Modeling of an internal combustion engine using Power-Oriented Graphs and electrical analogy

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Abstract: In this paper the Power-Oriented Graphs (POG) technique is used for modeling an internal combustion engine through electrical analogy. The aim of the authors, starting from an analogy with electrical systems, is to simplify the approach eliminating the space dynamics, while preserving the time dynamics. In this way, one can obtain an engine description similar to an electric circuit, with all the useful consequences in term of existence and numerical availability of the solution. The advantages are in the specific correspondence found between the engine components and variables with electrical counterparts . The main benefit achievable with this methodology is the simplicity to compose the whole engine model and customize it including the differential equations of the engine in state space form. The POG technique is a graphical modeling technique which uses only two basic blocks for modeling physical systems and the state space mathematical model of a system can be "directly" obtained from the corresponding POG representation.

Keywords: modeling technique; internal combustion engine model; Power-Oriented Graphs; engine dynamics;.

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Glossary

p	gas pressure $[N/m^2]$
T	gas temperature $[K]$
T_{sl}	temperature of lateral surface of cylinder $[K]$
T_{sb}	temperature of basic surface of cylinder $[K]$
$\dot{m}$	air mass flow rate $[kg/s]$
$\dot{Q}_{ext}$	heat flow rate $[kJ/s]$
V	volume $[m^3]$
θ	spark advance $[deg]$
c_x	specific heat capacities $(x = p$ or $v)[J/(kgK)]$
γ	ratio of specific heats , c_p/c_v $[-]$
h_i	enthalpy $[kJ/kg]$
m_{fuel}	fuel mass $[kg]$
η_{cb}	combustion efficiency $[-]$
λ	air/fuel ratio $[-]$
S	heat release $[W/kg]$
$\dot{m}_c$	choked air mass flow rate $[kg/s]$
$\dot{m}_{nc}$	not choked air mass flow rate $[kg/s]$
C_D	discharge coefficient $[-]$
A_T	effective flow area $[m^2]$

1 Introduction

Today's automotive systems face a competitive market that demands excellent performance and limited time of expensive tests. To achieve this, continual improvements are needed on both mechanical components and on system management software, i.e. control strategies.

Regarding the development of new and more efficient control strategies, automotive companies commonly adopt models that can be simulated on a computer to develop new control algorithms in order to reduce the effort and the cost of the testing phase, see Hu et al. (2009), Grubera et al. (2009), Petridis and A.T. Shenton (2003). This phase consists of running the control strategy to verify its performance and limits. To this aim, the models must be reliable and, as far as possible, easily achievable. Moreover models are usually shared among different people, therefore a common modeling language is needed to allow an easier and effective communication.

A modeling technique supported by physical properties together with a schematic representation based on some simple rules, would ease the writing of the models, simplify the formal check of models and allow a common modeling language to share the models. This problem is in common between automotive control systems and many other research fields and many possible solutions have been already proposed: the first one is the Bond Graphs modeling technique, see Gawthrop and Bevan (2007), A.K. Samantaray and S.K. Ghoshal and K. Medjaher and B. Ould Bouamama (2007) and the references therein. This modeling technique uses power interaction between subsystems as the basic concept for modeling. It has also a formal language to represent the basic components that may appear in a broad range of physical systems. However this technique has a few drawbacks that make it not completely suitable to satisfy the requirements mentioned above: the schematic representation needs more than ten symbols and it is not easily readable; the "power" variables are classified in "effort" and "flow" variables (note that this definition does not coincide with the definition of across-variable and through-variable) and finally the implementation of the Bond Graphs on a general purpose computer simulator may require a non trivial 'translation' (causality problem). The modeling technique we propose here is the Power-Oriented Graphs (POG) technique. As for Bond Graphs, the basic idea of the POG modeling technique is to use the power interaction between subsystems as basic concept for modeling, see Zanasi (1991) and Zanasi (2010). The POG schemes are particularly suitable to electro-hydraulic mechanical systems where the power flows through different energetic domains. Differently from the Bond Graphs technique, see Zanasi et al. (2008), the POG technique uses only two basic blocks (see Fig. 1), does not need to classify the power variables and always uses the integral causality. By this way, the POG schemes are easily readable, close to the computer implementation and allow reliable simulations using common computer simulator. Several examples of application to real systems are available. Some of them, related to automotive control systems, are listed below and are validated by comparing the simulation results with experimental measurements. Many examples involving the electrical, hydraulic and mechanical domains about vehicle systems and components can be found: common rail system in Morselli et al. (2002), clutches and gearboxes in Morselli et al. (2006).

In this work, the POG technique is used for modeling an internal combustion engine using the electric analogy, see Palma et al. (2008) and Deur et al. (2006), in order to obtain a mathematical approach useful for control purposes and easy to implement on computer simulators.

In the literature, particular interest is dedicated to engine modeling, both for analysis and control purpose. As an example, in Y. Nilsson and L. Eriksson and M. Gunnarsson (2008) to realize a fuel optimal control, the crucial component is the model for the engine torque. A model for the produced work that captures

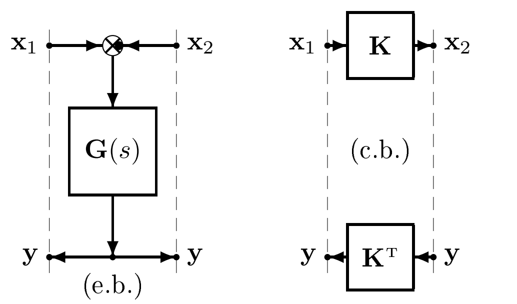

Figure 1 The POG basic blocks: the elaboration block (e.b.) on the left and the connection block (c.b.) on the right.

the important effects of ignition and compression ratio is proposed and investigated. In M. S. Sangha and J. B. Gomm and D. Yu (2008) and G. Lorini and A. Miotti and R. Scattolini (2008) a generic Spark Ignition (SI) Mean Value Engine Model (MVEM) is used for experimentation. In the former, the model is used for the classification of automotive engine air path faults from transient data, in the latter the MVEM is implemented in the synthesis of two control schemes with predictive capabilities.

In Garcia-Nieto et al. (2008), Kwiatkowski et al. (2009) and Huajin et al. (2009) different engine models aimed to control designs are presented. The main features of these models are the simplicity (typically linear models are adopted) and the accuracy necessary to reach the control goals. In internal combustion engines, the cyclic variation of in-cylinder pressure profile is one of the important factors that affect torque generation and fuel efficiency Li et al. (2010).

The focus of this paper is to formalize the analogy of the internal combustion engine with electrical systems, see Spring et al. (2007) and Grossi et al. (2009), realize the corresponding POG scheme and obtain from it the differential equations of the engine dynamics in the state space form. The authors, starting from an analogy with electrical systems, simplify the approach eliminating the space dynamics (multi-zone combustion and wave effects), while preserving the time dynamics (resonance and tuning phenomena). In this way they can obtain an engine description similar to an electrical (although not linear) circuit, with all the useful consequences in term of existence and numerical availability of the solution. The advantages are in the specific correspondence that is found between the engine components and variables (as throttle valve, cylinder, inertial flows), with electrical counterparts (current, voltage, resistance).

The main benefits achievable with this methodology is the simplicity to compose the whole engine model and customize it including all the latest devices. So it is possible to easily implement both a baseline engine and a high complex automotive system. The POG model can be directly implemented on a common computer simulator. Moreover, it allows to obtain a new engine state space mathematical model for control application.

The paper is organized as follows. Section 2 states the basic properties of the POG modeling technique and Section 3 shows the engine model features. An internal combustion engine model using POG is described in Section 4. Finally, simulation results are shown in Section 5 and compared with experimental data. Some conclusions end the paper.

2 The bases of Power-Oriented Graphs

The "Power-Oriented Graphs" (POG) technique is a graphical modeling technique that uses the "power interaction" between subsystems as basic element for modeling physical systems. The POG technique has a "modular" structure which essentially uses only the two blocks shown in Fig. 1 named "elaboration block" (e.b.) and "connection block" (c.b.). The basic characteristic of this modular structure is the direct correspondence between pairs of system variables and real power flows: the product of the two variables involved in each dashed line of the graph has the physical meaning of "power flowing through the section". The circle present in the e.b. is a summation element and the black spot represents a minus sign that multiplies the entering variable. There is no restriction on variables $\mathbf{x}$ and $\mathbf{y}$ other than the fact that their inner product $\langle \mathbf{x}, \mathbf{y} \rangle = \mathbf{x}^{\mathrm{T}}\mathbf{y}$ must have the physical meaning of a "power". The e.b. and the c.b. are suitable for representing both scalar and vectorial systems. In the vectorial case, $\mathbf{G}(s)$ and $\mathbf{K}$ are matrices: $\mathbf{G}(s)$ is always a square matrix composed by positive real transfer functions; matrix $\mathbf{K}$ can also be rectangular. There is a direct correspondence between the POG schemes and the corresponding state space dynamic equations. For example, the system

$$\begin{cases} \mathbf{L}\,\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \\ \mathbf{y} = \mathbf{B}^{\mathrm{T}}\mathbf{x} \end{cases} \qquad \mathbf{L} = \mathbf{L}^{\mathrm{T}} > 0 \tag{1}$$

can be represented by the POG scheme shown in Fig. 2. Note that every physical system respecting causality constraints can be written in the form (1). More details

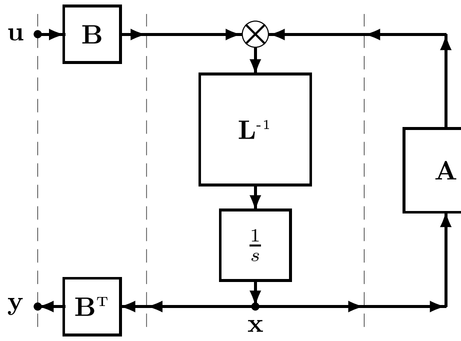

Figure 2 POG block scheme of a generic dynamic system.

on Power-Oriented Graphs are reported in Zanasi (1991), Morselli and Zanasi (2006) and Zanasi (2010).

3 Engine Model Description

It is widely known in the literature that there is an analogy (see Maxwell and Firestone analogies in Firestone (1933)) between dynamics of a simple linear mechanical system and that of a simple lumped-parameter linear electric circuit. The Bond Graphs and POG modeling techniques use different definitions for the power variables, classifying them into effort/flow (Bond Graphs) and through/across (POG). This definition however leads to different analogies among the different energetic domains. In the frame of POG we refer to Firestone analogy as it better preserves topology aspects. The dynamics of actual mechanical and mechatronics systems such

Table 1 Variables

element	q	i	v
electric	charge / flow	current	voltage
	q $[C]$ / ϕ $[Vs]$	i $[A]$	v $[V]$
hydraulic	volume / hydr. flow	flow rate	pressure
	V $[m^3]$ / ϕ_h $[Ns/m^2]$	$\dot{m}$ $[kg/s]$	p $[N/m^2]$
mechanic	angle / momentum	torque	speed
rotational	θ $[rad]$ / P $[kg\,rad/s]$	T $[Nm]$	ω $[rad/s]$

Table 2 Parameters

element	parameters		
electric	resistance	inductance	capacity
	$R = \frac{v}{i}$	$L = \frac{v}{\frac{di}{dt}}$	$C = \frac{i}{\frac{dv}{dt}}$
hydraulic	pneumatic resistance	hydraulic inductance	pneumatic capacity
	$R = \frac{p}{\dot{m}}$	$L = \frac{p}{\frac{d\dot{m}}{dt}}$	$C = \frac{\dot{m}}{\frac{dp}{dt}}$
mechanic	friction	elasticity	inertia
rotational	$B = \frac{T}{\omega}$	$\frac{1}{K} = \frac{\omega}{\frac{dT}{dt}}$	$J = \frac{T}{\frac{d\omega}{dt}}$

as an internal combustion engine or its components have never been analyzed intensively based on their corresponding electro-mechanical. Indeed, there are a lot of papers that have discussed directly any possibility of expressing Lagrange's equation of motion for nonlinear mechanical systems via lumped-parameter circuits like electric circuits, see Arimoto and Nakayama (1996).

A physical system can be decomposed in elementary subsystems or elements. For each elementary subsystem it is possible to define three variables named:

- q quantity, i.e. energy variable of the element;
- i through-variable, equal to dq/dt, i.e. the power variable that flows through the element;
- v variable, i.e. the power variable acting at the extremes of the element.

As an example, Table 1 reports these variables for the electric, hydraulic and mechanical elements. The main feature is that each variable can be considered constant despite the others or, alternatively, the following ratios can be considered constant

$$\frac{v}{i}, \frac{v}{\frac{di}{dt}}, \frac{i}{\frac{dv}{dt}} \tag{2}$$

as shown in Table 2. Moreover, it is possible to demonstrate that, for a network of elements and in particular conditions, the variables are related by Kirchhoff laws, as follows:

- for each node of the network, the algebraic sum of the through-variables is zero.
- for each mesh of the network, the algebraic sum of the across-variables is zero.

Now, considering the engine formed by mechanical components as throttle valve, manifolds, cylinders and crank shaft, crossed by a gas, the approach proposed in this work is based on the analogy among the electrical, simpler to model, and the mechanical and hydraulic elements, see Palma et al. (2008).

Regarding the mechanical systems, an analogy can be found among speed and torque with electric voltage and current respectively, it results in considering the inverse of the mechanical friction B as an electric resistance R and, similarly, the inertia J as a capacitor C and the inverse of the stiffness K as an electric inductance L. The same considerations can be done for the hydraulic elements. Here the analogies are between the gas flow rate $\dot{m}$ with the current i, the pressure p with the voltage v and the volume V with the charge q. Then, the electrical resistance corresponds to the pneumatic resistance, that is the resistance of gas flowing across an orifice, and the relationship between volume and pressure to an electric capacitor, see De Rinaldis and Scherpen (2005).

In this scenario, all the parts composing the engine can find an equivalent electrical circuit or element as detailed described in the following sections. Moreover, in order to exactly describe the operation of electrical circuit, it is necessary to use the Maxwell equations. These can capture both the dynamics of the electrical quantities, such as currents and voltages, and the related electromagnetical phenomena, as transmission and radiation. Fortunately, if the size of the circuit is small compared to the wavelength of the electrical variables (i.e. the ratio between the light speed and the frequency of the pulsating events), these electromagnetical phenomena can be neglected. As a consequence, the partial differential relationships of the Maxwell equations can be simplified to the widely used electrical engineering equations, that are the Kirchhoff laws and the current/voltage relationships of circuit components. Similarly, the same approach can be extended to the internal combustion engine, providing that the wavelength (in this case the ratio between the sound speed and the frequency of its pulsating events) is large enough compared to the length size of the engine. As an example, a four cylinder four stroke engine, running at 3000 rpm, generates intake pulses at 100 Hz, resulting in a wavelength of 3.4 m. Considering the engine size of approximately 1 m, the lumped parameter approach seems to be reasonable, see Palma et al. (2008) and Spring et al. (2007).

The engine is seen as an array of cylinders, having common connections with an intake and an exhaust manifold. The connections are regulated by valves opening. According to the previous section, it is possible to distinguish separate subsystems interconnected each others, such as the intake manifold equipped with throttle valve, the exhaust manifold and cylinders. From the phenomenological point of view, the elements composing the engine can be classified in the following categories: volumes, orifices, inertial effects and combustion. In the following, each category is introduced and the relationships involving the variables of interest

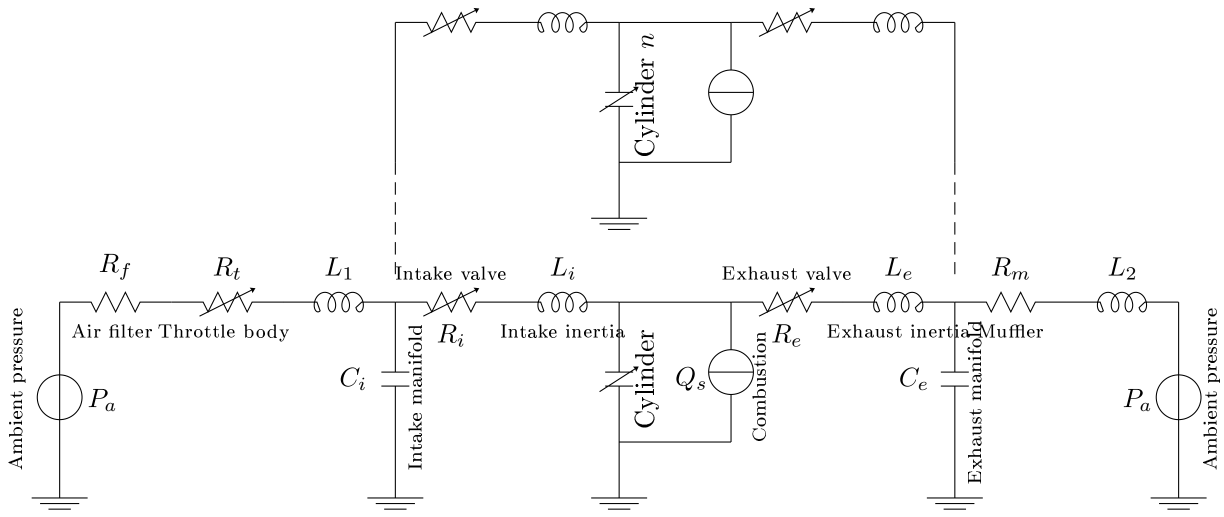

Figure 3 Internal combustion engine equivalent circuit.

are reported. Moreover a brief explanation of the POG section is given, according to Section 4.

3.1 Volumes

Here are grouped the intake and exhaust manifold and cylinders, respectively as constant and variable volumes. The electric counterpart is the quantity of charge stored in a capacitor. Applying the corresponding current/voltage relationship and considering the analogies with pressure and temperature inside the volume, it is possible to obtain the classical equations. Starting from ideal gas equation

$$pV = mRT \tag{3}$$

where R is the specific gas constant and m is the mass of the gas, it is possible to obtain the following relations:

$$\dot{p} = \frac{R\gamma}{V}\left[\sum_i \dot{m}_i T_i - T\sum_j \dot{m}_j + \frac{\gamma-1}{R\gamma}\dot{Q}_{ext} - \frac{p\dot{V}}{R}\right] \tag{4}$$

$$\begin{aligned}\dot{T} = \tfrac{R\gamma T}{pV}\Big[&\textstyle\sum_i \dot{m}_i T_i(1-\tfrac{T}{\gamma T_i}) - T\sum_j \dot{m}_j(1-\tfrac{1}{\gamma T_j})+\\ &+\tfrac{\gamma-1}{R\gamma}\dot{Q}_{ext} - \tfrac{p\dot{V}}{R}(1-\tfrac{1}{\gamma T})\Big]\end{aligned} \tag{5}$$

where i represents the entering mass flow and j the outgoing mass flow. For sake of brevity, the details on how to obtain equations (4) and (5) are omitted.

It is remarked that, regarding the intake and exhaust manifolds, since the volume V is constant, the derivative terms in the equation disappear. These elements have their corresponding POG elaboration blocks between power sections 4 and 5 for the intake manifold and between sections 16 and 17 for the exhaust manifold.

3.2 Orifices

The orifices are responsible for the pressure drops along the gas path. They are modeled as variable resistances causing equivalent voltage drops. The size of the orifice is variable and regulated by valve opening, as throttle valve, air bypass, intake and exhaust valves.

The electrical resistance is governed by a static relationship between voltage and current, corresponding to a static relationship between the analogue variables, i.e. pressure and flow rate, according to the well known equations in Hendricks (1989), Colin et al. (2009) and Setlur et al. (2005)

$$\begin{cases} \dot{m}_c = \frac{C_D A_T p_0}{\sqrt{RT}}\gamma_{\frac{1}{2}}(\frac{2}{\gamma+1})^{\frac{\gamma+1}{2(\gamma-1)}} & \frac{p_T}{p_0} > 1 \\ \dot{m}_{nc} = \frac{C_D A_T p_0}{\sqrt{RT}}(\frac{p_T}{p_0})^{\frac{1}{\gamma}}\{\frac{2\gamma}{\gamma-1}[1-(\frac{p_T}{p_0})^{\frac{\gamma-1}{\gamma}}]\} & \frac{p_T}{p_0} < 1 \end{cases} \tag{6}$$

where p_T and p_0 are respectively the pressure upstream and downstream the orifice. In the POG scheme orifices are represented with static elaboration blocks named R_f, R_t, $\mathbf{R}_i$, $\mathbf{R}_e$ and R_m (see Section 4 for symbols explanation).

3.3 Inertial effects

The inertial phenomena can be considered as minor efforts but not completely negligible. They describe the reduction or the increase of the pressure upstream the valve of a quantity proportional to the derivative of the mass flow through the same valve. Here, they are modeled as a linear inductance, regulated by a differential relationship between voltage and current, corresponding to the following equation

$$p_{corr} = p - k\ddot{m} \tag{7}$$

where p_{corr} is the manifold pressure k is a parameter to be set and $\ddot{m}$ is the time derivative of the air mass flow. It is remarked that this kind of relationship is not present in literature. In order to justify the adopted choice, both the analogy with the electrical circuit and the simulation results illustrated later on the paper can be adduced. The POG dynamic blocks representing inertial effects are placed between power sections ③ and ④, sections ⑦ and ⑧ (intake inertia), sections ⑭ and ⑮ (exhaust inertia) and between sections ⑱ and ⑲.

3.4 Combustion description

The combustion process constitutes the most meaningful and complex phenomenon occurring into the engine. In order to model the in-cylinder cycle pressure, an equivalent electric circuit has been adopted. The circuit is formed by a variable capacitor, representing the cylinder volume, equipped by an impulsive current generator. In intake condition, the piston downstroke causes a pressure drop through the valve (it generates a voltage difference at the capacitor extremities and, consequently, generates a current flow through the capacitor). During the combustion phase, when the intake and exhaust valves are closed, the current generator develops an impulsive current flow, simulating the pressure increase in combustion chamber during this phase, see Powell (1979). This phenomenon corresponds to the well known combustion process, i.e. an impulsive increase of the in-cylinder pressure caused by the combustion results in a torque generation and in mass flow through the exhaust valves. The equation regarding this process is described by the following relationship

$$\begin{aligned}\dot{Q}_e &= h_1 A_l(\theta)(T_{sl} - T) + h_2 A_b(T_{sb} - T) + \\ &+ m_{fuel}\,\eta_{cb}\,\eta_{burn}(\lambda, p) S(\theta, p) Q_{HV}\end{aligned} \tag{8}$$

where h_1 and h_2 are parameters to be set, A_l and A_b the lateral and base area respectively and Q_{HV} is the fuel lower heating value (expressed in J/kg), see Heywood (1988). It is remarked that equation (8) represents the heat power generated by the combustion affecting the pressure (4) and temperature (5). The cyclic variation has been implemented as a function of the operating conditions (η_{burn} in (8)) of engine and, partially, equipped with a random behavior needed to represent the combustion cycle-by-cycle and cylinder-by-cylinder irregularity. In the POG scheme the dynamic elaboration block modeling the array of cylinders and combustion is given between sections ⑫ and ⑬.

3.5 Equivalent circuit

Based on the analogies depicted in the previous paragraph, the whole engine can be represented by the circuit shown in Fig. 3.

The model starts describing the dynamic of the air crossing the intake manifold, i.e. driven by the ambient pressure (a voltage generator), the air mass passes the filter (a resistance) and the throttle body (a variable resistance) and arrives into the cylinder through the intake valves (a new variable resistance).

The cylinders are described by a parallel of “n” combustion equivalent circuits (variable capacitors for the cylinders and an impulsive current generator for the spark plug), with “n” the number of cylinders composing the engine. Finally, the gas mixture is discharged into the exhaust manifold through the exhaust valves (a variable resistance) and ends into the ambient crossing the muffler (a resistance). For sake of completeness, it is possible to introduce the inductors representing the inertial effects of the current as it is explained in Section 3.3 that are able to describe the analogue effects of the fluid columns. The validity of the modeling choices adopted in this work has been tested with experimental data of Fiat engine 1.8 liters with 8 valves and with a Variable Valve Timing (VVT). The engine has been coupled to an eddy current dynamometer with low inertia 250 kW AC and it has been tested at Elasis Research Center. An AVL PUMA Open automation software has been used to measure engine variables of interest during different engine operation modes. The authors use this automation package system for this test bed regarding it capability to run real-time models from MATLAB-Simulink.

4 POG model of the internal combustion engine

The POG scheme of the combustion engine is reported in Fig. 4. The meaning of the model parameters is the following:

R_f	:	filter resistance;
R_t	:	throttle variable resistance;
L_1, L_2	:	inductors representing inertial effects;
C_i	:	intake manifold capacity;
$\mathbf{R}_i$	:	intake valve variable resistance;
$\mathbf{L}_i$	:	intake valve inertia;
$\mathbf{C}_c$	:	cylinder capacity;
$\mathbf{R}_e$	:	exhaust valve variable resistance;
$\mathbf{L}_e$	:	exhaust valve inertia;
C_e	:	exhaust manifold capacity;
R_m	:	muffler resistance;
J_m	:	motor inertia;
A	:	cylinder area.

Note that upright type denotes scalar parameters, while bold type denotes matrices. The scheme blocks between power sections ① to ⑧ and ⑫ to ⑲ are in the same order as the components appear in the equivalent electric circuit of Fig.3, according to the direction of the power flow. The power flows into this scheme according to the following rule: for each section of the POG scheme the power is entering (outgoing) if the path from the input to the output has an even (odd) number of ‘minus’ signs. Blocks between power sections ⑨ and ⑫ represent the mechanical part that is in parallel with the hydraulic

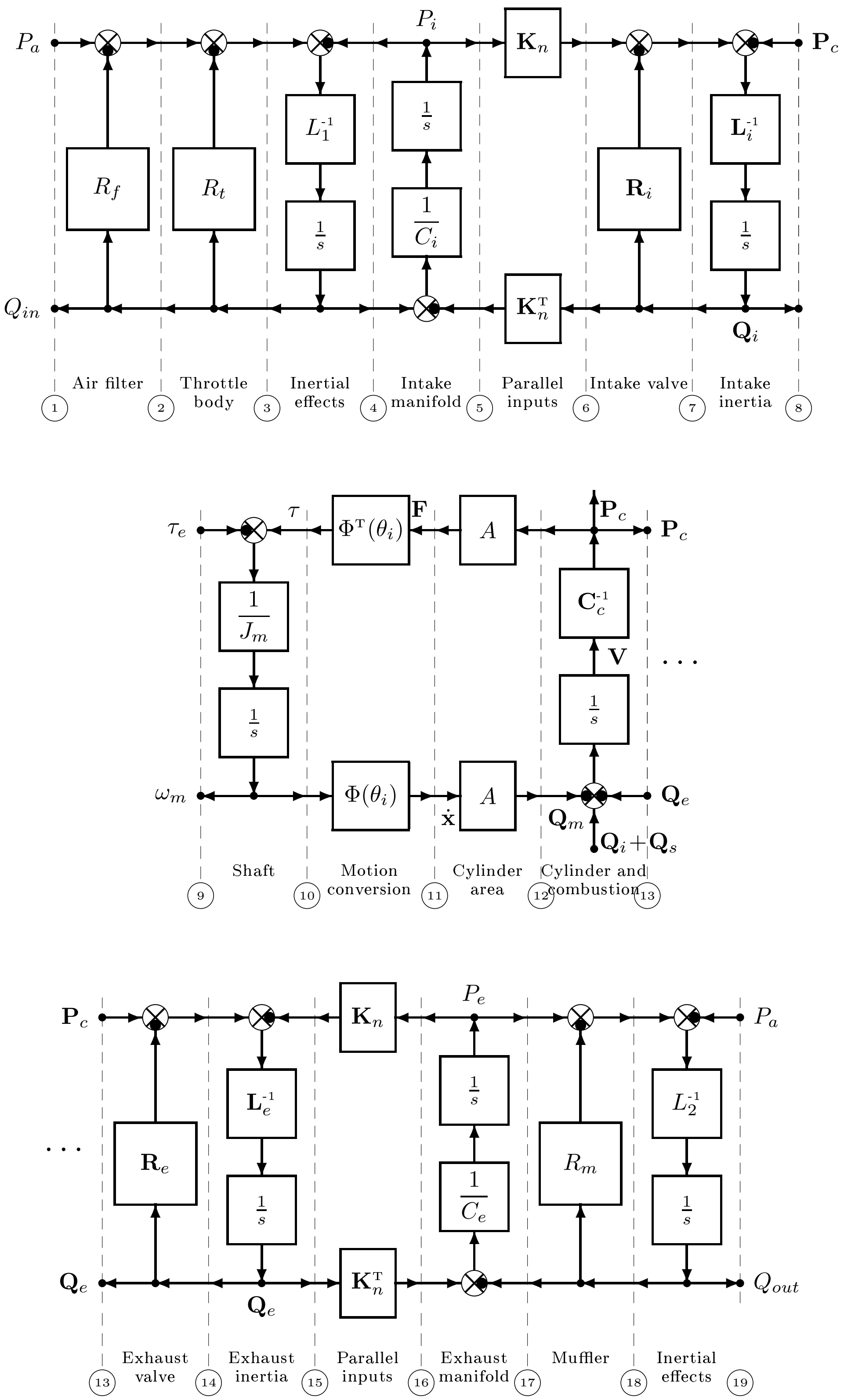

Figure 4 POG scheme of the combustion engine.

part between sections ① and ⑧. This part is not considered in the equivalent electric circuit of Fig. 3, but here it is necessary in order to give the exact energetic meaning to the model. The model has eight dynamic elements. The dynamic elements such as inductances integrate the across-variable and give the through-variable, while dynamic elements such as capacitors and inertias integrate the through-variable and give the across-variable. The state vector components can be chosen as the power variables in output from the dynamic elements taken in the order as they appear in the POG scheme:

$$\mathbf{x} = \left[Q_{in}\; P_i\; \mathbf{Q}_i\; \mathbf{P}_c\; \mathbf{Q}_e\; P_e\; Q_{out}\; \omega_m \right]^{\mathrm{T}},$$

where Q_{in} is the mass flow in the air filter and throttle body, P_i is the pressure given by the intake manifold, $\mathbf{Q}_i$ is the vector of mass flows in the intake valve entering the cylinder, $\mathbf{P}_c$ is the cylinder pressure, $\mathbf{Q}_e$ is the vector of mass flows in the exhaust valve outgoing the cylinder, P_e is the pressure given by the exhaust manifold, Q_{out} is the mass flow passing through the muffler and ω_m is the velocity of the motor shaft. The input vector is:

$$\mathbf{u} = \left[P_a\; \tau_e \right]^{\mathrm{T}},$$

where P_a is the ambient pressure and τ_e is the external torque applied to the motor shaft. The connection matrix $\mathbf{K}_n$ is a column unitary vector of dimension n:

$$\mathbf{K}_n = \left[1\,1\,1 \ldots 1 \right]^{\mathrm{T}}$$

where n is the number of cylinders. This matrix allows to pass from a scalar to a vectorial system (and viceversa) that is the parallel of n cylinders. The connection blocks between power sections ⑩ and ⑫ allow to pass from the hydraulic to the mechanical domain and viceversa. Function $\Phi(\theta_i)$ is defined as

$$\Phi(\theta_i) = \left[\varphi(\theta_1)\, \varphi(\theta_2) \ldots \varphi(\theta_n) \right]^{\mathrm{T}}$$

where θ_i is the angle between the piston rod and the motion direction of each cylinder, $\varphi(\theta)$ converts the rotational motion of the shaft into the linear motion of the piston (and viceversa) and it is defined as:

$$\varphi(\theta) = -l_1 \sin(\theta) \left(1 + \frac{\cos(\theta)}{l_2 \sqrt{1 - \left(\frac{l_1}{l_2} \sin(\theta) \right)^2}} \right),$$

where l_1 and l_2 are the lengths of the piston rod and the crank respectively.

The differential equations of the motor can be written in the state space form as:

$$\mathbf{L}\,\dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} + \mathbf{B}\,\mathbf{u}$$

and the system matrices can be obtained straightforward from the POG scheme. Matrix $\mathbf{L}$, named *energy matrix*, is always symmetric and positive definite. In this case it can be obtained putting on its diagonal the coefficients of the dynamic elements of the system, it is to say all inductors, capacitors and inertias taken in the order as they appear in the POG scheme from left to right (as the mechanical part is in parallel, the inertia can be put in the last position in matrix $\mathbf{L}$). The input matrix $\mathbf{B}$ can be obtained from the scheme following the paths connecting the three inputs to each component of vector $\dot{\mathbf{x}}$. The system matrices are given by:

$$\mathbf{L} = \begin{bmatrix} L_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & C_i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{L}_i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{C}_c & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbf{L}_e & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & C_e & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & L_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & J_r \end{bmatrix}, \; \mathbf{B} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -1 & 0 \\ 0 & -1 \end{bmatrix} \tag{9}$$

and

$$\mathbf{A} = \begin{bmatrix} -(R_f+R_t) & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -\mathbf{K}_n^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{K}_n & -R_i & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & -\varphi(\theta)A \\ 0 & 0 & 0 & 1 & -R_e & -\mathbf{K}_n & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbf{K}_n^{\mathrm{T}} & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -R_m & 0 \\ 0 & 0 & 0 & \varphi(\theta)A & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{10}$$

The system matrix $\mathbf{A}$ can always be represented as the sum of a symmetric part $\mathbf{A}_s = \frac{A+A^{\mathrm{T}}}{2}$ and a skew-symmetric part $\mathbf{A}_w = \frac{A-A^{\mathrm{T}}}{2}$. The symmetric part $\mathbf{A}_s$ is a function of the static parameters of the system and it represents the system dissipations. The dissipating power of the system is given by a quadratic form function of matrix $\mathbf{A}_s$ and state vector $\mathbf{x}$. The skew-symmetric part $\mathbf{A}_w$ is a function of the connecting parameters between the system elements and it represents energy redistribution within the system, so it is not responsible for neither storing nor dissipating energy and the quadratic form based on this matrix is always zero.

The POG modeling technique has been chosen in this work in order to shorten the simulation time and therefore exploit the proposed model for control purposes. In Palma et al. (2008) the same engine was modeled with a lumped parameter technique – in-cylinder engine model – but even if that model was more accurate and guaranteed the simulation of mass flows and fuel flows of each cylinder, the simulation time was about 4 – 6 minutes for each engine point on a Intel Core 2 vpro 2, 26 Ghz processor. The model implemented in this paper has a simulation time of every engine stroke of less than 5 s.

5 Simulation results

The POG scheme given in Fig. 4 can be easily implemented in Simulink with no “translation” effort, as it needs only a few kind of functional blocks, such as *Gain*, *Integrator* and *Summator*. This is an important

Table 3 Parameters values

$$
\begin{aligned}
R_f + R_t &= 280\,[\Omega] \\
L_1, L_2 &= 0.1, 0.01\,[H] \\
\mathbf{R}_i &= 1\,[\Omega] \\
\mathbf{L}_i &= 0.01\,[H] \\
\mathbf{C}_c &= 450\,[cm^3/Pa] \\
\mathbf{R}_e &= 0.013\,[\Omega] \\
\mathbf{L}_e &= 0.01\,[H] \\
C_e &= 1.2\,[m^3/Pa] \\
R_m &= 9.9\,[\Omega] \\
J_m &= 1.2987\,[kg\,m^2] \\
A &= 0.0161\,[m^2]
\end{aligned}
$$

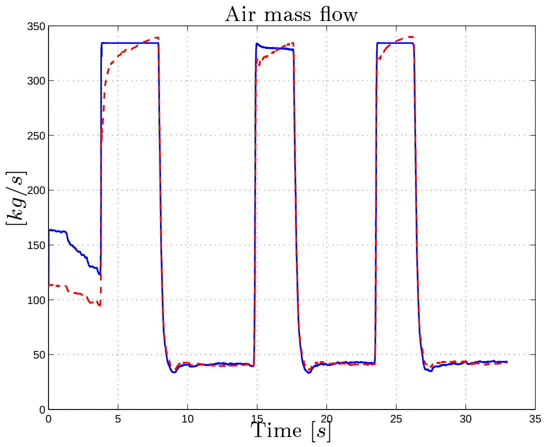

Figure 5 Mass flow in the air filter and throttle body (experimental data in red dashed, simulation result in blue solid line).

advantage of the POG approach. The parameters of the model have been identified (see Table 3 for the values of the main parameters of the model). In order to identify the model parameters, an automatic calibration tool has been exploited that, with the help of a multi-map optimizer, is able to calibrate all parameters simultaneously, finding an optimal solution in terms of mean square percentage estimation error, see Cook and Powell (1998) and Palladino et al. (2008). In the simulations the function $\Phi(\theta)$ has been substituted according with the following formula

$$P_{av} = \frac{2\pi\, n_R\, \tau_{av}}{C_c}$$

that gives the relation between the mean effective pressure P_{av} and the mean effective torque τ_{av}, where n_R is the number of crank revolutions for each power stroke per cylinder, see Heywood (1988) and Hrovat and Sun (1997). Figures 5 to 9 show experimental data in red dashed line and simulation results in blue solid line and the percentage errors (they are all related to the same experiment). Firstly, in Fig. 5 the performances of the proposed model are evaluated comparing the simulated air mass flow in the air filter and throttle valve with the measured signals during selected experiments.

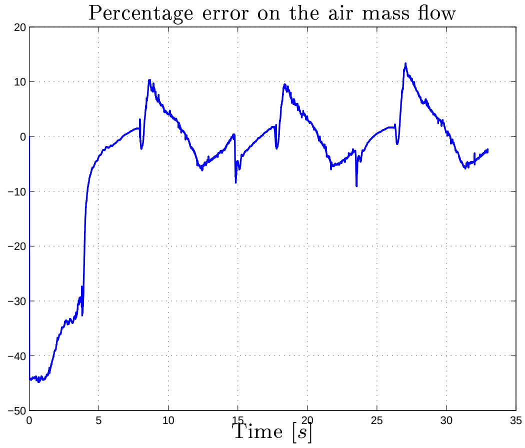

Figure 6 Percentage error on the air mass flow.

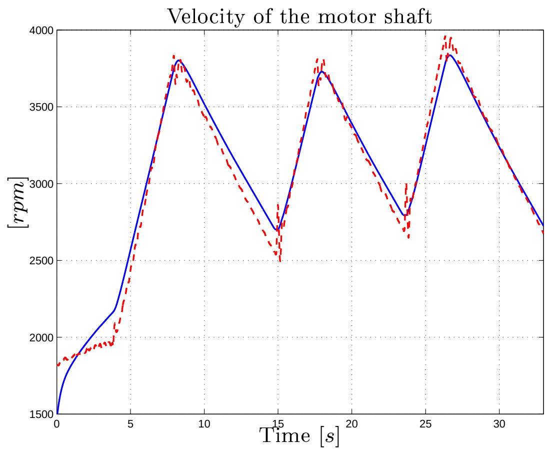

Figure 7 Velocity of the motor shaft (experimental data in red dashed line, simulation result in blue solid line).

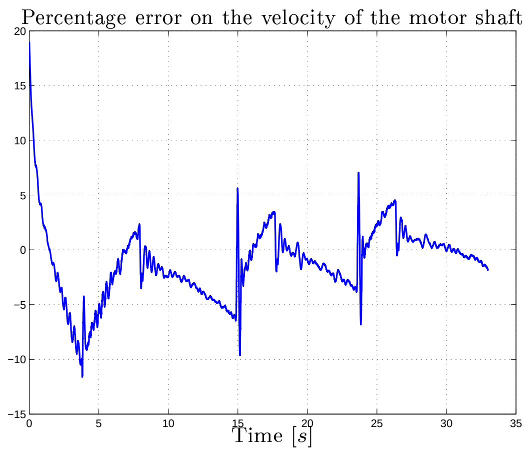

Figure 8 Percentage error on the velocity of the motor shaft.

It is important to note that the experiments used for the validation phase are different from those used for the calibration of model parameters. The experimental data are collected by a 1.4 liter Fiat engine, equipped with VVT system and with a Turbo Compressor (TC). The experiments are designed to evaluate the behavior

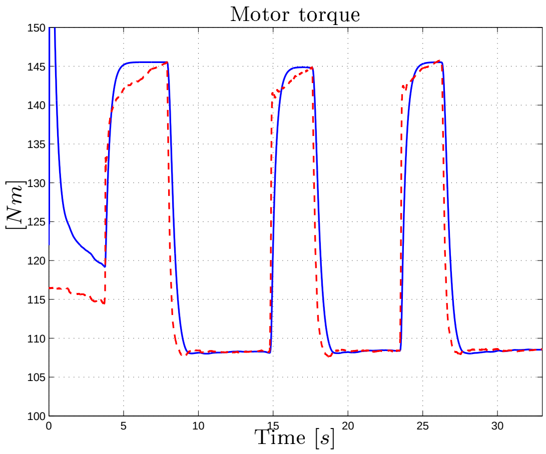

Figure 9 Motor torque (experimental data in red dashed line, simulation result in blue solid line).

of the model during tip-in and tip-out maneuvers, particularly critical to test and so difficult to obtain an accurate parameters identification. The tip-in and tip-out maneuvers are an accelerator pedal maneuvers referred to the action of a driver pressing/releasing the pedal from a depressed position to a zero or near zero position and viceversa. The maneuvers have been realized in clutch down, so in low load condition. However, Fig. 7 and Fig. 9 highlight again the good performance during the experiments conducted with abrupt step on the throttle valve, at first closed and then completely opened (Widely Open Throttle, WOT), other details are omitted for confidential reason.

Even if the proposed model cannot catch all the system dynamics, due to the various simplifying hypotheses, it is able to reproduce the main dynamics of the system. Moreover simulations are very fast with respect to simulations based on the mean value model and this is an important improvement for control purposes.

6 Conclusion and future work

The paper has described the Power-Oriented Graphs (POG) technique for modeling Spark-Ignition engine based on the equivalence with electric circuit. Previous mean value models were more detailed and accurate, but very complex from both mathematical and computational point of view and not control-oriented. The approach proposed here exhibits some advantages in comparison with other graphical techniques: even if it is based on some simplifying hypotheses, it allows to write the control equations, to realize very compact schemes which can be easily translated into Simulink models and the computational cost of simulations is very low. The POG model has been tested by comparing simulation results with experimental data, showing a good level of reliability and accuracy. Thanks to its properties, the POG modeling technique is a good alternative to model automotive systems and to define the differential equations of the engine in the state space form in simple tasks. As future work, the aim of the authors is to use the POG engine model, here presented, in the synthesis of control schemes with predictive capabilities, in order to test fuel consumption control strategies on the spark ignition engine.

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