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Modeling of Multi-phase Permanent Magnet Synchronous Motors under Open-phase Fault Condition

Marco Fei, Roberto Zanasi and Federica Grossi

***Abstract*— This paper deals with the modeling of multi-phase permanent magnet synchronous motors under open-phase fault condition. Multi-phase electrical motors offer high reliability thanks to their capability to operate safely even in case of faults such the loss of some phase. In this paper the model of a multi phase PM machine in the case of open phase fault condition is proposed. The model is suitable for generic number of phases, generic shape of the rotor flux and the open circuit fault can occur to any phase. The motor model in fault condition is presented for faults occurring on one single phase or more than one phase (both adjacent and not adjacent phases). The use of a model is very useful both for simulations and implementation of fault-tolerant control strategies.**

I. INTRODUCTION

Multi-phase machines offer some advantages and greater number of degrees of freedom compared to three-phase machines, see [1] and [2]. One of these advantages is the better fault tolerance and this is very important in propulsion and traction applications where high reliability is a very important issue. Many kinds of faults can occur in multi-phase PM machines and the realization of a model can be helpful in analysis and simulation of the machine in faulty operation mode. A great number of fault-tolerant control strategies have been proposed, see [3], [4] and [5], in order to make the motor able to operate safely even in case of fault in particular obtaining ripple-free torque and minimizing losses. In [10] the modeling and control of a three-phase PMSM under supply fault conditions is investigated. However in the literature any effective model for the faulty motor has been provided yet.

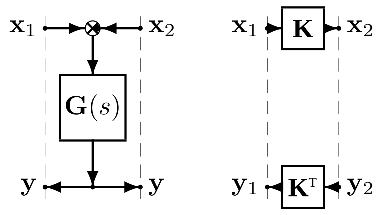
In this paper the model of a multi-phase PM machine in case of open circuited phase is proposed. The model is as general as possible, for a generic number of stator phases and for generic periodic shape of the rotor flux. The open circuit fault can occur to any of the phases. The model is obtained with the Power-Oriented Graphs modeling technique and can be directly implemented in a general-type simulator. Thanks to this model the different control strategies for faulty operation of the motor can be tested before the implementation on a real machine.

The paper is organized as follows. Sec. II introduces the main features of POG technique, Sec. III shows the details of the dynamic model of the m_s-phase synchronous motors, in Sec. IV the model of the motor in open phase fault condition is presented and Sec. V is devoted to simulation results. Conclusions are given in Sec. VI.

M. Fei, R. Zanasi and F. Grossi are with the Information Engineering Department, University of Modena and Reggio Emilia, Via Vignolese 905, 41100 Modena, Italy, e-mail: {marco.fei, roberto.zanasi, federica.grossi}@unimore.it.

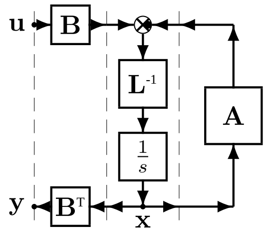
II. POWER-ORIENTED GRAPHS BASIC PRINCIPLES

The Power-Oriented Graphs technique, see [7], is an energy-based technique suitable for modeling physical systems. The POG are block diagrams combined with a particular modular structure essentially based on the use of the two blocks shown in Fig. 1.a and Fig. 1.b: the *elaboration block* (e.b.) stores and/or dissipates energy, the *connection block* (c.b.) redistributes the power within the system without storing nor dissipating energy. The c.b. transforms the power variables with the constraint $\mathbf{x}_1^{\mathrm{T}}\mathbf{y}_1 = \mathbf{x}_2^{\mathrm{T}}\mathbf{y}_2$. The circle in the e.b. is a summation element and the black spot represents a minus sign that multiplies the entering variable. The POG keep a direct correspondence between the dashed sections of the graphs and real power sections of the modeled system: the scalar product $\mathbf{x}^{\mathrm{T}}\mathbf{y}$ of the two *power vectors* $\mathbf{x}$ and $\mathbf{y}$ involved in each dashed line of a POG, see Fig. 1, has the physical meaning of *the power flowing through that particular section*. Another important aspect of the POG technique is the direct correspondence between the POG representations and the corresponding state space descriptions. For example, the POG scheme shown in Fig. 2 can be represented by the state space equations given in (1) where the *energy matrix* $\mathbf{L}$ is symmetric and positive definite: $\mathbf{L} = \mathbf{L}^{\mathrm{T}} > 0$. The dynamic model (1) can be transformed and reduced to system (2) using a "congruent" transformation



a) *e. b.* b) *c. b.*

Fig. 1. POG basic blocks: a) *elaboration block*; b) *connection block*.



$$\begin{cases} \mathbf{L}\,\dot{\mathbf{x}} = -\mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \\ \mathbf{y} = \mathbf{B}^{\mathrm{T}}\mathbf{x} \end{cases} \tag{1}$$

$$\Downarrow (\mathbf{x} = \mathbf{T}\mathbf{z})$$

$$\begin{cases} \overline{\mathbf{L}}\dot{\mathbf{z}} = -\overline{\mathbf{A}}\mathbf{z} + \overline{\mathbf{B}}\mathbf{u} \\ \mathbf{y} = \overline{\mathbf{B}}^{\mathrm{T}}\mathbf{z} \end{cases} \tag{2}$$

Fig. 2. POG scheme of a generic dynamic system in the complex domain.

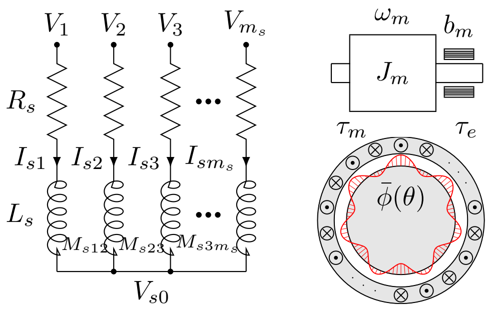


Fig. 3. Basic structure of a multi-phase synchronous motor.

m_s	number of motor phases
p	number of polar expansions
θ, θ_m	electric and rotor angular positions: $\theta = p\,\theta_m$
ω, ω_m	electric and rotor angular velocities: $\omega = p\,\omega_m$
R_s	i-th stator phase resistance
L_s	i-th stator phase self induction coefficient
M_{s0}	maximum value of mutual inductance
$\phi_c(\theta)$	total rotor flux chained with stator phase 1
φ_c	maximum value of function $\phi_c(\theta)$
J_m	rotor moment of inertia
b_m	rotor linear friction coefficient
τ_m	electromotive torque acting on the rotor
τ_e	external load torque acting on the rotor
γ_s	basic angular displacement ($\gamma_s = 2\pi/m_s$)

TABLE I

MAIN PARAMETERS OF A MULTI-PHASE SYNCHRONOUS MOTOR.

$\mathbf{x} = \mathbf{T}\mathbf{z}$ (matrix $\mathbf{T}$ can also be rectangular and time-varying) where $\overline{\mathbf{L}} = \mathbf{T}^{\mathrm{T}}\mathbf{L}\mathbf{T}$, $\overline{\mathbf{A}} = \mathbf{T}^{\mathrm{T}}\mathbf{A}\mathbf{T} + \mathbf{T}^{\mathrm{T}}\mathbf{L}\dot{\mathbf{T}}$ and $\overline{\mathbf{B}} = \mathbf{T}^{\mathrm{T}}\mathbf{B}$.

A. Notations

The full and diagonal matrices will be denoted as follows:

$$\underset{1:n}{\overset{i}{\left\|\left[\right.\right.}} R_{i,j} \underset{1:m}{\overset{j}{\left.\left.\right]\right\|}} = \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1m} \\ R_{21} & R_{22} & \cdots & R_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ R_{n1} & R_{n2} & \cdots & R_{nm} \end{bmatrix}, \quad \underset{1:n}{\overset{i}{\left\|\left[\right.\right.}} R_i \left.\left.\right]\right\| = \begin{bmatrix} R_1 & & & \\ & R_2 & & \\ & & \ddots & \\ & & & R_n \end{bmatrix}$$

The symbols $\underset{1:n}{\overset{i}{\left\|\left[\right.\right.}} R_i \left.\left.\right]\right\|$ and $\left\|\left[\right.\right. R_i \underset{1:n}{\overset{i}{\left.\left.\right]\right\|}}$ will denote the column and row matrices. The symbol $\sum_{n=a:d}^{b} c_n = c_a + c_{a+d} + c_{a+2d} + \ldots$ will be used to represent the sum of a succession of numbers c_n where the index n ranges from a to b with increment d. The symbol $\mathbf{0}$ and $\mathbf{1}$ will be used to represent zeros and ones block matrices of proper dimensions.

III. ELECTRICAL MOTORS MODELING

The basic structure of a multi-phase synchronous motor is shown in Fig. 3. In this paper we refer to a permanent magnet synchronous motor with an *odd* number m_s of star connected concentrated windings [8]-[9] characterized by the parameters shown in Tab. I. Saturation of iron will be neglected in this analysis. Let us introduce the following current and voltage stator vectors:

$${}^t\mathbf{I}_s = \begin{bmatrix} I_{s1} & I_{s2} & \cdots & I_{sm_s} \end{bmatrix}^{\mathrm{T}}, \quad {}^t\mathbf{V}_s = \begin{bmatrix} V_{s1} & V_{s2} & \cdots & V_{sm_s} \end{bmatrix}^{\mathrm{T}} \quad (3)$$

where V_{si} is the i-th phase voltage: $V_{si} = V_i - V_{s0}$.
Using a "Lagrangian" approach, see [9], the dynamic model S_t of the considered electric motor with respect to the external fixed frame Σ_t is the following:

$$\left[\begin{array}{c|c} {}^t\mathbf{L}_s & \mathbf{0} \\ \hline \mathbf{0} & J_m \end{array}\right] \left[\begin{array}{c} {}^t\dot{\mathbf{I}}_s \\ \hline \dot{\omega}_m \end{array}\right] = - \left[\begin{array}{c|c} {}^t\mathbf{R}_s & {}^t\mathbf{K}_\tau(\theta) \\ \hline -{}^t\mathbf{K}_\tau^{\mathrm{T}}(\theta) & b_m \end{array}\right] \left[\begin{array}{c} {}^t\mathbf{I}_s \\ \hline \omega_m \end{array}\right] + \left[\begin{array}{c} {}^t\mathbf{V}_s \\ \hline -\tau_e \end{array}\right] \quad (4)$$

where ${}^t\mathbf{R}_s = R_s\,\mathbf{I}_{m_s}$ and matrix ${}^t\mathbf{L}_s$ is defined as:

$${}^t\mathbf{L}_s = L_{s0}\,\mathbf{I}_{m_s} + {}^t\mathbf{M}_s = L_{s0}\,\mathbf{I}_{m_s} + M_{s0} \underset{1:m_s}{\overset{i}{\left[\!\left[\right.\right.}} \cos((i-h)\gamma_s) \underset{1:m_s}{\overset{h}{\left.\left.\right]\!\right]}}$$

with $L_{s0} = L_s - M_{s0}$. The rotor flux linkage vector is:

$$\boldsymbol{\Phi}_c(\theta) = \varphi_c \underset{1:m_s}{\overset{h}{\left\|\left[\right.\right.}} \sum_{n=1:2}^{\infty} a_n \cos[n(\theta - h\,\gamma_s)] \left.\left.\right]\right\| \quad (5)$$

where the parameters a_n are the coefficients of the normalized periodic rotor flux function $\bar{\phi}(\theta)$ expressed in Fourier series $\bar{\phi}(\theta) = \phi_c(\theta)/\varphi_c = \sum_{n=1:2}^{\infty} a_n \cos(n\theta)$. The torque vector ${}^t\mathbf{K}_\tau(\theta)$ is:

$${}^t\mathbf{K}_\tau(\theta) = \frac{\partial\, {}^t\boldsymbol{\Phi}_c(p\,\theta_m)}{\partial\theta_m} = p\,\varphi_c \underset{1:m_s}{\overset{h}{\left\|\left[\right.\right.}} -\sum_{n=1:2}^{\infty} n\,a_n \sin\left(n(\theta - h\gamma_s)\right) \left.\left.\right]\right\|. \quad (6)$$

The torque τ_m and the back-electromotive force ${}^t\mathbf{E}_s$ are:

$$\tau_m = {}^t\mathbf{K}_\tau^{\mathrm{T}}\,{}^t\mathbf{I}_s\,, \qquad {}^t\mathbf{E}_s = {}^t\mathbf{K}_\tau \omega_m = \underset{1:m_s}{\overset{h}{\left[\!\left[\right.\right.}} E_{sh} \left.\left.\right]\!\right].$$

The POG block scheme of the synchronous motor in the fixed reference frame Σ_t, see eq. (4), is shown in Fig. 4. The *elaboration blocks* between power sections ① and ② represent the *Electrical part* of the system, while the blocks between sections ③ and ④ represent the *Mechanical part* of the system. The *connection block* between sections ② and ③ represents the energy and power conversion between the electrical and mechanical parts of the motor.

IV. ELECTRICAL MOTORS MODELING IN OPEN-PHASE FAULT CONDITION

In this section the model of the motor in open-phase fault condition is introduced. The simulation of an open-phase fault has been proposed in [10] where the open circuited phase failure is simulated by a variable resistor as opening element. However using this method a system time constant becomes very high then the simulation needs a very small step thus increasing the simulation time. To overcome this problem we propose to simulate the open-phase failure by supplying the faulty phase with an additional voltage such that its steady-state current is zero. In order to exploit this approach in the fixed reference frame it is necessary to compute the common voltage V_{s0}, but this calculation is quite complex. Moreover the star connection constraint must be taken into account. To overcome these problems a transformation is now introduced. Let us consider the

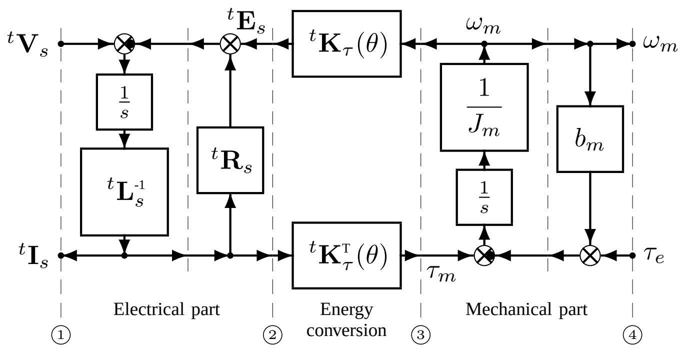


Fig. 4. POG block scheme of the dynamic model of a multi-phase synchronous motor in the fixed reference frame Σ_t.

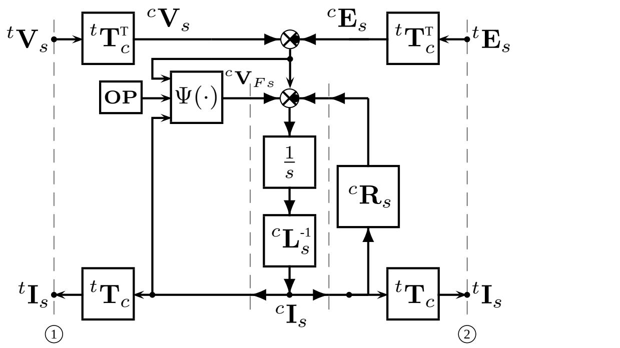


Fig. 5. POG scheme of the electrical part of the faulty motor.

following transformation matrix ${}^{\underline{t}}\mathbf{T}_c$ and its inverse:

$$ {}^{\underline{t}}\mathbf{T}_c = \left[\begin{array}{c|c} \mathbf{I}_{m_s-1} & \mathbf{0} \\ \hline -\mathbf{1} & 1 \end{array}\right], \qquad {}^{\underline{t}}\mathbf{T}_c^{-1} = \left[\begin{array}{c|c} \mathbf{I}_{m_s-1} & \mathbf{0} \\ \hline \mathbf{1} & 1 \end{array}\right]. $$

Applying the transformation ${}^{\underline{t}}\mathbf{T}_c$ to the electrical part of system (4), one obtains the following transformed current and voltage vectors ${}^{\underline{c}}\mathbf{I}_s$ and ${}^{\underline{c}}\mathbf{V}_s$:

$$ {}^{\underline{c}}\mathbf{I}_s = {}^{\underline{t}}\mathbf{T}_c^{-1}\, {}^t\mathbf{I}_s = \left[\begin{array}{cccc|c} I_{s1} & I_{s2} & \ldots & I_{sm_s-1} & \sum_{i=1}^{m_s} I_{si} \end{array}\right]^{\mathrm{T}} $$

$$ {}^{\underline{c}}\mathbf{V}_s = {}^{\underline{t}}\mathbf{T}_c^{\mathrm{T}}\, {}^t\mathbf{V}_s = \left[\begin{array}{c} V_{s1}-V_{sm_s} \\ V_{s2}-V_{sm_s} \\ \vdots \\ V_{sm_s-1}-V_{sm_s} \\ \hline V_{sm_s}-V_{s0} \end{array}\right]. $$

Note that the last component of current vector ${}^{\underline{c}}\mathbf{I}_s$ represents the star-connection constraint and that only the last component of voltage vector ${}^{\underline{c}}\mathbf{V}_s$ is referred to the common voltage V_{s0}. When the multi-phase motor is star-connected the last component of vector ${}^{\underline{c}}\mathbf{I}_s$ is zero, therefore to satisfy this constraint it is necessary to eliminate the last column of matrix ${}^{\underline{t}}\mathbf{T}_c$ and consider only matrix ${}^t\mathbf{T}_c$ defined as:

$$ {}^t\mathbf{T}_c = {}^{\underline{t}}\mathbf{T}_c\, \mathbf{S}_{m_s} = \begin{bmatrix} \mathbf{I}_{m_s-1} \\ -\mathbf{1} \end{bmatrix} \quad \text{where} \quad \mathbf{S}_{m_s} = \begin{bmatrix} \mathbf{I}_{m_s-1} \\ \mathbf{0} \end{bmatrix}. $$

Applying transformation ${}^t\mathbf{T}_c$ to the electrical part of system (4), one obtains the following transformed system:

$$ {}^c\mathbf{L}_s\, {}^c\dot{\mathbf{I}}_s = -\,{}^c\mathbf{R}_s\, {}^c\mathbf{I}_s - {}^c\mathbf{E}_s + {}^c\mathbf{V}_s \tag{7} $$

characterized by the following transformed vectors ${}^c\mathbf{I}_s = \mathbf{S}_{m_s}^{\mathrm{T}}\, {}^{\underline{t}}\mathbf{T}_c^{-1}\, {}^t\mathbf{I}_s$, ${}^c\mathbf{V}_s = {}^t\mathbf{T}_c^{\mathrm{T}}\, {}^t\mathbf{V}_s$, ${}^c\mathbf{E}_s = {}^t\mathbf{T}_c^{\mathrm{T}}\, {}^t\mathbf{E}_s$ defined as:

$$ {}^c\mathbf{I}_s = \underset{1:m_s-1}{\overset{i}{[\![\; {}^cI_{si} \;]\!]}}, \quad {}^c\mathbf{V}_s = \underset{1:m_s-1}{\overset{i}{[\![\; {}^cV_{si} \;]\!]}}, \quad {}^c\mathbf{E}_s = \underset{1:m_s-1}{\overset{i}{[\![\; {}^cE_{si} \;]\!]}}, $$

where: ${}^cI_{si} = I_{si}$, ${}^cV_{si} = V_{si} - V_{sm_s}$, ${}^cE_{si} = E_{si} - E_{sm_s}$. The transformed matrices ${}^c\mathbf{R}_s = {}^t\mathbf{T}_c^{\mathrm{T}}\, {}^t\mathbf{R}_s\, {}^t\mathbf{T}_c$ and ${}^c\mathbf{L}_s = {}^t\mathbf{T}_c^{\mathrm{T}}\, {}^t\mathbf{L}_s\, {}^t\mathbf{T}_c$ have the following form:

$$ {}^c\mathbf{R}_s = \underset{1:m_s-1\;\; 1:m_s-1}{\overset{i \qquad k}{[\![\; {}^cR_{si,k} \;]\!]}} = R_s \underset{1:m_s-1\;\; 1:m_s-1}{\overset{i \qquad k}{\left[\!\left[\; 1+\delta_{i,k} \;\right]\!\right]}}, \quad {}^c\mathbf{L}_s = \underset{1:m_s-1\;\; 1:m_s-1}{\overset{i \qquad k}{[\![\; {}^cL_{si,k} \;]\!]}} $$

where $\delta_{i,k} = \{1 \text{ if } i = k,\ 0 \text{ otherwise}\}$ and

$$ {}^cL_{si,k} = L_{s0}(1+\delta_{i,k}) + 4M_{s0}\sin(k\tfrac{\gamma_s}{2})\sin(i\tfrac{\gamma_s}{2})\cos((i-k)\tfrac{\gamma_s}{2}). $$

Note that the transformed inductance matrix ${}^c\mathbf{L}_s$ is symmetric with respect to both the main and secondary diagonal:

$$ {}^cL_{si,k} = {}^cL_{sk,i} = {}^cL_{s(m-i)+1,(m-k)+1} = {}^cL_{s(m-k)+1,(m-i)+1}. $$

We consider two different fault cases: open-phase fault occurring on a single phase and on two phases (adjacent or not adjacent).

A. Open-phase fault on a single phase

Let i denote the index of the faulty phase. The i-th equation of system (7) can be written as:

$$ \sum_{k=1}^{m_s-1} {}^cL_{si,k}\, {}^c\dot{I}_{sk} = -\sum_{k=1}^{m_s-1} {}^cR_{si,k}\, {}^cI_{sk} - {}^cE_{si} + {}^cV_{si}. \tag{8} $$

When phase i is open the corresponding current I_{si} must be zero. To achieve this condition it is sufficient to add to phase i the following additional voltage ${}^{f_1}V_i$:

$$ {}^{f_1}V_i = \sum_{k=1,k\neq i}^{m_s-1} {}^cL_{si,k}\, {}^c\dot{I}_{sk} + \sum_{k=1,k\neq i}^{m_s-1} {}^cR_{si,k}\, {}^cI_{sk} + {}^cE_{si} - {}^cV_{si}. \tag{9} $$

In fact, adding (9) to the left part of equation (8) one obtains:

$$ {}^cL_{si,i}\, {}^c\dot{I}_{si} = -2R_s\, {}^cI_{si}. \tag{10} $$

This equation guarantees ${}^cI_{si} = I_{si} = 0$ in steady state condition without modifying the value of R_s. Putting ${}^c\dot{I}_{si} = {}^cI_{si} = 0$ in system (7) and eliminating the i-th equation one obtains the following reduced system:

$$ {}^{f_1}\mathbf{L}_s\, {}^{f_1}\dot{\mathbf{I}}_s = -\,{}^{f_1}\mathbf{R}_s\, {}^{f_1}\mathbf{I}_s - {}^{f_1}\mathbf{E}_s + {}^{f_1}\mathbf{V}_s. \tag{11} $$

This system can also be obtained from (7) using the transformation ${}^c\mathbf{I}_s = {}^{f_1}\mathbf{T}_c^{\mathrm{T}}\, {}^{f_1}\mathbf{I}_c$, with matrix ${}^{f_1}\mathbf{T}_c$ defined as:

$$ {}^{f_1}\mathbf{T}_c = \left[\mathbf{e}_1^{m_s-1} \ldots \mathbf{e}_{i-1}^{m_s-1}\ \mathbf{e}_{i+1}^{m_s-1} \ldots \mathbf{e}_{m_s-1}^{m_s-1}\right] \tag{12} $$

where $\mathbf{e}_h^{m_s-1}$ is the h-th vector of the standard basis of space $\mathbb{R}^{m_s-1}$. The transformed vectors ${}^{f_1}\mathbf{I}_s$, ${}^{f_1}\mathbf{V}_s$ and ${}^{f_1}\mathbf{E}_s$, belonging to $\mathbb{R}^{m_s-2}$, are obtained from vectors ${}^c\mathbf{I}_s$, ${}^c\mathbf{V}_s$ and ${}^c\mathbf{E}_s$ eliminating their i-th component:

$$ {}^{f_1}\mathbf{I}_s = {}^{f_1}\mathbf{T}_c^{\mathrm{T}}\, {}^c\mathbf{I}_s, \quad {}^{f_1}\mathbf{V}_s = {}^{f_1}\mathbf{T}_c^{\mathrm{T}}\, {}^c\mathbf{V}_s, \quad {}^{f_1}\mathbf{E}_s = {}^{f_1}\mathbf{T}_c^{\mathrm{T}}\, {}^c\mathbf{E}_s. $$

Similarly, the transformed inductance and resistance matrices ${}^{f_1}\mathbf{R}_s$ and ${}^{f_1}\mathbf{L}_s$ are obtained from ${}^{c}\mathbf{R}_s$ and ${}^{c}\mathbf{L}_s$ eliminating the i-th row and the i-th column:

$$ {}^{f_1}\mathbf{R}_s = {}^{f_1}\mathbf{T}_c^{\mathrm{T}}\, {}^{c}\mathbf{R}_s\, {}^{f_1}\mathbf{T}_c \ , \qquad {}^{f_1}\mathbf{L}_s = {}^{f_1}\mathbf{T}_c^{\mathrm{T}}\, {}^{c}\mathbf{L}_s\, {}^{f_1}\mathbf{T}_c. $$

Equation (9) can be rewritten in vectorial notation as:

$$ {}^{f_1}V_i = (\mathbf{e}_i^{m_s-1})^{\mathrm{T}} \Big[{}^{c}\mathbf{L}_s\, {}^{f_1}\mathbf{T}_c\, {}^{f_1}\dot{\mathbf{I}}_s + {}^{c}\mathbf{R}_s\, {}^{f_1}\mathbf{T}_c\, {}^{f_1}\mathbf{I}_s + {}^{c}\mathbf{E}_s - {}^{c}\mathbf{V}_s \Big]. \tag{13} $$

where the time derivative of the reduced current vector ${}^{f_1}\dot{\mathbf{I}}_s$ can be calculated from the reduced system (11) as follows:

$$ {}^{f_1}\dot{\mathbf{I}}_s = {}^{f_1}\mathbf{L}_s^{-1}(-\,{}^{f_1}\mathbf{R}_s\, {}^{f_1}\mathbf{I}_s - {}^{f_1}\mathbf{E}_s + {}^{f_1}\mathbf{V}_s). \tag{14} $$

So, the open-phase condition ${}^{c}I_{si} = I_{si} = 0$ can be obtained adding the following voltage vector ${}^{c}\mathbf{V}_{Fs}$ to system (7):

$$ {}^{c}\mathbf{V}_{Fs} = \big[\, 0 \cdots 0 \;\; {}^{f_1}V_i \;\; 0 \cdots 0 \,\big]^{\mathrm{T}} \tag{15} $$

where ${}^{f_1}V_i$ is obtained substituting (14) in (13).

B. Open-phase fault on two phases

Using the same method it is possible to simulate also a second fault occurring at the j-th phase. In order to obtain the additional voltage ${}^{f_2}V_j$ it is necessary to consider the following reduced system:

$$ {}^{f_2}\mathbf{L}_s\, {}^{f_2}\dot{\mathbf{I}}_s = -\,{}^{f_2}\mathbf{R}_s\, {}^{f_2}\mathbf{I}_s - {}^{f_2}\mathbf{E}_s + {}^{f_2}\mathbf{V}_s \tag{16} $$

obtained from (7) using a transformation matrix ${}^{f_2}\mathbf{T}_c$ which is equal to the identity matrix of dimension m_s without the i-th and j-th columns. The reduced vectors ${}^{f_2}\mathbf{I}_s = {}^{f_2}\mathbf{T}_c^{\mathrm{T}}\, {}^{c}\mathbf{I}_s$, ${}^{f_2}\mathbf{V}_s = {}^{f_2}\mathbf{T}_c^{\mathrm{T}}\, {}^{c}\mathbf{V}_s$ and ${}^{f_2}\mathbf{E}_s = {}^{f_2}\mathbf{T}_c^{\mathrm{T}}\, {}^{c}\mathbf{E}_s$, belonging to $\mathbb{R}^{m_s-3}$, are obtained from vectors ${}^{c}\mathbf{I}_s$, ${}^{c}\mathbf{V}_s$ and ${}^{c}\mathbf{E}_s$ eliminating the i-th and j-th components, while the reduced inductance and resistance matrices ${}^{f_2}\mathbf{R}_s = {}^{f_2}\mathbf{T}_c^{\mathrm{T}}\, {}^{c}\mathbf{R}_s\, {}^{f_2}\mathbf{T}_c$ and ${}^{f_2}\mathbf{L}_s = {}^{f_2}\mathbf{T}_c^{\mathrm{T}}\, {}^{c}\mathbf{L}_s\, {}^{f_2}\mathbf{T}_c$ are obtained from ${}^{c}\mathbf{R}_s$ and ${}^{c}\mathbf{L}_s$ eliminating the i-th and j-th rows, and the i-th and j-th columns. The additional voltage ${}^{f_2}V_j$ to be added to the j-th phase of system (7) is:

$$ {}^{f_2}V_j = (\mathbf{e}_j^{m_s-1})^{\mathrm{T}} \Big[{}^{c}\mathbf{L}_s\, {}^{f_2}\mathbf{T}_c\, {}^{f_2}\dot{\mathbf{I}}_s + {}^{c}\mathbf{R}_s\, {}^{f_2}\mathbf{T}_c\, {}^{f_2}\mathbf{I}_s + {}^{c}\mathbf{E}_s - {}^{c}\mathbf{V}_s \Big]. \tag{17} $$

where the time derivative of the reduced current vector in double fault condition ${}^{f_2}\dot{\mathbf{I}}_s$ can be calculated as:

$$ {}^{f_2}\dot{\mathbf{I}}_s = {}^{f_2}\mathbf{L}_s^{-1}(-\,{}^{f_2}\mathbf{R}_s\, {}^{f_2}\mathbf{I}_s - {}^{f_2}\mathbf{E}_s + {}^{f_2}\mathbf{V}_s). $$

In this case, in order to have the current ${}^{c}I_{si} = I_{si} = 0$, the additional voltage ${}^{f_1}V_i$ given in (13) must be recalculated taking into account the additional voltage ${}^{f_2}V_j$ computed in (17) for phase j, then the vector ${}^{f_1}\dot{\mathbf{I}}_s$ must be recalculated adding vector ${}^{f_1}\mathbf{V}_{Fs}$ to the voltage vector ${}^{f_1}\mathbf{V}_s$:

$$ {}^{f_1}\dot{\mathbf{I}}_s = {}^{f_1}\mathbf{L}_s^{-1}(-\,{}^{f_1}\mathbf{R}_s\, {}^{f_1}\mathbf{I}_s - {}^{f_1}\mathbf{E}_s + {}^{f_1}\mathbf{V}_s + {}^{f_1}\mathbf{V}_{Fs}) \tag{18} $$

Vector ${}^{f_1}\mathbf{V}_{Fs} \in \mathbb{R}^{m_s-1}$ is: ${}^{f_1}\mathbf{V}_{Fs} = \big[\, 0 \cdots 0 \;\; {}^{f_2}V_j \;\; 0 \cdots 0 \,\big]^{\mathrm{T}}$ where the only component different from zero of ${}^{f_1}\mathbf{V}_{Fs}$ is the j-th component ${}^{f_2}V_j$ related to the second broken phase. Note that it is still possible to calculate ${}^{f_1}V_i$ using (13) because the adding term $(\mathbf{e}_i^{m_s-1})^{\mathrm{T}}\, {}^{f_1}\mathbf{T}_c\, {}^{f_1}\mathbf{V}_{Fs}$ is zero. Now the voltage vector ${}^{c}\mathbf{V}_{Fs}$ added to system (7) is:

$$ {}^{c}\mathbf{V}_{Fs} = \big[\, 0 \cdots 0 \;\; {}^{f_1}V_i \;\; 0 \cdots 0 \;\; {}^{f_2}V_j \;\; 0 \cdots 0 \,\big]^{\mathrm{T}} \tag{19} $$

where the only two components different from zero are ${}^{f_1}V_i$ and ${}^{f_1}V_j$. Note that in (19) the relative position of the two non-zero elements depends on the values of indices i and j. The POG block scheme representing the dynamics of the electrical part of the motor in fault condition is shown in Fig. 5: this part substitutes the electrical part of the POG model motor shown in Fig. 4. The element denoted by $\mathbf{OP}$ is an internal input which defines the instants at which the faults occur and the indexes of the open phases. The block $\Psi(\cdot)$ is the function that calculates the voltage vector ${}^{c}\mathbf{V}_{Fs}$ added to the system starting from the vectors ${}^{c}\mathbf{I}_s$, ${}^{c}\mathbf{V}_s$ and ${}^{c}\mathbf{E}_s$. In healthy condition vector ${}^{c}\mathbf{V}_{Fs}$ is zero, while in faulty condition it is given by equations (15) or (19) and it is the voltage vector which guarantees zero currents through the faulty phases.

C. Example: 5-phase motor

Let consider the example of a 5-phase motor where a first failure occurs at time t_1 on the phase $i = 2$ and a second failure occurs at time $t_2 > t_1$ on the phase $j = 4$. The transformation matrices ${}^{t}\mathbf{T}_c$, ${}^{f1}\mathbf{T}_c$ and ${}^{f2}\mathbf{T}_c$ have the following structure:

$$ {}^{t}\mathbf{T}_c = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & -1 & -1 & -1 \end{bmatrix}, \quad {}^{f_1}\mathbf{T}_c = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad {}^{f_2}\mathbf{T}_c = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}. $$

The transformed system (7) can be written explicitly as:

$$ \begin{bmatrix} L_{11} & L_{12} & L_{13} & L_{14} \\ L_{12} & L_{22} & L_{23} & L_{13} \\ L_{13} & L_{23} & L_{22} & L_{12} \\ L_{14} & L_{13} & L_{12} & L_{11} \end{bmatrix} \begin{bmatrix} \dot{I}_{s1} \\ \dot{I}_{s2} \\ \dot{I}_{s3} \\ \dot{I}_{s4} \end{bmatrix} = - \begin{bmatrix} 2R_s & R_s & R_s & R_s \\ R_s & 2R_s & R_s & R_s \\ R_s & R_s & 2R_s & R_s \\ R_s & R_s & R_s & 2R_s \end{bmatrix} \begin{bmatrix} I_{s1} \\ I_{s2} \\ I_{s3} \\ I_{s4} \end{bmatrix} - \begin{bmatrix} E_{s1}-E_{s5} \\ E_{s2}-E_{s5} \\ E_{s3}-E_{s5} \\ E_{s4}-E_{s5} \end{bmatrix} + \begin{bmatrix} V_{s1}-V_{s5} \\ V_{s2}-V_{s5} \\ V_{s3}-V_{s5} \\ V_{s4}-V_{s5} \end{bmatrix}. \tag{20} $$

When $t_1 \le t \le t_2$, using the congruent transformation ${}^{f_1}\mathbf{I}_s = {}^{f_1}\mathbf{T}_c\, {}^{c}\mathbf{I}_s$ one obtains the following reduced system:

$$ \underbrace{\begin{bmatrix} L_{11} & L_{13} & L_{14} \\ L_{13} & L_{22} & L_{12} \\ L_{14} & L_{12} & L_{11} \end{bmatrix}}_{{}^{f_1}\mathbf{L}_s} \underbrace{\begin{bmatrix} \dot{I}_{s1} \\ \dot{I}_{s3} \\ \dot{I}_{s4} \end{bmatrix}}_{{}^{f_1}\dot{\mathbf{I}}_s} = - \underbrace{\begin{bmatrix} 2R_s & R_s & R_s \\ R_s & 2R_s & R_s \\ R_s & R_s & 2R_s \end{bmatrix}}_{{}^{f_1}\mathbf{R}_s} \underbrace{\begin{bmatrix} I_{s1} \\ I_{s3} \\ I_{s4} \end{bmatrix}}_{{}^{f_1}\mathbf{I}_s} - \underbrace{\begin{bmatrix} E_{s1}-E_{s5} \\ E_{s3}-E_{s5} \\ E_{s4}-E_{s5} \end{bmatrix}}_{{}^{f_1}\mathbf{E}_s} + \underbrace{\begin{bmatrix} V_{s1}-V_{s5} \\ V_{s3}-V_{s5} \\ V_{s4}-V_{s5} \end{bmatrix}}_{{}^{f_1}\mathbf{V}_s}. $$

which is obtained from the previous one eliminating the 2nd row and the 2nd column. The time derivative ${}^{f1}\dot{\mathbf{I}}_s$ of the current vector necessary to calculate the voltage ${}^{f_1}V_2$ defined in (13) can be obtained using eq. (14). The voltage ${}^{f_1}V_2$ has the following structure:

$$ \begin{aligned} {}^{f_1}V_2 &= (\mathbf{e}_2^4)^{\mathrm{T}} \Big[{}^{c}\mathbf{L}_s\, {}^{f_1}\mathbf{T}_c\, {}^{f_1}\dot{\mathbf{I}}_s + {}^{c}\mathbf{R}_s\, {}^{f_1}\mathbf{T}_c\, {}^{f_1}\mathbf{I}_s + {}^{c}\mathbf{E}_s - {}^{c}\mathbf{V}_s \Big] \\ &= L_{12}\dot{I}_{s1} + L_{23}\dot{I}_{s3} + L_{13}\dot{I}_{s4} + R_s I_{s1} + R_s I_{s3} + \\ &\quad + R_s I_{s4} + E_{s2} - E_{s5} - V_{s2} + V_{s5}. \end{aligned} $$

The vector ${}^{c}\mathbf{V}_{Fs}$ to be added to system (7) is: ${}^{c}\mathbf{V}_{Fs} = \begin{bmatrix} 0 & {}^{f_1}V_2 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$. In this case equation (10) is: $L_{22}\,{}^{c}\dot{I}_{s2} = -2R_s\,{}^{c}I_{s2}$ and the condition ${}^{c}I_{s2} = I_{s2} = 0$ is reached with settling time $T_a = 3\,L_{22}/2R_s$.

When $t > t_2$ the reduced system (11) has the following structure:

$$\underbrace{\begin{bmatrix} L_{11} & L_{13} \\ L_{13} & L_{22} \end{bmatrix}}_{{}^{f_2}\mathbf{L}_s}\underbrace{\begin{bmatrix} \dot{I}_{s1} \\ \dot{I}_{s3} \end{bmatrix}}_{{}^{f_2}\dot{\mathbf{I}}_s} = -\underbrace{\begin{bmatrix} 2R_s & R_s \\ R_s & 2R_s \end{bmatrix}}_{{}^{f_2}\mathbf{R}_s}\underbrace{\begin{bmatrix} I_{s1} \\ I_{s3} \end{bmatrix}}_{{}^{f_2}\mathbf{I}_s} - \underbrace{\begin{bmatrix} E_{s1}-E_{s5} \\ E_{s3}-E_{s5} \end{bmatrix}}_{{}^{f_2}\mathbf{E}_s} + \underbrace{\begin{bmatrix} V_{s1}-V_{s5} \\ V_{s3}-V_{s5} \end{bmatrix}}_{{}^{f_2}\mathbf{V}_s}.$$

which can be obtained from (20) eliminating the 2nd and 4th rows and columns, i.e. by applying the transformation ${}^{f_2}\mathbf{T}_c$. The voltage ${}^{f_2}V_4$ is given by:

$$\begin{aligned} {}^{f_2}V_4 &= (\mathbf{e}_4^4)^{\mathrm{T}}\left[{}^{c}\mathbf{L}_s\,{}^{f_2}\mathbf{T}_c\,{}^{f_2}\dot{\mathbf{I}}_s + {}^{c}\mathbf{R}_s\,{}^{f_2}\mathbf{T}_c\,{}^{f_2}\mathbf{I}_s + {}^{c}\mathbf{E}_s - {}^{c}\mathbf{V}_s\right] \\ &= L_{14}\dot{I}_{s1} + L_{12}\dot{I}_{s3} + R_s I_{s1} + R_s I_{s3} + \\ &\quad + E_{s4} - E_{s5} - V_{s4} + V_{s5} \end{aligned}$$

The new additional voltage ${}^{f_1}V_2$ applied to the first broken phase 2 must be recalculated considering also the additional voltage ${}^{f_2}V_4$ given to phase 4. In this case the new time derivative of current vector ${}^{f1}\dot{\mathbf{I}}_s$ is obtained using (18) and considering the following reduced system:

$$\begin{bmatrix} L_{11} & L_{13} & L_{14} \\ L_{13} & L_{22} & L_{12} \\ L_{14} & L_{12} & L_{11} \end{bmatrix}\begin{bmatrix} \dot{I}'_{s1} \\ \dot{I}'_{s3} \\ \dot{I}'_{s4} \end{bmatrix} = -\begin{bmatrix} 2R_s & R_s & R_s \\ R_s & 2R_s & R_s \\ R_s & R_s & 2R_s \end{bmatrix}\begin{bmatrix} I_{s1} \\ I_{s3} \\ I_{s4} \end{bmatrix} - \begin{bmatrix} E_{s1}-E_{s5} \\ E_{s3}-E_{s5} \\ E_{s4}-E_{s5} \end{bmatrix} + \begin{bmatrix} V_{s1}-V_{s5} \\ V_{s3}-V_{s5} \\ V_{s4}-V_{s5} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ {}^{f_2}V_4 \end{bmatrix}.$$

Hence the new voltage ${}^{f_1}V_2$ has the following structure:

$$\begin{aligned} {}^{f_1}V_2 = {} & L_{12}\dot{I}'_{s1} + L_{23}\dot{I}'_{s3} + L_{13}\dot{I}'_{s4} + R_s I_{s1} + R_s I_{s3} + \\ & + R_s I_{s4} + E_{s2} - E_{s5} - V_{s2} + V_{s5}, \end{aligned}$$

in which the new components $\dot{I}'_{s1}$, $\dot{I}'_{s3}$ and $\dot{I}'_{s4}$ of vector ${}^{f1}\dot{\mathbf{I}}_s$ obtained from (18) are different from the previous one $\dot{I}_{s1}$, $\dot{I}_{s3}$ and $\dot{I}_{s4}$. Using ${}^{f_2}V_4$ and ${}^{f_1}V_2$ calculated above the vector ${}^{c}\mathbf{V}_{Fs}$ to be added to system (7) is ${}^{c}\mathbf{V}_{Fs} = \begin{bmatrix} 0 & {}^{f_1}V_2 & 0 & {}^{f_2}V_4 \end{bmatrix}^{\mathrm{T}}$. In this way the steady state condition $I_{s2} = I_{s4} = 0$ is achieved.

V. SIMULATION

The POG model of the electric motor shown in Fig. 4 with the electrical part modeled as in Fig. 5 has been implemented in Simulink. The motor considered for simulations is a 5-phase permanent magnet synchronous motor. The main electrical and mechanical parameters are the following: $m_s = 5$, $p = 1$, $R_s = 2\,\Omega$, $L_s = 0.03$ H, $M_{s0} = 0.02$ H, $\varphi_r = 0.02$ Wb, $J_m = 1.6$ kg m^2, $b_m = 0.8$ Nm s/rad, $V_{max} = 100$ V, $a_1 = 0.87$, $a_3 = 0.13$ and the external torque $\tau_e = 0$ Nm. The motor is controlled using a vector-field control described in [6]. The simulation results shown in Fig. 6-8 correspond to the case of two open-phase faults occurring on two non adjacent phases, in particular phase $i = 2$ opens at time $t = 8$ s and phase $j = 4$ opens at time $t = 12$ s. Fig. 6 shows the stator currents ${}^{t}\mathbf{I}_s$, the sum of stator currents (always equal to zero because the motor is star-connected) and the number of faulty phases ($i = 2$, $j = 4$). The zoom of the stator currents ${}^{t}\mathbf{I}_s$ when the faults

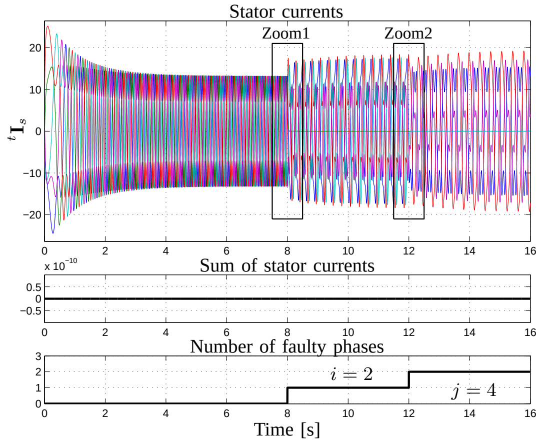


Fig. 6. a) Stator currents ${}^{t}\mathbf{I}_s$, b) Sum of stator currents, c) Number of faulty phases ($i = 2$, $j = 4$).

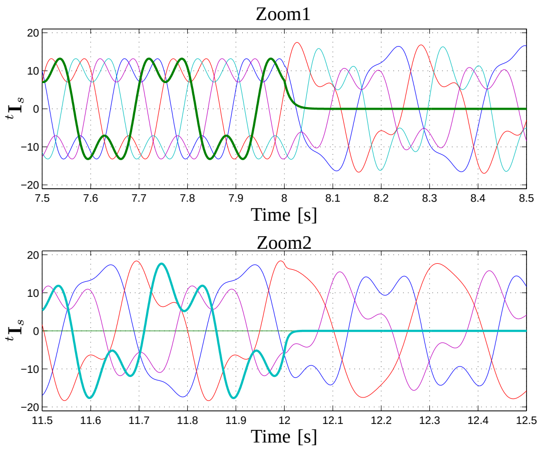


Fig. 7. Zoom of stator currents when the faults occur: phase $i = 2$ opens at time $t = 8$ s and phase $j = 4$ opens at time $t = 12$ s.

occur is shown in Fig. 7: note that at time $t = 8$ s the phase 2 opens and the current I_{s2} goes to zero, then at time $t = 12$ s the phase 4 opens and the current I_{s4} goes to zero. The motor velocity ω_m and the motor torque τ_m (with its mean value) are shown in Fig. 8. Note that when the first open-phase fault occurs the mean value of the torque reduces to the 82.9% and the torque ripple is 56.6%. When the second fault occurs on a non adjacent phase the mean value of the torque reduces to the 64.9% and the torque ripple is 57.6%. The second set of simulation results is shown in Fig. 9-11, corresponding to the case of open-phase fault occurring on two adjacent phases: phase $i = 3$ opens at $t = 8$ s and phase $j = 2$ opens at $t = 12$ s. Fig. 9 shows the stator currents ${}^{t}\mathbf{I}_s$, the sum of stator currents and the number of faulty phases. The zoom of stator currents when the faults occur is shown in Fig. 10. The motor velocity ω_m and the motor torque τ_m (with its mean value) are shown in Fig. 11. Note that when the first open-phase fault occurs the mean value of the torque reduces to the 82.8% and the torque ripple is 56.7%. When the second fault occurs on an adjacent phase the torque mean value reduces to the 54.9% and the torque ripple is 192.4%.

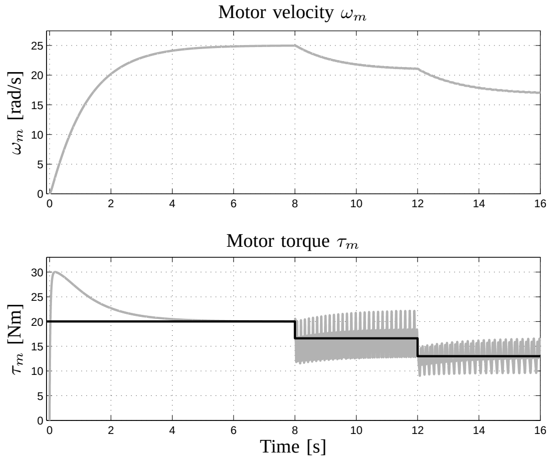


Fig. 8. Motor velocity ω_m and motor torque τ_m (the solid line indicates the mean value of the torque).

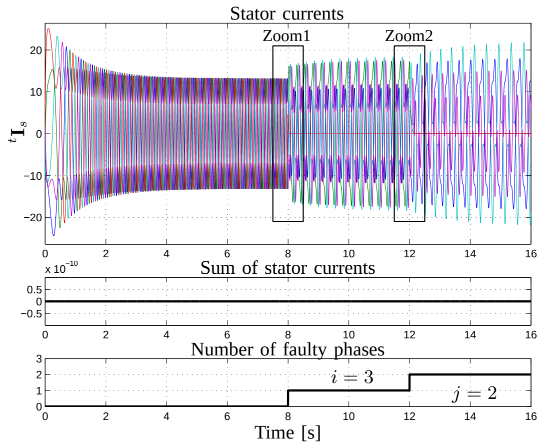


Fig. 9. a) Stator currents ${}^t\mathbf{I}_s$, b) Sum of stator currents, c) Number of faulty phases ($i = 3$, $j = 2$).

VI. CONCLUSIONS

In this paper the modeling of multi-phase permanent magnet synchronous motors under open-phase fault condition has been investigated. The proposed model is suitable for multi-phase motors with generic number of phases and the open circuit failure can occur to any of the phases. The model of the motor can be used for failures occurring on one single phase or more phases and for adjacent and not adjacent phases. The use of the POG modeling technique allows to directly implement the model in Simulink. Using the proposed approach the simulation time is quite short. Some simulation results show the effectiveness of the proposed model in case of open-phase fault of two adjacent and non adjacent phases of a five phase motor. The use of this model is very useful both for simulations and implementation of fault-tolerant control strategies.

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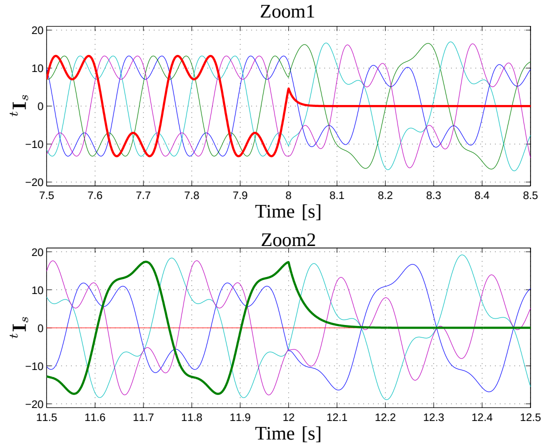

Fig. 10. Zoom of stator currents when the faults occur: phase $i = 3$ opens at time $t = 8$ s and phase $j = 2$ opens at time $t = 12$ s.

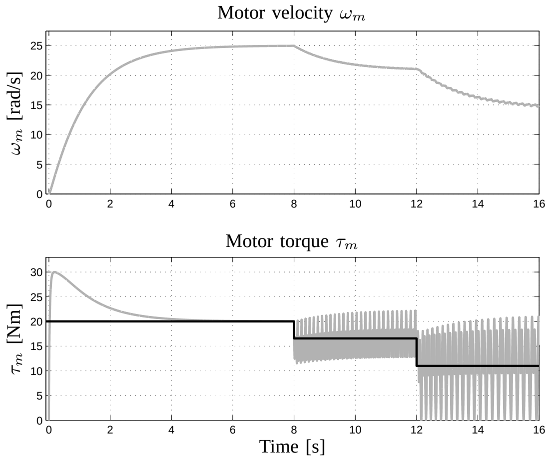

Fig. 11. Motor velocity ω_m and motor torque τ_m (the solid line indicates the mean value of the torque).

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