quad (1)$$

where n is the number of decades considered in the logarithmic plot. In [3] it is $n = 6$. This approach gives good results in particular with respect to the problem of detecting the intersections with the (negative) real axis, but it requires to set the minimum value of $|G(j\omega)|$ that can be represented and any value of $|G(j\omega)|$ lower than this minimum value is located in the center of the diagram (i.e. the origin of the complex plane). In the considered case in [3] any value lower than 10^{-6} is set to $10^{-6} = -120$ dB and any intersection of the polar plot with the real axis in this region is not shown. In order to avoid any loss of information, the value of parameter n must be chosen according to the particular form of the considered function.

In order to face this problem this paper propose some different logarithmic scaling for Nyquist diagrams.

III. Open Logarithmic Nyquist Plot

The amplitude M of the frequency response $M = |G(j\omega)|$ can be logarithmically deformed in "base 2" using the following function $N_2(M)$:

$$N_2(M) = M^{\log_{10} 2}$$

The function $N_2(M)$ transforms the powers of 10 in powers of 2 as follows:

$$M = 10^n \;\rightarrow\; N_2(M) = (10^n)^{\log_{10} 2} = 10^{\log_{10} 2^n} = 2^n$$

The points with constant modulus defined by function $N_2(M)$ are shown in Fig. 2 using blue dotted lines. This kind of diagram is called "Open Logarithmic Nyquist plot".

Note that this logarithmic Nyquist plot makes the region about the origin larger. Similarly, the region of points with modulus greater than one is contracted. However the polar diagram obtained with function $N_2(M)$ has the drawback to be unlimited for $M \rightarrow \infty$.

IV. Closed Logarithmic Nyquist Plot

In order to have a limited logarithmic polar plot when the amplitude M of the frequency response tends to infinity, the following function $L_2(M)$ is introduced:

$$L_2(M) = \begin{cases} M^{\log_{10} 2} & \text{for } \; M \leq 1 \\ 2 - M^{-\log_{10} 2} & \text{for } \; M > 1 \end{cases} \quad (2)$$

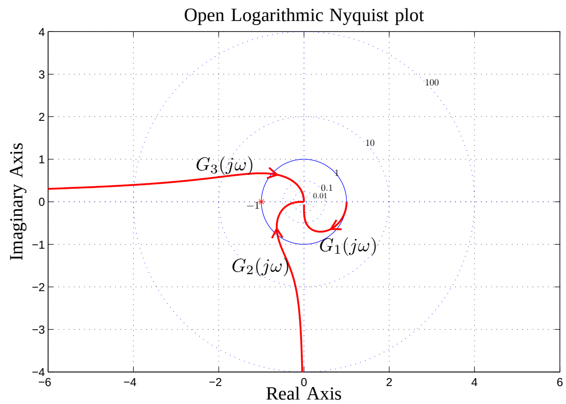

Fig. 2. Open Logarithmic Nyquist plot of $G_1(s)$, $G_2(s)$ and $G_3(s)$ obtained using function $N_2(M)$.

This kind of diagram is called "Closed Logarithmic Nyquist plot" (CLN plot). The CLN plots of $G_1(s)$, $G_2(s)$ and $G_3(s)$ obtained using function $L_2(M)$ are shown in Fig. 3.

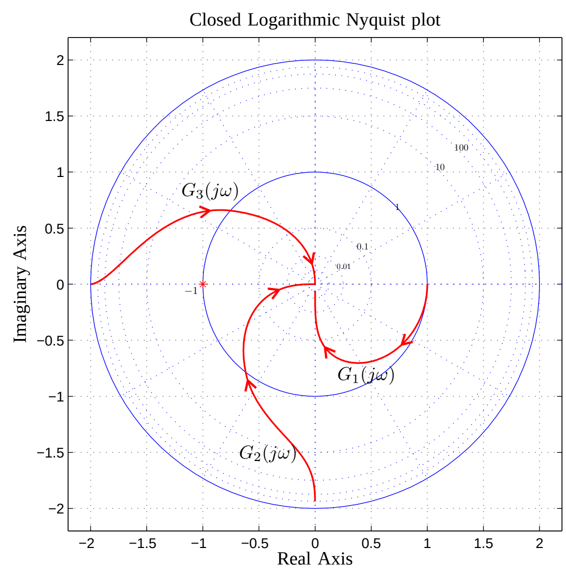

Fig. 3. Closed Logarithmic Nyquist plots of $G_1(s)$, $G_2(s)$ and $G_3(s)$ obtained using function $L_2(M)$.

The main properties of the Closed Logarithmic Nyquist plot are the following:

- The CLN plot is entirely contained within a circle of radius 2.
- The phase $\varphi(\omega) = \arg G(j\omega)$ of the frequency response is preserved as in the ordinary linear polar plot.
- The unit circle is preserved as in the ordinary linear polar plot.
- The points of the plot with modulus $M(\omega) = |G(j\omega)|$ lower than 1 are expanded, i.e. the diagram in the vicinity of the origin is logarithmically expanded.
- The points of the plot having modulus $M(\omega) = |G(j\omega)|$ greater than 1 are contracted, i.e. the diagram

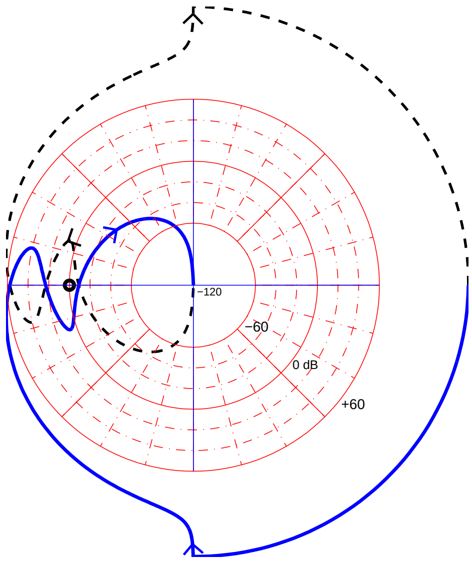

Fig. 4. Log-polar plot of $G_4(s)$ obtained with the MATLAB function `nyqlog` proposed in [3].

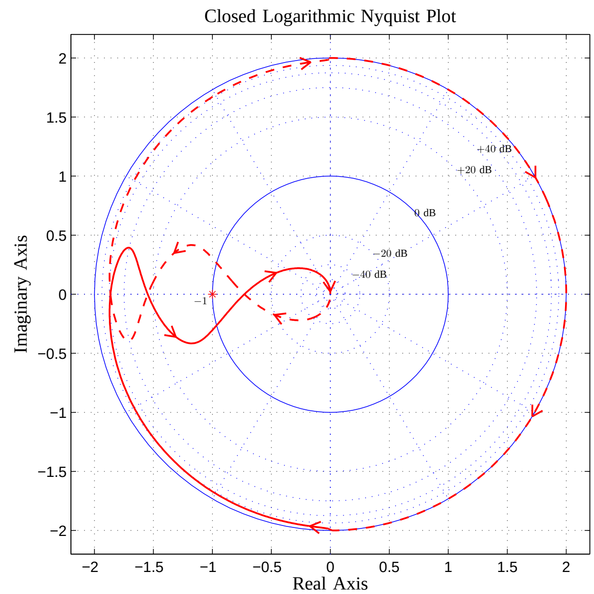

Fig. 5. Closed Logarithmic Nyquist plot of $G_4(s)$.

is logarithmically contracted for amplitudes between 1 and infinity.

- The points of the plot with infinite modulus $M(\omega) = |G(j\omega)| = \infty$ lie onto the circle of radius 2.
- The CLN plots of systems with one simple pole at the origin do not have asymptotes, as opposite to the standard linear Nyquist plot where vertical asymptotes are present for systems with one pole at the origin.
- All the intersections of the CLN diagram with the real axis are shown.
- The gain margin and phase margin of function $G(j\omega)$ can be extracted from the CLN plot.
- The Nyquist stability criterion holds for CLN plots.

In this paper we consider only functions without delays and without poles on the imaginary axis except for poles at the origin. Anyway the CLN plot can easily be extended also to these cases. Consider the following function, given as an example in [3]:

$$G_4(s)=\frac{200(1+3s)(1+2s)}{s(1+50s)(1+10s)(1+0.5s)(1+0.1s)}$$

The "log-polar" plot of $G_4(s)$ obtained with the MATLAB function `nyqlog(sys)` provided by the author of [3] is shown in Fig. 4. The CLN plot of $G_4(s)$ is shown in Fig. 5.

A. Closed Logarithmic Nyquist plots: general case

The Closed Logarithmic Nyquist plot can be obtained also using a base n different from base 2 used in (2). The general case of the function $L_2(M)$ is the following:

$$L_n(M) = \begin{cases} M^{\log_{10} n} & \text{if } \quad M \leq 1 \\ 2 - M^{-\log_{10} n} & \text{if } \quad M > 1 \end{cases} \tag{3}$$

V. Qualitative drawing of the Nyquist curve

The Closed Logarithmic Nyquist plot can also be manually drawn in a qualitative way using the procedure introduced in [5] for the standard Nyquist plot and briefly reported here. Consider a generic transfer function $G(s)$ expressed in one of the following three equivalent forms:

- Polynomial form:

$$G(s) = \frac{b_m\, s^m + b_{m-1}\, s^{m-1} + \ldots + b_1\, s + b_0}{s^h\,(a_n\, s^n + a_{n-1}\, s^{n-1} + \ldots + a_1\, s + a_0)},$$

- Zero-pole-gain form:

$$G(s) = K_1\, \frac{(s-z_1)\,(s-z_2)\,\cdots\,(s-z_m)}{s^h\,(s-p_1)\,(s-p_2)\,\cdots\,(s-p_n)},$$

- Time-constant form:

$$G(s) = K\, \frac{(1+\tau_1' s)\,(1+\tau_2' s)\cdots\,(1+\tau_m' s)}{s^h\,(1+\tau_1 s)\,(1+\tau_2 s)\cdots\,(1+\tau_n s)}.$$

The initial and final points of the polar plot can be obtained by computing the functions $G_0(s)$ and $G_\infty(s)$ that approximate $G(s)$ for $|s| \simeq 0^+$ and $|s| \simeq \infty$, respectively. The approximating function $G_0(s)$ is obtained from $G(s)$ by neglecting all the s terms except for zeros and poles at the origin:

$$G_0(s) = \lim_{|s|\to 0^+} G(s) = \frac{K}{s^h} \tag{4}$$

The function $G_\infty(s)$ is deduced from $G(s)$ by considering in the numerator and denominator polynomials only the monomials of s of higher degree, i.e.

$$G_\infty(s) = \lim_{|s|\to\infty} G(s) = \frac{K_1}{s^r} \tag{5}$$

where $r = h+n-m$ is the relative degree of function $G(s)$.

The qualitative Nyquist plot of a generic transfer function $G(s)$ can be manually drawn using the following procedure, see [5].

1. *Initial point*. The initial point of the diagram when $\omega = 0^+$can be determined by computing magnitude M_0 and phase φ_0 of the approximating function $G_0(j\omega)$ for $\omega \to 0^+$:

$$M_0 = |G_0(j\omega)| = \lim_{\omega\to 0^+} \frac{|K|}{\omega^h} = \begin{cases} \infty & \text{if } h > 0 \\ |K| & \text{if } h = 0 \\ 0 & \text{if } h < 0 \end{cases}$$

$$\varphi_0 = \arg\{G_0(j\omega)\} = \arg(K) - h\frac{\pi}{2}$$

Note that in the proposed CLN plot the points with infinite modulus are represented onto the external circle of radius 2.

2. *Phase shift (lead or lag) for $\omega \simeq 0^+$*. For $\omega \simeq 0^+$ the Nyquist plot starts with a phase shift concordant with the following parameter Δ_τ:

$$\begin{aligned} \Delta_\tau &= \frac{b_1}{b_0} - \frac{a_1}{a_0} = -\sum_{i=1}^{m} \frac{1}{z_i} + \sum_{i=1}^{n} \frac{1}{p_i} \\ &= \sum_{i=1}^{m} \tau_i^{'} - \sum_{i=1}^{n} \tau_i \end{aligned} \tag{6}$$

Parameter $\Delta_\tau > 0$ implies an initial phase lead with respect to φ_0 while $\Delta_\tau < 0$ produces an initial phase lag with respect to φ_0.

3. *Final point*. The final point of the diagram can be determined by computing magnitude M_∞ and phase φ_∞ of the approximating function $G_\infty(j\omega)$ for $\omega \to \infty$:

$$M_\infty = |G_\infty(j\omega)| = \lim_{\omega\to \infty} \frac{|K_1|}{\omega^r} = \begin{cases} 0 & \text{if } r > 0 \\ |K_1| & \text{if } r = 0 \\ \infty & \text{if } r < 0 \end{cases}$$

$$\varphi_\infty = \arg\{G_\infty(j\omega)\} = \arg(K_1) - r\frac{\pi}{2}$$

4. *Phase shift (lead or lag) for $\omega \simeq \infty$*. For $\omega \simeq \infty$ the Nyquist plot ends with a phase shift concordant with the following parameter Δ_p:

$$\begin{aligned} \Delta_p &= \frac{a_{n-1}}{a_n} - \frac{b_{m-1}}{b_m} = -\sum_{i=1}^{m} \frac{1}{\tau_i^{'}} + \sum_{i=1}^{n} \frac{1}{\tau_i} \\ &= \sum_{i=1}^{m} z_i - \sum_{i=1}^{n} p_i \end{aligned} \tag{7}$$

Parameter $\Delta_p > 0$ implies a final phase lead with respect to φ_∞, while $\Delta_p < 0$ produces a final phase lag with respect to φ_∞.

5. *Presence of asymptotes*. For systems with $h = 1$ the linear polar curve exhibits a vertical asymptote while in the proposed CLN plot no asymptotes exist.

6. *Total phase variation*. When ω varies from 0^+ to ∞, the rotation angle of $G(j\omega)$ about the origin can be calculated as follows

$$\Delta\varphi = -\frac{\pi}{2}(n_{p,s} - n_{p,u} - n_{z,s} + n_{z,u}) \tag{8}$$

where $n_{p,s}$, $n_{z,s}$, $n_{p,u}$ and $n_{z,u}$ denote the number of "stable" and "unstable" poles and zeros of $G(s)$ respectively. Note that the computation of $\Delta\varphi$ only requires the knowledge of the amount of stable and unstable poles and zeros but not their numerical value. As a consequence, if poles and zeros are not explicitly given, e.g. in the polynomial form of $G(s)$, it is sufficient to apply the Routh-Hurwitz criterion to both numerator and denominator of $G(s)$ to find $n_{p,s}$, $n_{z,s}$, $n_{p,u}$ and $n_{z,u}$.

7. *Computation of $G(j\omega)$ for some frequency values*. A precise meaningful drawing of the Nyquist plot needs the computation of the intersections of the curve with the real axis. These points are of great importance for many aims: stability analysis of linear systems, stability margins evaluation, stability analysis of systems with static nonlinearities, etc. A direct computation of such intersections using classical methods may require complex calculations[1]. The intersections with the real axis can also be obtained in a simpler way by applying the Routh-Hurwitz criterion to the characteristic equation $1 + kG(s) = 0$ of the feedback system. As a matter of fact, if $k^\star$ denotes a value of gain k which makes the system marginally stable, and therefore nullifies an element of the first column of the Routh table, an intersection of the Nyquist plot with the real axis occurs in $G(j\omega) = -\frac{1}{k^\star}$.

8. *Nyquist plot drawing for ω from 0^+ to ∞*. Once that initial and final segments of the polar curve are known, the qualitative Nyquist plot can be obtained by connecting them with a continuous curve which performs a rotation of an angle $\Delta\varphi$ about the origin and crosses the points computed in the previous step. Obviously, the exact computation of some points of the frequency response contributes to improve the precision of the polar plot, but a qualitative sketch/analysis only based on initial and final directions and on the phase shift $\Delta\varphi$ may be sufficient for many goals, including stability analysis and correct interpretation of the diagrams obtained with numerical software.

9. *Complete Nyquist plot*. The frequency response for negative values of ω is the complex-conjugate of function $G(j|\omega|)$. Therefore, the Nyquist plot for ω ranging from $-\infty$ to 0^- can be deduced from that of $G(j\omega)$ obtained for positive values of ω by reflecting this plot with respect to the real axis. Finally, if the curve starts from infinity for $\omega \simeq 0$, i.e. from the external circle in the CLN plot, the complete CLN plot

[1] Usually the intersection with the real axis are deduced by computing $G(j\omega^\star)$ for those frequencies $\omega^\star$ which satisfy relation $\text{Im}\{G(j\omega^\star)\} = 0$.

is obtained by connecting point $G(j0^-)$ with point $G(j0^+)$ with h clockwise semicircles lying on the external circle of radius 2 (h is the number of poles at the origin).

A. Example

Consider the following function

$$G_5(s) = \frac{20(\frac{1}{10}s + 1)}{s(\frac{1}{3}s + 1)(\frac{1}{2}s + 1)}$$

The approximating functions and the initial and final points are:

$$G_0(s) = \frac{20}{s} \quad \rightarrow \quad \begin{cases} M_0 = \infty \\ \varphi_0 \; = -\frac{\pi}{2} \end{cases}$$

$$G_\infty(s) = \frac{12}{s^2} \quad \rightarrow \quad \begin{cases} M_\infty = 0 \\ \varphi_\infty \; = -\pi \end{cases}$$

The values of the two parameters Δ_τ and Δ_p are

$$\Delta_\tau = \frac{1}{10} - \left(\frac{1}{3} + \frac{1}{2}\right) = -0.73 < 0$$

$$\Delta_p = -10 - (-3 - 2) = -5 < 0$$

therefore the initial point of CLN plot of $G_5(s)$ has a phase lag with respect to $\varphi_0 = -\frac{\pi}{2}$ and the final point has a phase lag with respect to $\varphi_\infty = -\pi$, see Fig. 6. Considering Fig. 6 a straightforward application of the Nyquist stability criterion suggests that the system $G_5(s)$ in a unity feedback configuration is unstable (the complete polar plot encircles the critical point $-1 + j\,0$) while it will be stable only for values of the gain $0 < k < k^\star$ (when the complete polar plot does not encircle the critical point $-1 + j\,0$).

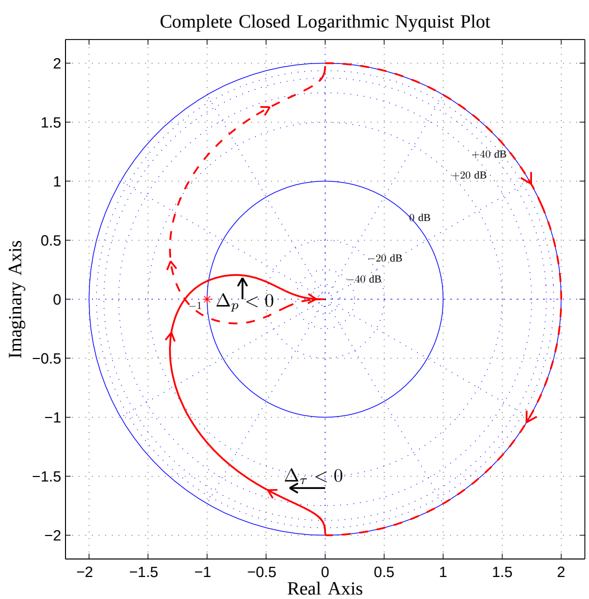

Fig. 6. Complete Closed Logarithmic Nyquist plot of $G_5(s)$.

VI. Lead and Lag Compensators Design

The design of first order lead and lag compensators can be performed by means of graphical methods based on Nyquist plot, see [6], [7]. These methods can also be addressed using the Closed Logarithmic Nyquist plot. The structure of the considered first order compensator is the following:

$$C(s) = \frac{1 + \tau_1 s}{1 + \tau_2 s}. \qquad (9)$$

When $\tau_1 > \tau_2$ function $C(s)$ represents a lead compensator, when $\tau_1 < \tau_2$ it represents a lag compensator. The admissible domains $\mathcal{D}_d$ and $\mathcal{D}_g$ corresponding to a desired phase margin $M_\varphi = 60^o$ on the linear and logarithmic Nyquist planes are shown in Fig.7: point B is $B = e^{j(\pi + M_\varphi)}$, domain $\mathcal{D}_d$ (light red) is the set of all the points that can be moved to point B by using a lead compensator and domain $\mathcal{D}_g$ (light green) is the set of all the points that can be moved to point B by using a lag compensator.

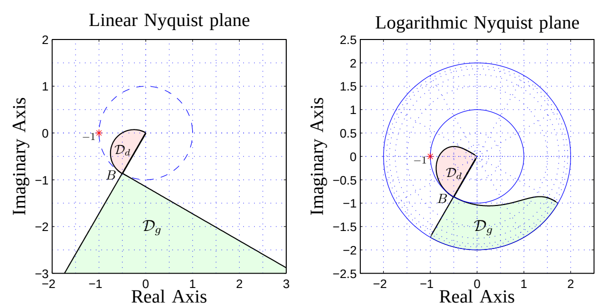

Fig. 7. Admissible domains $\mathcal{D}_d$ and $\mathcal{D}_g$ for lead and lag compensators on the linear and logarithmic Nyquist planes.

Let us consider, for example, the design of a lag compensator $C_g(s)$ able to stabilize system $G_5(s)$ imposing a phase margin $M_\varphi = 60^o$, see Fig. 8. The point A must belong to the admissible domain $\mathcal{D}_g$ and to the frequency response $G_5(j\omega)$. The logarithmic module $\bar{M}_A$ and the phase φ_A of point $A = (-0.56, -1.58)$ on the CLN plot are $\bar{M}_A = 1.67$ and $\varphi_A = 250.4°$. The module M_A of point A on the linear Nyquist plot can be determined using the inverse of function $L_2(M)$ defined in (2):

$$M_A = L_2^{\text{-}1}(\bar{M}_A) = (2 - \bar{M}_A)^{-\log_2 10} = 40.6.$$

From points A and B one obtains the parameters:

$$M = \frac{M_B}{M_A} = 0.0246, \qquad \varphi = \varphi_B - \varphi_A = -10.4°$$

where $M_B = 1$ and $\varphi_B = 240°$ are the module and the phase of point B. Parameters τ_1 and τ_2 of the compensator (9) which moves point A in B are obtained using the following inversion formulae [6]:

$$\tau_1 = \frac{M - \cos\varphi}{\omega \sin\varphi} \qquad \tau_2 = \frac{\cos\varphi - \frac{1}{M}}{\omega \sin\varphi} \qquad (10)$$

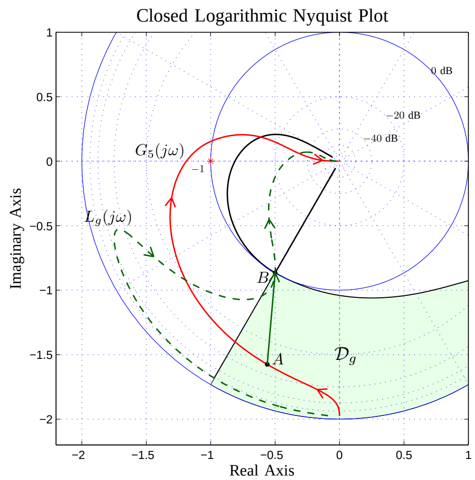

Fig. 8. Design of the lag compensator $C_g(s)$ on the logarithmic Nyquist plane: plots of functions $G_5(s)$ and $L_g(s) = C_g(s)G_5(s)$.

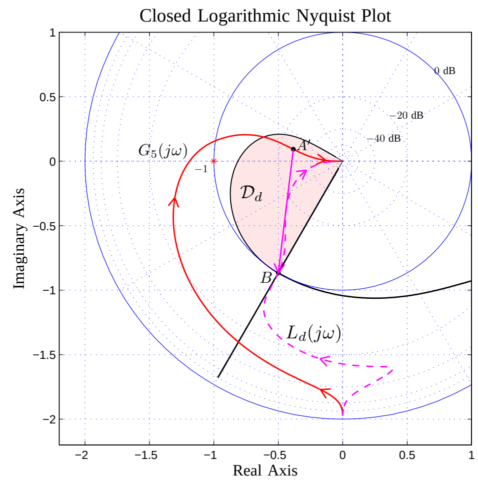

Fig. 9. Design of the lead compensator $C_d(s)$ on the logarithmic Nyquist plane: plots of functions $G_5(s)$ and $L_d(s) = C_d(s)G_5(s)$.

where $\omega = \omega_A = 0.474$ is the frequency of point A: $\tau_1 = 11.2$ and $\tau_2 = 462.9$. The lag compensator is:

$$C_g(s) = \frac{1 + 11.2\,s}{1 + 462.9\,s}.$$

The frequency response of function $L_g(s) = C_g(s)G_5(s)$ is shown in Fig. 8. In a similar way one can design a lead compensator $C_d(s)$ able to impose the phase margin $M_\varphi = 60^o$, see Fig. 9. In this case the point $A' = (-0.383, 0.093)$ is chosen within domain $\mathcal{D}_d$: $\bar{M}_{A'} = 0.394$ and $\varphi_{A'} = 166.4°$. The linear module M_A of point A' can be determined using the inverse of function $L_2(M)$ defined in (2):

$$M_{A'} = L_2^{\text{-}1}(\bar{M}_{A'}) = (\bar{M}_{A'})^{\log_2 10} = 0.0455.$$

From points A' and B one obtains the parameters: $M = M_B/M_{A'} = 21.97$, $\varphi = \varphi_B - \varphi_{A'} = 73.6°$. Substituting M, φ and $\omega = \omega_{A'} = 17.3$ in (10) one obtains the following lead compensator

$$C_d(s) = \frac{1 + 1.3092\,s}{1 + 0.0143\,s}.$$

The frequency response of function $L_d(s) = C_d(s)G_5(s)$ is shown in Fig. 9. The step response of the closed loop systems $C_g(s)G_5(s)$ and $C_d(s)G_5(s)$ are shown in Fig. 10.

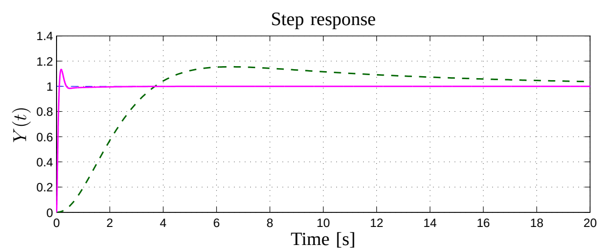

Fig. 10. Step response of closed loop systems $C_g(s)G_5(s)$ (dashed line) and $C_d(s)G_5(s)$ (solid line).

VII. Implementation of MATLAB function

The CLN plot is implemented in Matlab in function `Closed_Logarithmic_Nyquist(sys)` available at `http://www.mathworks.com/matlabcentral/fileexchange/43768`.

VIII. Conclusion

In this paper a new logarithmic Nyquist plot called "Closed Logarithmic Nyquist (CLN) plot" is proposed where the amplitude is in a logarithmic scale and the diagram is entirely contained in a circle of finite radius. Using CLN plot has many advantages: no need of zooming in and out as for linear Nyquist plot, the frequency response curve is entirely shown in a finite circle, the Nyquist stability criterion can be applied, the properties of polar plots (gain and phase margins, intersection points with the real axis, encirclements of the critical point) are maintained, it can be used in the design of first order lead and lag compensator and for many other control purposes. A MATLAB function implementation has been made available for users.

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