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Field Oriented Control of a Multi-Phase Asynchronous Motor with Harmonic Injection

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Abstract: The paper presents a new complex dynamic modeling of multi-phase asynchronous motors and investigates the Field-Oriented control applied to this type of machines. The new dynamic model is obtained in the state space using a complex rectangular transformation. The obtained model is complex, reduced-order and takes into account also the odd harmonic injection of the motor. The Field-Oriented control is considered and discussed in the multi-phase general case and then implemented in Matlab/Simulink considering a specific numeric case. The simulation results validate the obtained complex model and the implemented control design.

Keywords: Multi-phase asynchronous motors, Harmonic injection, Field Oriented control.

1. INTRODUCTION

The interest in multi-phase asynchronous machines has considerably increased during last years, see Jones and Levi (2002) and Levi et al. (2007), especially in traction and propulsion research fields where high-power performances are needed. Moreover, the advantages of the odd harmonic order injection is well known in literature for its providing an higher torque density, see Toliyat et al. (1991) and Duran et al. (2008). Different Field Oriented control strategies have been implemented and discussed in the specific case of 5-phases machines, see for instance Duran et al. (2008) and Xu et al. (2001).

This paper presents a new complex and general dynamic model of a multi-phase asynchronous motor with an arbitrary number of phases and the odd order harmonic injection, and then it extends the Indirect Rotor Field-Oriented (IRFO) control to the multi-phase general case. The dynamic equations of the system have been obtained using a "complex and congruent" state space transformation and graphically represented using the Power-Oriented Graphs modeling technique. The paper is organized as follows: Section 2 briefly introduces the basic properties of the POG technique in the complex case. Section 3 presents and describes the new complex reduced dynamic equations of the system and the corresponding model. Section 4 reports the IRFO control equations in the general multi-phase case. Last Section 5 shows some simulation results.

2. POWER-ORIENTED GRAPHS

The Power-Oriented Graphs, see Zanasi (1991) and Zanasi (2010), is a graphical modeling technique suitable for modeling physical systems. The POG are normal block diagrams combined with a particular modular structure essentially based on the use of the two blocks shown in

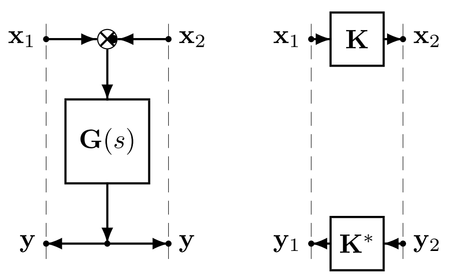

Fig. 1. POG: a) *elaboration block*; b) *connection block*.

Fig. 1: the *elaboration block* stores and/or dissipates energy (i.e. springs, masses, dampers, capacities, inductances, resistances, etc.); the *connection block* redistributes the power within the system without storing or dissipating energy (i.e. any type of gear reduction, transformers, etc.). The POG schemes can be used both for scalar and vectorial systems, and for real and complex variables. In the vectorial case, $\mathbf{G}(s)$ and $\mathbf{K}$ are matrices: $\mathbf{G}(s)$ is always a square matrix of positive real transfer functions; matrix $\mathbf{K}$ can also be rectangular, time varying and function of other state variables. The circle present in the e.b. is a summation element and the black spot represents a minus sign that multiplies the entering variable. The main feature of the Power-Oriented Graphs is to keep a direct correspondence between the dashed sections of the graphs and real power sections of the modeled systems: the real part of the scalar product $\mathbf{x}^*\mathbf{y}$ of the two *power vectors* $\mathbf{x}$ and $\mathbf{y}$ involved in each dashed line of a power-oriented graph, see Fig. 1, has the physical meaning of *the power flowing through that particular section.* From the POG schemes one can directly obtain the state space equations of the system: $\mathbf{L}\,\dot{\mathbf{x}} = -\mathbf{A}\mathbf{x}+\mathbf{B}\mathbf{u}$, $\mathbf{y} = \mathbf{B}^*\mathbf{x}$. The *energy matrix* $\mathbf{L}$ is always symmetric and positive definite:

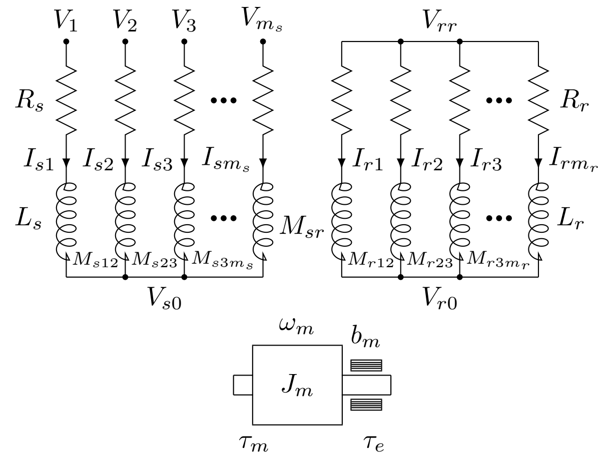

Fig. 2. Structure of a multi-phase asynchronous motor.

$\mathbf{L} = \mathbf{L}^* > 0$. When an eigenvalue of matrix $\mathbf{L}$ tends to zero (or to infinity), the system degenerates towards a smaller dynamic system. The dynamic equations $\overline{\mathbf{L}}\dot{\mathbf{z}} = -\overline{\mathbf{A}}\mathbf{z} + \overline{\mathbf{B}}\mathbf{u}$ and $\mathbf{y} = \overline{\mathbf{B}}^*\mathbf{z}$ of the "reduced" system can always be obtained from the original one using a "congruent" transformation $\mathbf{x} = \mathbf{T}\mathbf{z}$ (matrix $\mathbf{T}$ can also be complex and/or rectangular) where $\overline{\mathbf{L}} = \mathbf{T}^*\mathbf{L}\mathbf{T}$, $\overline{\mathbf{A}} = \mathbf{T}^*\mathbf{A}\mathbf{T} = \mathbf{T}^*\mathbf{L}\dot{\mathbf{T}}$ and $\overline{\mathbf{B}} = \mathbf{T}^*\mathbf{B}$. When matrix $\mathbf{T}$ is rectangular, the system is transformed and reduced at the same time.

2.1 Notations

In this paper the following notations are used to denote, respectively, full, diagonal, column and row matrices:

$$\underset{1:n}{\overset{i}{[\![}}\, R_{i,j}\, \underset{1:m}{\overset{j}{]\!]}} = \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1m} \\ R_{21} & R_{22} & \cdots & R_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ R_{n1} & R_{n2} & \cdots & R_{nm} \end{bmatrix}, \quad \underset{1:n}{\overset{i}{[\!\!\lceil}}\, R_i\,]\!\!\rfloor = \begin{bmatrix} R_1 & & \\ & \ddots & \\ & & R_n \end{bmatrix},$$

$$\underset{1:n}{\overset{i}{[\![}}\, R_i\,]\!] = [\, R_1\ R_2\ \cdots\ R_n\,]^{\mathrm{T}}, \qquad [\![\, R_j\, \underset{1:m}{\overset{j}{]\!]}} = [\, R_1\ R_2\ \cdots\ R_m\,].$$

The symbol $\delta(n)|_k^m$ denote the following function:

$$\delta(n)|_k^m = \begin{cases} 1 & \text{if } n \in [k,\, k \pm m,\, k \pm 2m, \ldots] \\ 0 & \text{in the other cases} \end{cases}$$

where $n, k, m \in \mathbb{Z}$. The symbol $\mathbf{I}_m$ denotes an identity matrix of order m.

3. COMPLEX DYNAMIC MODEL OF THE MOTOR

The basic structure of a multi-phase star-connected asynchronous motor is shown in Fig. 2. The electrical and mechanical parameters of the system are shown in Table 1. All the electrical parameters of the machine have been obtained connecting in series the p polar couples of the motor. Let ${}^t\mathbf{V}_s$, ${}^t\mathbf{I}_s$, ${}^t\mathbf{V}_r$ and ${}^t\mathbf{I}_r$ denote the stator and rotor voltage/current vectors in the external reference frame Σ_t:

$${}^t\mathbf{V}_s = \begin{bmatrix} V_{s1} \\ V_{s2} \\ \vdots \\ V_{sm_s} \end{bmatrix}, \; {}^t\mathbf{I}_s = \begin{bmatrix} I_{s1} \\ I_{s2} \\ \vdots \\ I_{sm_s} \end{bmatrix}, \; {}^t\mathbf{V}_r = \begin{bmatrix} V_{r1} \\ V_{r2} \\ \vdots \\ V_{rm_r} \end{bmatrix}, \; {}^t\mathbf{I}_r = \begin{bmatrix} I_{r1} \\ I_{r2} \\ \vdots \\ I_{rm_r} \end{bmatrix}$$

m_s	: number of stator phases;
m_r	: number of rotor phases;
p	: number of rotor and stator polar expansions;
γ_s	: stator angular phase displacement ($\gamma_s = \frac{2\pi}{m_s}$);
γ_r	: rotor angular phase displacement ($\gamma_r = \frac{2\pi}{m_r}$);
θ_m	: rotor angular position;
ω_m	: rotor angular velocity;
θ_s	: stator voltage angular position;
ω_s	: stator voltage frequency;
θ	: electric angle ($\theta = p\,\theta_m$);
R_s	: stator phases resistance;
L_s	: stator phases self inductance;
M_{s0}	: maximum mutual inductance of the stator phases;
R_r	: rotor phases resistance;
L_r	: rotor phases self inductance;
M_{r0}	: maximum mutual inductance of the rotor phases;
M_{sr0}	: maximum value of the mutual inductance between stator and rotor phases;
J_m	: rotor inertia momentum;
b_m	: rotor linear friction coefficient;
τ_m	: electromotive torque acting on the rotor;
τ_e	: external load torque acting on the rotor.

Table 1. Electrical and mechanical parameters of a multi-phase asynchronous motor.

where $V_{si} = V_i - V_{s0}$ for $i \in \{1, 2, \ldots, m_s\}$ and $V_{ri} = V_{rr} - V_{r0}$ for $i \in \{1, 2, \ldots, m_r\}$. Using the following generalized state vector ${}^t\dot{\mathbf{q}}$ and extended input vector ${}^t\mathbf{V}$:

$${}^t\dot{\mathbf{q}} = \begin{bmatrix} {}^t\mathbf{I}_s \\ {}^t\mathbf{I}_r \\ \hline \omega_m \end{bmatrix} = \begin{bmatrix} {}^t\mathbf{I}_e \\ \omega_m \end{bmatrix}, \qquad {}^t\mathbf{V} = \begin{bmatrix} {}^t\mathbf{V}_s \\ {}^t\mathbf{V}_r \\ \hline -\tau_e \end{bmatrix} = \begin{bmatrix} {}^t\mathbf{V}_e \\ -\tau_e \end{bmatrix}$$

and applying the "Lagrangian" approach discussed in Zanasi et al. (2009), one obtains the following dynamic equations of the multi-phase asynchronous motors:

$$\frac{d}{dt}\left(\underbrace{\left[\begin{array}{c|c} {}^t\mathbf{L}_e & 0 \\ \hline 0 & J_m \end{array}\right]}_{{}^t\mathbf{L}({}^t\mathbf{q})} \underbrace{\left[\begin{array}{c} {}^t\mathbf{I}_e \\ \hline \omega_m \end{array}\right]}_{{}^t\dot{\mathbf{q}}} \right) = - \underbrace{\left[\begin{array}{c|c} {}^t\mathbf{R}_e + {}^t\mathbf{F}_e & {}^t\mathbf{K}_e \\ \hline -{}^t\mathbf{K}_e^{\mathrm{T}} & b_m \end{array}\right]}_{{}^t\mathbf{R} + {}^t\mathbf{W}} \underbrace{\left[\begin{array}{c} {}^t\mathbf{I}_e \\ \hline \omega_m \end{array}\right]}_{{}^t\dot{\mathbf{q}}} + \underbrace{\left[\begin{array}{c} {}^t\mathbf{V}_e \\ \hline -\tau_e \end{array}\right]}_{{}^t\mathbf{V}} \quad (1)$$

The structure of the matrices ${}^t\mathbf{L}({}^t\mathbf{q})$, ${}^t\mathbf{R}$ and ${}^t\mathbf{W}$ in (1) are given in Zanasi and Azzone (2010). In order to take into account the odd order harmonic injection of the motor, the self and mutual inductance matrices ${}^t\mathbf{L}_s$, ${}^t\mathbf{L}_r$ and ${}^t\mathbf{M}_{sr}$ are supposed to have the following structure:

$${}^t\mathbf{L}_s = L_{s0}\,\mathbf{I}_{m_s} + M_{s0} \underset{1:m_s}{\overset{i}{[\![}} \sum_{n=1:2}^{m_s-2} a_n^s \cos(n\,(i-j)\gamma_s) \underset{1:m_s}{\overset{j}{]\!]}},$$

$${}^t\mathbf{L}_r = L_{r0}\,\mathbf{I}_{m_r} + M_{r0} \underset{1:m_r}{\overset{i}{[\![}} \sum_{n=1:2}^{m_r-2} a_n^r \cos(n\,(i-j)\gamma_r) \underset{1:m_r}{\overset{j}{]\!]}},$$

$${}^t\mathbf{M}_{sr}(\theta) = M_{sr0} \underset{0:m_r-1}{\overset{i}{[\![}} \sum_{n=1:2}^{m_{sr}-2} a_n^{sr} \cos(n(\theta + i\gamma_r - j\gamma_s)) \underset{0:m_s-1}{\overset{j}{]\!]}}$$

where $m_{sr} = \min\{m_s, m_r\}$, $L_{s0} = L_s - M_{s0}$ and $L_{r0} = L_r - M_{r0}$. The coefficients a_n^s, a_n^r and a_n^{sr} of the Fourier series are supposed to satisfy the following constraints:

$$\sum_{n=1:2}^{m_s-2} |a_n^s| \le 1, \qquad \sum_{n=1:2}^{m_r-2} |a_n^r| \le 1, \qquad \sum_{n=1:2}^{m_{sr}-2} |a_n^{sr}| \le 1.$$

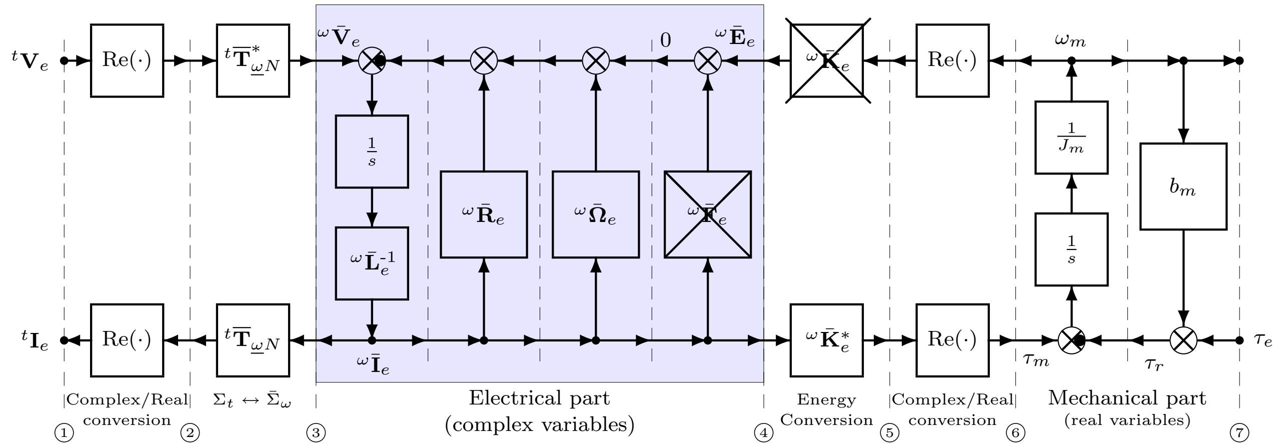

Fig. 3. POG graphical representation of a multi-phase asynchronous motor in the transformed rotating frame $\bar{\Sigma}_\omega$.

Let ${}^t\tilde{\mathbf{T}}_{\underline{\omega}N} \in \mathbb{C}^{m\times(m+1)/2}$ denote the following matrix:

$$ {}^t\tilde{\mathbf{T}}_{\underline{\omega}N}(m,\theta) = {}^t\tilde{\mathbf{T}}_{\underline{\omega}}(m,\theta)\,\mathbf{N}_m = \begin{bmatrix} {}^t\tilde{\mathbf{T}}_\omega & \mathbf{z}_m \end{bmatrix} \mathbf{N}_m \qquad (2) $$

where ${}^t\tilde{\mathbf{T}}_\omega(m,\theta) \in \mathbb{C}^{m\times(m-1)/2}$ is a "complex" matrix:

$$ {}^t\tilde{\mathbf{T}}_\omega(m,\theta) = \sqrt{\frac{1}{m}} \mathop{\Big\|\Big[}\limits_{0:m-1}^{h} e^{j\,k(\theta-h\gamma_m)} \mathop{\Big]\Big\|}\limits_{1:2:m-2}^{k} $$

with $\gamma_m = \frac{2\pi}{m}$, and where vector $\mathbf{z}_m \in \mathbb{R}^m$ and matrix $\mathbf{N}_m \in \mathbb{C}^{(m+1)/2\times(m+1)/2}$ are defined as follows:

$$ \mathbf{z}_m = \mathop{\Big\|\Big[}\limits_{0:m-1}^{h} \sqrt{\tfrac{1}{m}} \Big]\Big\|, \qquad \mathbf{N}_m = \begin{bmatrix} \sqrt{2}\,\mathbf{I}_{\frac{m-1}{2}} & 0 \\ 0 & 1 \end{bmatrix}. $$

Based on matrix (2), the following transformation matrix ${}^t\mathbf{T}_\omega \in \mathbb{C}^{(m_s+m_r+1)\times(m_s+m_r+1)}$ can be defined as:

$$ {}^t\mathbf{T}_\omega = \left[\begin{array}{cc|c} {}^t\tilde{\mathbf{T}}_{\underline{\omega}N}(m_s,\theta_s) & 0 & 0 \\ 0 & {}^t\tilde{\mathbf{T}}_{\underline{\omega}N}(m_r,\theta_p) & 0 \\ \hline 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{c|c} {}^t\overline{\mathbf{T}}_{\underline{\omega}N} & 0 \\ \hline 0 & 1 \end{array}\right] $$

$$ = \left[\begin{array}{cc|c} {}^t\tilde{\mathbf{T}}_{\underline{\omega}}(m_s,\theta_s) & 0 & 0 \\ 0 & {}^t\tilde{\mathbf{T}}_{\underline{\omega}}(m_r,\theta_p) & 0 \\ \hline 0 & 0 & 1 \end{array}\right] \left[\begin{array}{cc|c} \mathbf{N}_{m_s} & 0 & 0 \\ 0 & \mathbf{N}_{m_r} & 0 \\ \hline 0 & 0 & 1 \end{array}\right] $$

$$ = \left[\begin{array}{c|c} {}^t\overline{\mathbf{T}}_{\underline{\omega}} & 0 \\ \hline 0 & 1 \end{array}\right] \left[\begin{array}{c|c} \overline{\mathbf{N}} & 0 \\ \hline 0 & 1 \end{array}\right] = {}^t\overline{\mathbf{T}}_\omega\,\mathbf{N} $$

where $\theta_p = \theta_s - \theta$. All the columns of matrix ${}^t\mathbf{T}_\omega$ are orthogonal complex vectors. The complex matrix ${}^t\mathbf{T}_\omega$ can be used to perform a "pseudo" state space transformation ${}^t\dot{\mathbf{q}} = {}^t\mathbf{T}_\omega\,{}^\omega\dot{\mathbf{q}}$ from the original external frame Σ_t to a new complex rotating frame $\bar{\Sigma}_\omega$. The dynamic equations in the new complex transformed frame $\bar{\Sigma}_\omega$ are:

$$ \underbrace{\left[\begin{array}{c|c} {}^\omega\bar{\mathbf{L}}_e & 0 \\ \hline 0 & J_m \end{array}\right]}_{{}^\omega\mathbf{L}} \underbrace{\begin{bmatrix} {}^\omega\dot{\bar{\mathbf{I}}}_e \\ \dot{\omega}_m \end{bmatrix}}_{{}^\omega\ddot{\mathbf{q}}} = - \underbrace{\left[\begin{array}{c|c} {}^\omega\bar{\mathbf{R}}_e + {}^\omega\bar{\mathbf{F}}_e + {}^\omega\bar{\mathbf{\Omega}}_e & {}^\omega\bar{\mathbf{K}}_e \\ \hline -\,{}^\omega\bar{\mathbf{K}}_e^* & b_m \end{array}\right]}_{{}^\omega\mathbf{R} + {}^\omega\mathbf{W}} \underbrace{\begin{bmatrix} {}^\omega\bar{\mathbf{I}}_e \\ \omega_m \end{bmatrix}}_{{}^\omega\dot{\mathbf{q}}} + \underbrace{\begin{bmatrix} {}^\omega\bar{\mathbf{V}}_e \\ -\tau_e \end{bmatrix}}_{{}^\omega\mathbf{V}} \qquad (4) $$

The state space transformation is called "pseudo" because the transformed matrices ${}^\omega\mathbf{L}$, ${}^\omega\mathbf{R}$ and ${}^\omega\mathbf{W}$ are obtained using matrix ${}^t\overline{\mathbf{T}}_\omega$:

$$ {}^\omega\mathbf{L} = {}^t\overline{\mathbf{T}}^*_\omega\,{}^t\mathbf{L}\,{}^t\overline{\mathbf{T}}_\omega, \quad {}^\omega\mathbf{R} = {}^t\overline{\mathbf{T}}^*_\omega\,{}^t\mathbf{R}\,{}^t\overline{\mathbf{T}}_\omega, \quad {}^\omega\mathbf{W} = {}^t\overline{\mathbf{T}}^*_\omega\,{}^t\mathbf{W}\,{}^t\overline{\mathbf{T}}_\omega $$

while the complex vectors ${}^\omega\mathbf{V}$ and ${}^\omega\dot{\mathbf{q}}$ are obtained using matrix ${}^t\mathbf{T}_\omega$:

$$ {}^\omega\mathbf{V} = {}^t\mathbf{T}^*_\omega\,{}^t\mathbf{V} = \begin{bmatrix} {}^\omega\bar{\mathbf{V}}_s \\ {}^\omega\bar{\mathbf{V}}_r \\ \hline -\tau_e \end{bmatrix} = \begin{bmatrix} {}^\omega\bar{\mathbf{V}}_e \\ -\tau_e \end{bmatrix}, \quad {}^\omega\dot{\mathbf{q}} = {}^t\mathbf{T}^*_\omega\,{}^t\dot{\mathbf{q}} = \begin{bmatrix} {}^\omega\bar{\mathbf{I}}_s \\ {}^\omega\bar{\mathbf{I}}_r \\ \hline \omega_m \end{bmatrix} = \begin{bmatrix} {}^\omega\bar{\mathbf{I}}_e \\ \omega_m \end{bmatrix} $$

where ${}^\omega\bar{\mathbf{V}}_r = 0$ because the rotor phases are short-circuited. The transformed vector ${}^\omega\bar{\mathbf{I}}_e$ has the following structure:

$$ {}^\omega\bar{\mathbf{I}}_e = {}^t\overline{\mathbf{T}}^*_{\underline{\omega}N}\,{}^t\mathbf{I}_e = \begin{bmatrix} {}^t\tilde{\mathbf{T}}^*_{\underline{\omega}N}(m_s,\theta_s)\,{}^t\mathbf{I}_s \\ \hline {}^t\tilde{\mathbf{T}}^*_{\underline{\omega}N}(m_r,\theta_p)\,{}^t\mathbf{I}_r \end{bmatrix} = \begin{bmatrix} {}^\omega\bar{\mathbf{I}}_s \\ {}^\omega I_{sm_s} \\ \hline {}^\omega\bar{\mathbf{I}}_r \\ {}^\omega I_{rm_r} \end{bmatrix} = \begin{bmatrix} {}^\omega\bar{\mathbf{I}}_s \\ 0 \\ \hline {}^\omega\bar{\mathbf{I}}_r \\ 0 \end{bmatrix} $$

where ${}^\omega I_{sm_s} = {}^\omega I_{rm_r} = 0$ because the stator and rotor phases are star-connected and vectors ${}^\omega\bar{\mathbf{I}}_s$ and ${}^\omega\bar{\mathbf{I}}_s$ are:

$$ {}^\omega\bar{\mathbf{I}}_s = \mathop{\Big\|\Big[}\limits_{1:2:m_s-2}^{k} {}^\omega\bar{I}_{sk} \Big]\Big\| = \mathop{\Big[\!\Big[}\limits_{1:2:m_s-2}^{k} I_{dsk} + j\,I_{qsk} \Big]\!\Big], $$

$$ {}^\omega\bar{\mathbf{I}}_s = \mathop{\Big\|\Big[}\limits_{1:2:m_r-2}^{k} {}^\omega\bar{I}_{rk} \Big]\Big\| = \mathop{\Big[\!\Big[}\limits_{1:2:m_r-2}^{k} I_{drk} + j\,I_{qrk} \Big]\!\Big]. $$

The transformed vector ${}^\omega\bar{\mathbf{V}}_e$ has the following structure:

$$ {}^\omega\bar{\mathbf{V}}_e = {}^t\overline{\mathbf{T}}^*_{\underline{\omega}N}\,{}^t\mathbf{V}_e = \begin{bmatrix} {}^t\tilde{\mathbf{T}}^*_{\underline{\omega}N}(m_s,\theta_s)\,{}^t\mathbf{V}_s \\ \hline {}^t\tilde{\mathbf{T}}^*_{\underline{\omega}N}(m_r,\theta_p)\,{}^t\mathbf{V}_r \end{bmatrix} = \begin{bmatrix} {}^\omega\bar{\mathbf{V}}_s \\ {}^\omega V_{sm_s} \\ \hline 0 \end{bmatrix} = \begin{bmatrix} {}^\omega\bar{\mathbf{V}}_s \\ 0 \\ \hline 0 \end{bmatrix} $$

where ${}^\omega V_{sm_s} = 0$ because the input stator voltages are balanced, and where vector ${}^\omega\bar{\mathbf{V}}_s$ is defined as:

$$ {}^\omega\bar{\mathbf{V}}_s = \mathop{\Big\|\Big[}\limits_{1:2:m_s-2}^{k} {}^\omega\bar{V}_{sk} \Big]\Big\| = \mathop{\Big\|\Big[}\limits_{1:2:m_s-2}^{k} V_{dsk} + j\,V_{qsk} \Big]\Big\|. $$

It can be easily proved that the transformed matrix ${}^\omega\mathbf{L}$ has the following symmetric constant structure:

$$ {}^\omega\mathbf{L} = \left[\begin{array}{cc|c} L_{s0} + \frac{m_s}{2}\,M_{s0}\,\mathbf{a}_s & M_{sre}\,\mathbf{a}_{sr}^{\mathrm{T}} & 0 \\ M_{sre}\,\mathbf{a}_{sr} & L_{r0} + \frac{m_r}{2}\,M_{r0}\,\mathbf{a}_r & 0 \\ \hline 0 & 0 & J_m \end{array}\right] $$

where $\mathbf{a}_s$, $\mathbf{a}_r$ and $\mathbf{a}_{sr}$ are real constant matrices (function of the Fourier series coefficients) defined as follows:

$$ \mathbf{a}_s = \mathop{\Big\|\Big[}\limits_{1:2:m_s-2}^{k} a_k^s \Big]\Big\|, \quad \mathbf{a}_r = \mathop{\Big\|\Big[}\limits_{1:2:m_r-2}^{k} a_k^r \Big]\Big\|, \quad \mathbf{a}_{sr} = \mathop{\Big\|\Big[}\limits_{1:2:m_r-2}^{k} a_k^{sr}\,\delta(k)|_l^\infty \mathop{\Big]\Big\|}\limits_{1:2:m_s-2}^{l}. $$

The energy redistribution matrix ${}^\omega\mathbf{W}$ has the following skew-symmetric structure:

$$\left[\begin{array}{cc|c} L_{s0}+\frac{m_s}{2}M_{s0}\mathbf{a}_s & M_{sre}\mathbf{a}_{sr}^{\mathrm{T}} & 0\\ M_{sre}\mathbf{a}_{sr} & L_{r0}+\frac{m_r}{2}M_{r0}\mathbf{a}_r & 0\\ \hline 0 & 0 & J_m\end{array}\right]\left[\begin{array}{c}{}^{\omega}\dot{\bar{\mathbf{I}}}_s\\ {}^{\omega}\dot{\bar{\mathbf{I}}}_r\\ \hline \dot{\omega}_m\end{array}\right]=-\left[\begin{array}{cc|c} R_s+j\omega_s\mathbf{k}_{m_s}(L_{s0}+\frac{m_s}{2}M_{s0}\mathbf{a}_s) & j\omega_s M_{sre}\mathbf{k}_{m_s}\mathbf{a}_{sr}^{\mathrm{T}} & 0\\ j\omega_p M_{sre}\mathbf{k}_{m_r}\mathbf{a}_{sr} & R_r+j\omega_p\mathbf{k}_{m_r}(L_{r0}+\frac{m_r}{2}M_{r0}\mathbf{a}_r) & 0\\ \hline j\frac{p}{2}M_{sre}{}^{\omega}\mathbf{I}_r^{*}\mathbf{k}_{m_r}\mathbf{a}_{sr} & -j\frac{p}{2}M_{sre}{}^{\omega}\mathbf{I}_s^{*}\mathbf{k}_{m_s}\mathbf{a}_{sr}^{\mathrm{T}} & b_m\end{array}\right]\left[\begin{array}{c}{}^{\omega}\bar{\mathbf{I}}_s\\ {}^{\omega}\bar{\mathbf{I}}_r\\ \hline \omega_m\end{array}\right]+\left[\begin{array}{c}{}^{\omega}\bar{\mathbf{V}}_s\\ 0\\ \hline -\tau_e\end{array}\right] \quad (3)$$

Fig. 4. The complex dynamic equations of a multi-phase asynchronous motor in the transformed rotating frame $\bar{\Sigma}_\omega$.

$$ {}^{\omega}\mathbf{W}=\left[\begin{array}{cc|c} j\omega_s\mathbf{k}_{m_s}(L_{s0}+\frac{m_s}{2}M_{s0}\mathbf{a}_s) & j(\omega_s-\frac{\omega}{2})M_{sre}\mathbf{k}_{m_s}\mathbf{a}_{sr}^{\mathrm{T}} & {}^{\omega}\bar{\mathbf{K}}_s\\ j(\omega_s-\frac{\omega}{2})M_{sre}\mathbf{k}_{m_r}\mathbf{a}_{sr} & j\omega_p\mathbf{k}_{m_r}(L_{r0}+\frac{m_r}{2}M_{r0}\mathbf{a}_r) & {}^{\omega}\bar{\mathbf{K}}_r\\ \hline -{}^{\omega}\bar{\mathbf{K}}_s^{*} & -{}^{\omega}\bar{\mathbf{K}}_r^{*} & 0\end{array}\right]$$

where $\mathbf{k}_m = \llbracket\; k \;\rrbracket_{1:2:m-2}^{k}$. In the new frame $\bar{\Sigma}_\omega$ the transformed torque vector ${}^{\omega}\bar{\mathbf{K}}_e^{*}$ has the following structure:

$$\begin{aligned} {}^{\omega}\bar{\mathbf{K}}_e^{*} &= \left[\ {}^{\omega}\bar{\mathbf{K}}_s^{*}\ \ {}^{\omega}\bar{\mathbf{K}}_r^{*}\ \right]\\ &= \left[\,-j\,\frac{p}{2}\,M_{sre}\,{}^{\omega}\bar{\mathbf{I}}_r^{*}\,\mathbf{k}_{m_r}\mathbf{a}_{sr}\ \middle|\ j\,\frac{p}{2}\,M_{sre}\,{}^{\omega}\bar{\mathbf{I}}_s^{*}\,\mathbf{k}_{m_s}\mathbf{a}_{sr}^{\mathrm{T}}\,\right].\end{aligned}$$

The mechanical torque τ_m can be expressed as follows:

$$\begin{aligned}\tau_m &= \mathrm{Re}\,(\,{}^{\omega}\bar{\mathbf{K}}_e^{*}\,{}^{\omega}\bar{\mathbf{I}}_e) = \mathrm{Re}\left(\left[\ {}^{\omega}\bar{\mathbf{K}}_s^{*}\ \ {}^{\omega}\bar{\mathbf{K}}_r^{*}\ \right]\begin{bmatrix}{}^{\omega}\bar{\mathbf{I}}_s\\ {}^{\omega}\bar{\mathbf{I}}_r\end{bmatrix}\right)\\ &= \frac{p}{2}\,M_{sre}\,\mathrm{Re}\left(\left[-j\,{}^{\omega}\bar{\mathbf{I}}_r^{*}\,\mathbf{k}_{m_r}\,\mathbf{a}_{sr}\ \middle|\ j\,{}^{\omega}\bar{\mathbf{I}}_s^{*}\,\mathbf{k}_{m_s}\,\mathbf{a}_{sr}^{\mathrm{T}}\right]\begin{bmatrix}{}^{\omega}\bar{\mathbf{I}}_s\\ {}^{\omega}\bar{\mathbf{I}}_r\end{bmatrix}\right)\\ &= pM_{sre}\sum_{k=1:2}^{m_{sr}-2} k a_k^{sr}(I_{drk}I_{qsk}-I_{dsk}I_{qrk}). \qquad (5)\end{aligned}$$

A POG graphical representation of system (4) is shown in Fig. 3: section ①-③ represents the state space transformation $\Sigma_t \leftrightarrow \bar{\Sigma}_\omega$. Function "Re(·)" denotes the "complex to real conversion" of the input. Section ③-④ represents the *Electrical part* of the system that, in this case, is described only by complex matrices and complex variables (the lightly shaded section of Fig. 3). The *Mechanical part* of the motor is described by section ⑥-⑦ which is characterized only by real values and real variables. Section ④-⑥ represents the energy and power conversion (without accumulation nor dissipation) between the Electrical and Mechanical parts.

It can be easily proved that in (4) the term ${}^{\omega}\bar{\mathbf{K}}_e\,\omega_m$ is simplified by the term ${}^{\omega}\bar{\mathbf{F}}_e\,{}^{\omega}\bar{\mathbf{I}}_e$, so the dynamics of the system can be rewritten in the following simplified form:

$$\underbrace{\left[\begin{array}{c|c}{}^{\omega}\bar{\mathbf{L}}_e & 0\\ \hline 0 & J_m\end{array}\right]}_{{}^{\omega}\mathbf{L}}\underbrace{\left[\begin{array}{c}{}^{\omega}\dot{\bar{\mathbf{I}}}_e\\ \hline \dot{\omega}_m\end{array}\right]}_{{}^{\omega}\ddot{\mathbf{q}}}=-\underbrace{\left[\begin{array}{c|c}{}^{\omega}\bar{\mathbf{R}}_e+{}^{\omega}\bar{\boldsymbol{\Omega}}_e & 0\\ \hline -{}^{\omega}\bar{\mathbf{K}}_e^{*} & b_m\end{array}\right]}_{{}^{\omega}\mathbf{R}+{}^{\omega}\mathbf{W}}\underbrace{\left[\begin{array}{c}{}^{\omega}\bar{\mathbf{I}}_e\\ \hline \omega_m\end{array}\right]}_{{}^{\omega}\dot{\mathbf{q}}}+\underbrace{\left[\begin{array}{c}{}^{\omega}\bar{\mathbf{V}}_e\\ \hline -\tau_e\end{array}\right]}_{{}^{\omega}\mathbf{V}} \quad (6)$$

The expanded form of system (6) is shown in Fig. 4 where:

$$M_{sre}=\frac{M_{sr0}\sqrt{m_s m_r}}{2}, \qquad \omega_p=\omega_s-\omega.$$

Let us now consider the case $m_s=m_r=m_{sr}$ and let $\mathbf{P}_\pi$ denote the following permutation matrix:

$$\mathbf{P}_\pi=\llbracket\ \mathbf{e}_h\quad \mathbf{e}_{m_s+h}\ \rrbracket_{1:\frac{m_{sr}-1}{2}}^{h} \qquad (7)$$

where $\mathbf{e}_h$ denotes a column vector of length $m_{sr}-1$ with 1 in the h^{th} position and 0 in every other position. Applying the transformation ${}^{\omega}\bar{\mathbf{I}}_e=\mathbf{P}_\pi\,{}^{\omega}\bar{\mathbf{I}}_{e_k}$ to the electrical part of system (6), one obtains the following reordered system:

$$\underbrace{\left[\begin{array}{c|c}{}^{\omega}\bar{\mathbf{L}}_{e_k} & 0\\ \hline 0 & J_m\end{array}\right]}_{{}^{\omega}\mathbf{L}_k}\underbrace{\left[\begin{array}{c}{}^{\omega}\dot{\bar{\mathbf{I}}}_{e_k}\\ \hline \dot{\omega}_m\end{array}\right]}_{{}^{\omega}\ddot{\mathbf{q}}_k}=-\underbrace{\left[\begin{array}{c|c}{}^{\omega}\bar{\mathbf{R}}_{e_k}+{}^{\omega}\bar{\boldsymbol{\Omega}}_{e_k} & 0\\ \hline -{}^{\omega}\bar{\mathbf{K}}_{e_k}^{*} & b_m\end{array}\right]}_{{}^{\omega}\mathbf{R}_k+{}^{\omega}\mathbf{W}_k}\underbrace{\left[\begin{array}{c}{}^{\omega}\bar{\mathbf{I}}_{e_k}\\ \hline \omega_m\end{array}\right]}_{{}^{\omega}\dot{\mathbf{q}}_k}+\underbrace{\left[\begin{array}{c}{}^{\omega}\bar{\mathbf{V}}_{e_k}\\ \hline -\tau_e\end{array}\right]}_{{}^{\omega}\mathbf{V}_k} \quad (8)$$

where ${}^{\omega}\bar{\mathbf{L}}_{e_k}=\mathbf{P}_\pi^{\mathrm{T}}\,{}^{\omega}\bar{\mathbf{L}}_e\,\mathbf{P}_\pi$, ${}^{\omega}\bar{\mathbf{R}}_{e_k}=\mathbf{P}_\pi^{\mathrm{T}}\,{}^{\omega}\bar{\mathbf{R}}_e\,\mathbf{P}_\pi$, ${}^{\omega}\bar{\boldsymbol{\Omega}}_{e_k}=\mathbf{P}_\pi^{\mathrm{T}}\,{}^{\omega}\bar{\boldsymbol{\Omega}}_e\,\mathbf{P}_\pi$, ${}^{\omega}\bar{\mathbf{I}}_{e_k}=\mathbf{P}_\pi^{\mathrm{T}}\,{}^{\omega}\bar{\mathbf{I}}_e$, ${}^{\omega}\bar{\mathbf{V}}_{e_k}=\mathbf{P}_\pi^{\mathrm{T}}\,{}^{\omega}\bar{\mathbf{V}}_e$:

$${}^{\omega}\bar{\mathbf{L}}_{e_k}=\left\llbracket\begin{bmatrix}L_{se_k} & M_{sre_k}\\ M_{sre_k} & L_{re_k}\end{bmatrix}\right\rrbracket_{1:2:m_{sr}-2}^{k},\quad {}^{\omega}\bar{\mathbf{I}}_{e_k}=\left\llbracket\begin{bmatrix}{}^{\omega}\bar{I}_{sk}\\ {}^{\omega}\bar{I}_{rk}\end{bmatrix}\right\rrbracket_{1:2:m_{sr}-2}^{k},\quad {}^{\omega}\bar{\mathbf{V}}_{e_k}=\left\llbracket\begin{bmatrix}{}^{\omega}\bar{V}_{sk}\\ {}^{\omega}\bar{V}_{rk}\end{bmatrix}\right\rrbracket_{1:2:m_{sr}-2}^{k},$$

$${}^{\omega}\bar{\mathbf{R}}_{e_k}=\left\llbracket\begin{bmatrix}R_s & 0\\ 0 & R_r\end{bmatrix}\right\rrbracket_{1:2:m_{sr}-2}^{k},\quad {}^{\omega}\bar{\boldsymbol{\Omega}}_{e_k}=\left\llbracket\begin{bmatrix}j\omega_s k L_{se_k} & j\omega_s k M_{sre_k}\\ j\omega_p k M_{sre_k} & j\omega_p k L_{re_k}\end{bmatrix}\right\rrbracket_{1:2:m_{sr}-2}^{k},$$

$${}^{\omega}\bar{\mathbf{K}}_{e_k}^{*}=\left\llbracket\ j\tfrac{p}{2}kM_{sre_k}\,{}^{\omega}\bar{I}_{rk}^{*}\ \middle|\ -j\tfrac{p}{2}kM_{sre_k}\,{}^{\omega}\bar{I}_{sk}^{*}\ \right\rrbracket_{1:2:m_{sr}-2}^{k}$$

and $L_{se_k}=L_{s0}+\frac{m_s}{2}\,a_k^{s}\,M_{s0}$, $L_{re_k}=L_{r0}+\frac{m_r}{2}\,a_k^{r}\,M_{r0}$, $M_{sre_k}=M_{sre}\,a_k^{sr}$. Equations (8) clearly show that a multi-phase asynchronous motor with an odd order harmonic injection can be mathematically described by $\frac{m_{sr}-1}{2}$ sets of decoupled equations in the frame $\bar{\Sigma}_{\omega_k}$, i.e. the system can be seen as the set of $\frac{m_{sr}-1}{2}$ three-phase asynchronous motors respectively supplied by balanced voltages at frequencies $k\,\omega_s$. The total mechanical torque of the motor τ_m is the sum of the torques generated by the $\frac{m_{sr}-1}{2}$ internal decoupled motors:

$$\begin{aligned}\tau_m &= \sum_{k=1:2}^{m_{sr}-2}\tau_{m_k} = \sum_{k=1:2}^{m_{sr}-2}\mathrm{Re}\,(\,{}^{\omega}\bar{\mathbf{K}}_{e_k}^{*}\,{}^{\omega}\bar{\mathbf{I}}_{e_k})\\ &= \mathrm{Re}\left(\sum_{k=1:2}^{m_{sr}-2}\left\llbracket -j\tfrac{p}{2}kM_{sre_k}\,{}^{\omega}\bar{I}_{rk}^{*}\ \middle|\ j\tfrac{p}{2}k\,M_{sre_k}\,{}^{\omega}\bar{I}_{sk}^{*}\right\rrbracket_{1:2:m_{sr}-2}^{k}\left\llbracket\begin{bmatrix}{}^{\omega}\bar{I}_{sk}\\ {}^{\omega}\bar{I}_{rk}\end{bmatrix}\right\rrbracket_{1:2:m_{sr}-2}^{k}\right)\\ &= pM_{sre}\sum_{k=1:2}^{m_{sr}-2} k a_k^{sr}(I_{drk}I_{qsk}-I_{dsk}I_{qrk}).\end{aligned}$$

Clearly, this expression is equal to the one given in (5).

4. INDIRECT FIELD ORIENTED CONTROL

Let ${}^{\omega}\bar{\boldsymbol{\Phi}}_e={}^{\omega}\bar{\mathbf{L}}_e\,{}^{\omega}\bar{\mathbf{I}}_e$ denote the fluxes vector in the frame $\bar{\Sigma}_\omega$. The steady-state equations of the electrical part of system (6) can also be written as follows:

$$\begin{bmatrix}{}^{\omega}\bar{\mathbf{V}}_s\\ 0\end{bmatrix}=\begin{bmatrix}{}^{\omega}\bar{\mathbf{R}}_s & 0\\ 0 & {}^{\omega}\bar{\mathbf{R}}_r\end{bmatrix}\begin{bmatrix}{}^{\omega}\bar{\mathbf{I}}_s\\ {}^{\omega}\bar{\mathbf{I}}_r\end{bmatrix}+\begin{bmatrix}j\,\omega_s\,\mathbf{k}_{m_s} & 0\\ 0 & j\,\omega_p\,\mathbf{k}_{m_r}\end{bmatrix}\begin{bmatrix}{}^{\omega}\bar{\boldsymbol{\Phi}}_s\\ {}^{\omega}\bar{\boldsymbol{\Phi}}_r\end{bmatrix}.$$

The considered multi-phase asynchronous motor has been controlled using the Indirect Rotor Field-Oriented (IRFO) control technique, see Leonard (2001) and Vas (1990), obtaining the following equations:

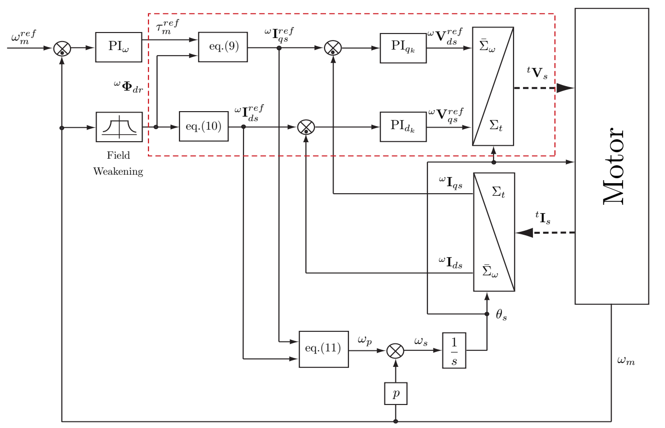

Fig. 5. Indirect Rotor Field-Oriented control including fundamental plus odd harmonic injection.

$$\tau_m = p\, M_{sre} \sum_{k=1:2}^{m_{sr}-2} \frac{k\, a_k^{sr}}{L_{re_k}}\, \Phi_{drk}\, I_{qsk} \qquad (9)$$

$${}^{\omega}\mathbf{\Phi}_{dr} = M_{sre}\, \mathbf{a}_{sr}\, {}^{\omega}\mathbf{I}_{ds} \qquad (10)$$

$$\omega_p = \frac{I_{qs1}}{T_r\, I_{ds1}} \qquad (11)$$

where ${}^{\omega}\mathbf{\Phi}_{dr} = \mathrm{Re}({}^{\omega}\bar{\mathbf{\Phi}}_r)$, ${}^{\omega}\mathbf{I}_{ds} = \mathrm{Re}({}^{\omega}\bar{\mathbf{I}}_s)$ and $T_r = L_{re}/R_r$ is the rotor constant. The corresponding IRFO control scheme is shown in Fig. 5: the PI$_\omega$ regulator controls the angular velocity ω_m generating a torque reference τ_m^{ref} and tracking a defined speed reference ω_m^{ref}. The PI$_{d_k}$ and PI$_{q_k}$ controllers regulate the rotor flux components ${}^{\omega}\mathbf{\Phi}_{dr}$ and the mechanical torque τ_m, respectively, according to the equations (9) and (10), and generating the voltage references ${}^{\omega}\mathbf{V}_{ds}^{ref}$ and ${}^{\omega}\mathbf{V}_{qs}^{ref}$.

In the previous section it has been shown that the multi-phase asynchronous motor can be described as a set of $\frac{m_{sr}-1}{2}$ decoupled machines. These machines can be controlled separately or, equivalently, one can control only the first machine, corresponding to the fundamental harmonic, and then scaling the obtained voltage references V_{ds1}^{ref} and V_{qs1}^{ref} by using a scaling coefficient, see Duran et al. (2008). The first solution is more flexible because one can define a custom control for each machine, but its implementation has an higher cost in terms of number of controllers and tuning. The second solution has a simpler structure, but it is limited to the first machine only. The trade-off that has to be considered is between the control degrees of freedom and the control computational and implementation costs.

5. SIMULATION RESULTS

The simulation results presented in this section have been obtained in Matlab/Simulink by implementing the proposed complex and reduced model of the system and using the IRFO control strategy. The following electrical and mechanical parameters have been considered: $m_s = 5$, $m_r = 5$, $p = 1$, $L_s = 0.12$ H, $M_{s0} = 0.1$ H, $R_s = 3\,\Omega$, $L_r = 0.12$ H, $M_{r0} = 0.1$ H, $R_s = 3\,\Omega$, $M_{sr0} = 0.09$ H, $J_m = 0.03$ kg m^2, $b_m = 0.05$ Nm s/rad and $V_{max} = 100$ V. V_{max} is the maximum value of the stator voltage vector ${}^{\omega}\mathbf{V}_s$. The following sum of balanced voltage inputs has been considered:

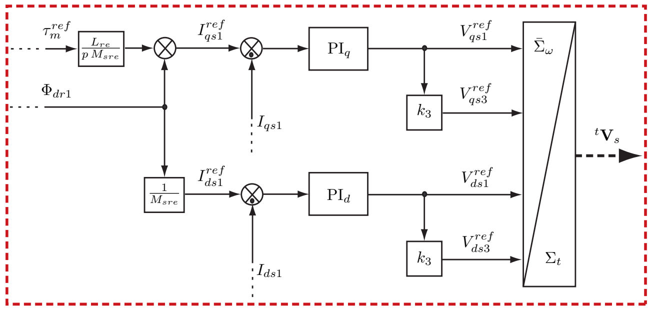

Fig. 6. Indirect Rotor Field-Oriented control (see Fig. 5) of a 5-phases machine: control of the fundamental harmonic and scaling of the third one.

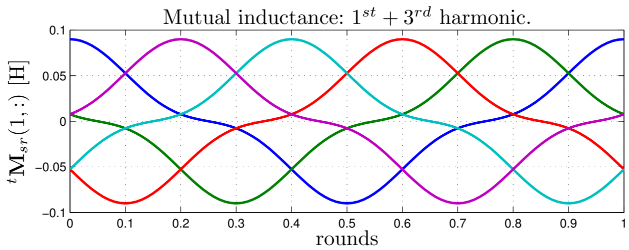

Fig. 7. Mutual inductance between the first stator phase and the rotor phases.

$${}^{t}\mathbf{V}_s = \mathop{[\![\, V_{sh}\,]\!]}_{1:5}^{h} = \sum_{k=1:2}^{3} \mathop{\left[\!\!\left[\, V_{mk}\, \cos(k\,(\theta_s - (h-1)\,\gamma_s))\, \right]\!\!\right]}_{1:5}^{h}$$

where k indicates the injected harmonic order and h the number of stator phases. The following Fourier series coefficients, to define the shape of the self and mutual inductances, have been used:

$$\mathbf{a}_s = \begin{bmatrix} 0.8 \\ 0.2 \end{bmatrix}, \quad \mathbf{a}_r = \begin{bmatrix} 0.8 \\ 0.2 \end{bmatrix}, \quad \mathbf{a}_{sr} = \begin{bmatrix} 0.8 \\ 0.2 \end{bmatrix}.$$

The IRFO control scheme used in simulation is shown in Fig. 6: only the control of the first subspace corresponding to the fundamental harmonic has been considered, while the third harmonic injection contribution has been obtained by scaling the voltage reference values $V_{ds3}^{ref} = k_3\, V_{ds1}^{ref}$ and $V_{qs3}^{ref} = k_3\, V_{qs1}^{ref}$ with $k_3 = 0.15$. The mutual inductance ${}^{t}\mathbf{M}_{sr}$ between the first stator phase and the rotor phases is shown in Fig. 7: the 1^{st} and 3^{rd} harmonic components are summed using the coefficients $\mathbf{a}_{sr}$ as weights.

The time behaviors of the stator and rotor currents ${}^{t}\mathbf{I}_s$ and ${}^{t}\mathbf{I}_r$ in the original reference Σ_t are shown in Fig. 8: their amplitudes and their frequencies change according to the velocity tracking of Fig. 10. The generated mechanical torque τ_m and the applied load torque profile τ_e are shown in Fig. 9: for $t \in [0, 1[$ s and $t \in [9, 10[$ s, the torque τ_m settles to the corresponding value of the load torque τ_e because a null velocity is tracked. For $t \in [1, 2[$ s and $t \in [8, 9[$ s, a positive and negative torque τ_m are generated, respectively, following the corresponding increase and decrease of the velocity ω_m (see Fig. 10). The control variables ω_m, I_{ds1} and I_{qs1} are reported in Fig. 10: their actual values (blue solid) optimally overlap the corresponding reference values (black dashed) providing low and limited control errors shown in Fig. 11.

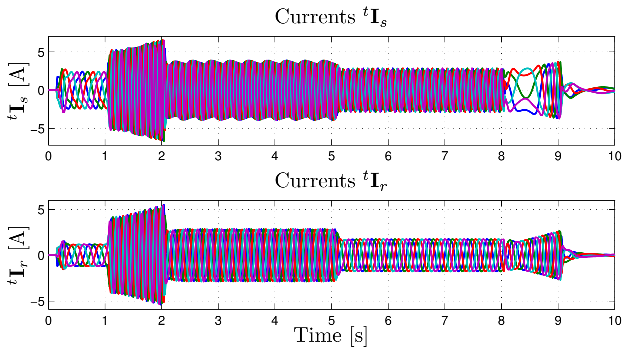

Fig. 8. Stator and rotor currents in the original reference frame Σ_t.

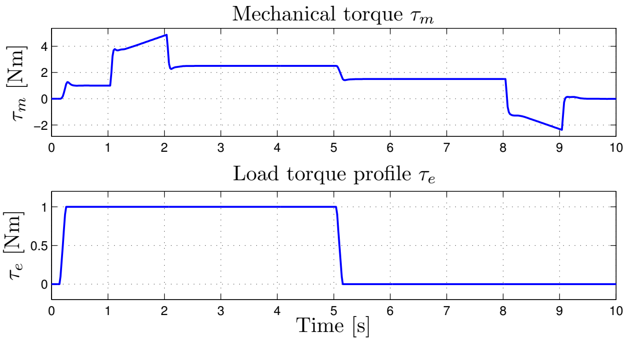

Fig. 9. Mechanical torque τ_m and load torque τ_e.

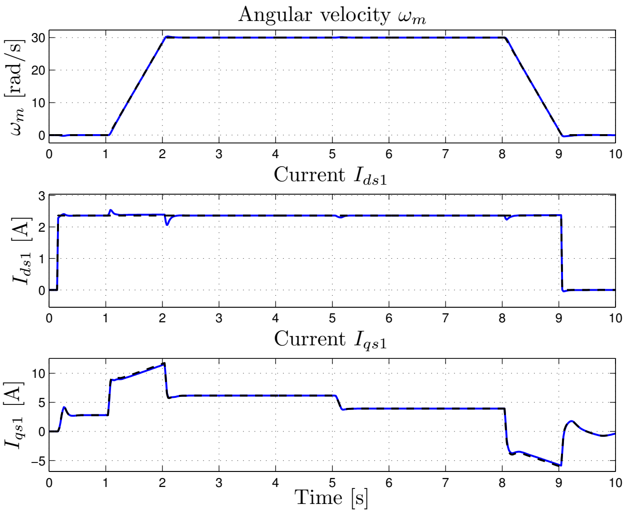

Fig. 10. IRFO control variables: actual values (blue solid) and reference values (black dashed).

6. CONCLUSION

In the paper a new complex dynamic model of a multi-phase asynchronous motor has been presented and a generalized form of the Field-Oriented control has been applied to the motor. The complex and reduced-order model has been obtained using a complex rectangular transformation and considering the odd harmonic injection. The Field-Oriented control has been considered and discussed in the multi-phase general case, and then it has been implemented in Matlab/Simulink considering a motor with 5 phases. The simulation results have shown the good behaviors of the presented model and the implemented control design.

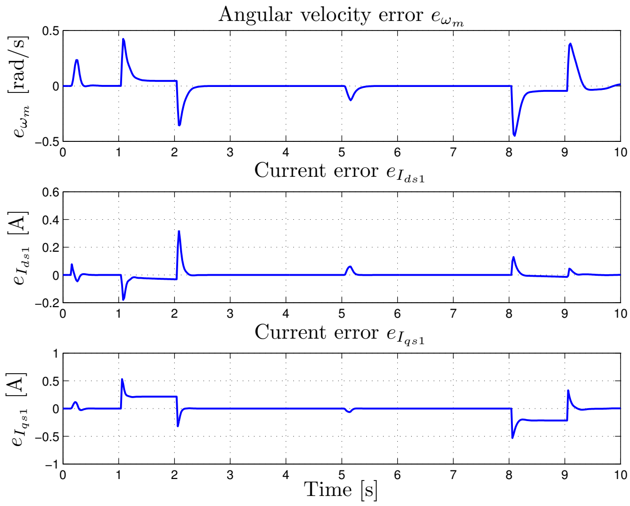

Fig. 11. IRFO control error variables e_{ω_m}, $e_{I_{ds1}}$ and $e_{I_{qs1}}$.

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