STAR AND DELTA COMPLEX DYNAMIC MODELS OF MULTI-PHASE SYNCHRONOUS MOTORS

R. Zanasi, M. Fei

Information Engineering Department, University of Modena and Reggio Emilia,
Via Vignolese 905, 41100 Modena, Italy
e-mail: roberto.zanasi@unimore.it , marco.fei@unimore.it

Abstract - In the paper the model and the control of multi-phase permanent magnet synchronous machines with an arbitrary number of star or delta connected phases and an arbitrary shape of the rotor flux are proposed. Using a vectorial approach the optimal current references minimizing the dissipation are obtained and used in the control law. This approach is suitable for both star and delta connected stator phases. Some simulation results end the paper.

Keyword - Multi-phase synchronous motor, Star and Delta connection, Power-Oriented Graphs.

1 INTRODUCTION

In the literature the dynamic model of multi-phase machines has been obtained applying a variety of transformations to the phase variables, see [1], [2] and [3]. In [1] the equivalence of a m-phase wye-connected machine to a set of $(m-1)/2$ fictitious independent two-phase machines is shown. This concept is also used in [2] in the case of five-phase machine. In [3] a complex transformation is used to reduce the number of the dynamic equations. In this paper the dynamic model of the motor is obtained applying a power invariant complex state-space transformation in the frame of the POG modeling technique, see [4],[5] and [6]. Since the type of stator connection modifies the relation between the terminal variables and the phase ones, in order to obtain a model as general as possible both the star and delta stator connections must be investigated. The main differences between the two types of connections are clearly shown in this paper. Moreover the optimal current references minimizing the dissipation are obtained using a vectorial approach. The proposed control law provides the desired torque and it is suitable whatever the type of stator connection is.

The paper is organized as follows. Sec. 2 shows the details of the complex POG dynamic model of the multi-phase synchronous motors, Sec. 3 contains the comparison between star and delta connected motors and Sec. 4 deals with the new vectorial torque control. Finally, some simulation results are presented in Sec. 5 and conclusions are given in Sec. 6.

1.1 NOTATIONS

Given a complex matrix $\mathbf{A}$, the conjugate matrix will be denoted by $\mathbf{A}^\circ$, the transpose matrix by $\mathbf{A}^{\mathrm{T}}$ and

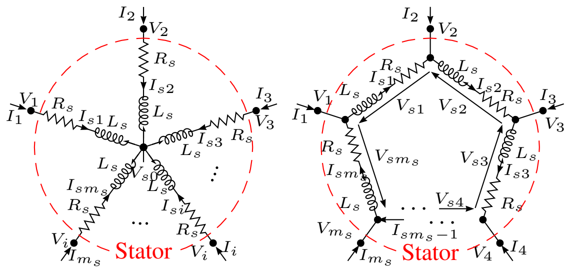


Fig. 1. Star and delta-connected stator phases.

the conjugate transpose matrix by $\mathbf{A}^*$. The following relations hold: $\mathbf{A}^* = (\mathbf{A}^\circ)^{\mathrm{T}} = (\mathbf{A}^{\mathrm{T}})^\circ$. The full, diagonal, column and row matrices will be denoted as follows:

$$\Big[\!\Big[R_{i,j} \Big]\!\Big]_{1:n}^{i}{}_{1:m}^{j} = \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1m} \\ R_{21} & R_{22} & \cdots & R_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ R_{n1} & R_{n2} & \cdots & R_{nm} \end{bmatrix}, \quad \Big[\!\Big[R_i \Big]\!\Big]_{1:n}^{i}, \quad \Big[\!\Big[R_i \Big]\!\Big|_{1:n}^{i}, \quad \Big[\!\Big[R_i \Big]\!\Big]_{1:n}^{i}.$$

The symbol $\sum_{n=a:d}^{b} c_n = c_a + c_{a+d} + c_{a+2d} + \ldots$ will be used to represent the sum of a succession of numbers c_n. The symbol $\mathbf{0}$ will be used to represent a zero block matrix of proper dimensions.

2 ELECTRICAL MOTORS MODELLING

In this paper we refer to a permanent magnet synchronous motor with an *odd* number m_s of concentrated winding in star or delta connection, see Fig. 1. The considered motor is characterized by the parameters shown in Tab.I. Let ${}^t\mathbf{I}_l$, ${}^t\mathbf{V}_l$, ${}^t\mathbf{I}_s$ and ${}^t\mathbf{V}_s$ denote the following current and voltage terminal and phase vectors:

$${}^t\mathbf{I}_l = \begin{bmatrix} I_1 & \cdots & I_{m_s} \end{bmatrix}^{\mathrm{T}}, \quad {}^t\mathbf{V}_l = \begin{bmatrix} V_1 & \cdots & V_{m_s} \end{bmatrix}^{\mathrm{T}},$$

m_s	number of motor phases
p	number of polar expansions
θ, θ_m	electric and rotor angular positions: $\theta = p\theta_m$
ω, ω_m	electric and rotor angular velocities: $\omega = p\omega_m$
R_s	i-th stator phase resistance
L_s	i-th stator phase self induction coefficient
M_{s0}	maximum value of mutual inductance
$\phi_c(\theta)$	total rotor flux chained with stator phase 1
φ_c	maximum value of function $\phi_c(\theta)$
J_m	rotor moment of inertia
b_m	rotor linear friction coefficient
τ_m	electromotive torque acting on the rotor
τ_e	external load torque acting on the rotor
γ_s	basic angular displacement ($\gamma_s = 2\pi/m_s$)

I. Parameters of the multi-phase synchronous motor.

$$ {}^t\mathbf{I}_s = \begin{bmatrix} I_{s1} & \cdots & I_{sm_s} \end{bmatrix}^{\mathrm{T}}, \; {}^t\mathbf{V}_s = \begin{bmatrix} V_{s1} & \cdots & V_{sm_s} \end{bmatrix}^{\mathrm{T}}. $$

The following equations relate the phase vectors ${}^t\mathbf{V}_s$ and ${}^t\mathbf{I}_s$ to the terminal vectors ${}^t\mathbf{V}_l$ and ${}^t\mathbf{I}_l$:

$$ {}^t\mathbf{I}_l = {}^t\mathbf{T}\,{}^t\mathbf{I}_s, \qquad {}^t\mathbf{V}_s = {}^t\mathbf{T}^{\mathrm{T}}\,{}^t\mathbf{V}_l - {}^t\mathbf{V}_{s0}, \quad (1) $$

where the connection matrix ${}^t\mathbf{T}$ and the constant vector ${}^t\mathbf{V}_{s0}$ depend on the type of stator connection:

$$ {}^t\mathbf{T} = \begin{cases} \mathbf{I}_{m_s} & \text{if star-connected} \\ {}^t\mathbf{T}_\triangle & \text{if delta-connected} \end{cases} $$

$$ {}^t\mathbf{T}_\triangle = \begin{bmatrix} 1 & 0 & 0 \cdots & -1 \\ -1 & 1 & 0 \cdots & 0 \\ 0 & -1 & 1 \cdots & 0 \\ \vdots & \vdots & \vdots \; \ddots & \vdots \\ 0 & 0 & 0 \cdots & 1 \end{bmatrix} \quad (2) $$

$$ {}^t\mathbf{V}_{s0} = \begin{cases} [1,\ 1,\ \ldots,\ 1]^{\mathrm{T}} V_{s0} & \text{if star-connected} \\ [0,\ 0,\ \ldots,\ 0]^{\mathrm{T}} & \text{if delta-connected} \end{cases} $$

Referring to the state-space vector $[\,{}^t\mathbf{I}_s^{\mathrm{T}} \quad \omega_m]^{\mathrm{T}}$ and using a "Lagrangian" approach, see [4], the dynamic equations S_t of the synchronous motor expressed with respect to the phase fixed frame Σ_t are:

$$ \left[\begin{array}{c|c} {}^t\mathbf{L}_s & \mathbf{0} \\ \hline \mathbf{0} & J_m \end{array}\right] \left[\begin{array}{c} {}^t\dot{\mathbf{I}}_s \\ \hline \dot{\omega}_m \end{array}\right] = - \left[\begin{array}{c|c} {}^t\mathbf{R}_s & {}^t\mathbf{K}_\tau \\ \hline -{}^t\mathbf{K}_\tau^{\mathrm{T}} & b_m \end{array}\right] \left[\begin{array}{c} {}^t\mathbf{I}_s \\ \hline \omega_m \end{array}\right] + \left[\begin{array}{c} {}^t\mathbf{V}_s \\ \hline -\tau_e \end{array}\right] \quad (3) $$

where ${}^t\mathbf{R}_s = R_s\,\mathbf{I}_{m_s}$, inductance matrix ${}^t\mathbf{L}_s$ is:

$$ {}^t\mathbf{L}_s = L_{s0}\mathbf{I}_{m_s} + M_{s0} \left\| \left[\sum_{n=1:2}^{m_s-2} a_{Mn} \cos(n(i-h)\gamma_s) \right] \right\|_{1:m_s}^{i} {}_{1:m_s}^{h} \quad (4) $$

and torque vector ${}^t\mathbf{K}_\tau$ is:

$$ {}^t\mathbf{K}_\tau(\theta) = p\varphi_c \left\| \left[-\sum_{n=1:2}^{\infty} n a_n \sin[n(\theta - h\gamma_s)] \right] \right\|_{0:m_s-1}^{h} \quad (5) $$

According to [2], in (4) the first $m_s - 2$ odd components $a_{Mn}M_{s0}$ of the mutual inductance have been also considered. The parameters a_n in (5) are the coefficients of the periodic normalized rotor flux function $\bar{\phi}(\theta)$ expressed in Fourier series:

$$ \bar{\phi}(\theta) = \frac{\phi_c(\theta)}{\varphi_c} = \sum_{n=1:2}^{\infty} a_n \cos(n\theta). \quad (6) $$

The original m_s-dimension model in the fixed frame Σ_t is transformed and reduced to a $(m_s+1)/2$-dimension complex model in the complex rotating frame $\overline{\Sigma}_\omega$ using the following complex transformation matrix ${}^t\overline{\mathbf{T}}_{\omega N} \in C^{m_s \times \frac{m_s-1}{2}}$ (see [3] and [5]):

$$ {}^t\overline{\mathbf{T}}_{\underline{\omega} N} = {}^t\overline{\mathbf{T}}_{\underline{\omega}}\,\mathbf{N} \quad (7) $$

where ${}^t\overline{\mathbf{T}}_{\underline{\omega}}$ and $\mathbf{N}$ are defined as follows:

$$ {}^t\overline{\mathbf{T}}_{\underline{\omega}} = \sqrt{\frac{1}{m_s}} \left[\left\| \left[e^{jk(\theta - h\gamma_s)} \right] \right\|_{0:m_s-1}^{h} {}_{1:2:m_s-2}^{k} \quad \left[\!\left[1 \right]\!\right]_{0:m_s-1}^{h} \right], $$
$$ \mathbf{N} = \begin{bmatrix} \sqrt{2}\,\mathbf{I}_{\frac{m_s-1}{2}} & \mathbf{0} \\ \mathbf{0} & 1 \end{bmatrix}. \quad (8) $$

Applying the complex pseudo-transformation ${}^t\mathbf{I}_s = {}^t\overline{\mathbf{T}}_{\underline{\omega} N}\,{}^{\underline{\omega}}\overline{\mathbf{I}}_s$ to system (3), one obtains the following transformed system $\overline{S}_\omega$ in the complex and reduced rotating frame $\overline{\Sigma}_\omega$:

$$ \left[\begin{array}{c|c} {}^{\underline{\omega}}\overline{\mathbf{L}}_s & \mathbf{0} \\ \hline \mathbf{0} & J_m \end{array}\right] \left[\begin{array}{c} {}^{\underline{\omega}}\dot{\overline{\mathbf{I}}}_s \\ \hline \dot{\omega}_m \end{array}\right] = - \left[\begin{array}{c|c} {}^{\underline{\omega}}\overline{\mathbf{Z}}_s & {}^{\underline{\omega}}\overline{\mathbf{K}}_{\tau N} \\ \hline -{}^{\underline{\omega}}\overline{\mathbf{K}}_{\tau N}^{*} & b_m \end{array}\right] \left[\begin{array}{c} {}^{\underline{\omega}}\overline{\mathbf{I}}_s \\ \hline \omega_m \end{array}\right] + \left[\begin{array}{c} {}^{\underline{\omega}}\overline{\mathbf{V}}_s \\ \hline -\tau_e \end{array}\right] \quad (9) $$

where the complex matrix ${}^{\underline{\omega}}\overline{\mathbf{Z}}_s = {}^{\underline{\omega}}\overline{\mathbf{R}}_s + {}^{\underline{\omega}}\overline{\mathbf{L}}_s\,{}^{\underline{\omega}}\overline{\mathbf{J}}_s$ is obtained using ${}^{\underline{\omega}}\overline{\mathbf{R}}_s = {}^t\overline{\mathbf{T}}_{\underline{\omega}}^{*}\,{}^t\mathbf{R}_s\,{}^t\overline{\mathbf{T}}_{\underline{\omega}} = R_s\mathbf{I}_{\frac{m_s+1}{2}}$, ${}^{\underline{\omega}}\overline{\mathbf{L}}_s = {}^t\overline{\mathbf{T}}_{\underline{\omega}}^{*}\,{}^t\mathbf{L}_s\,{}^t\overline{\mathbf{T}}_{\underline{\omega}}$ and ${}^{\underline{\omega}}\overline{\mathbf{J}}_s = {}^t\overline{\mathbf{T}}_{\underline{\omega}}^{*}\,{}^t\dot{\overline{\mathbf{T}}}_{\underline{\omega}}$:

$$ {}^{\underline{\omega}}\overline{\mathbf{L}}_s = \begin{bmatrix} \left\| \left[L_{sk} \right] \right\|_{1:2:m_s-2}^{k} & \mathbf{0} \\ \mathbf{0} & L_{s0} \end{bmatrix}, \qquad {}^{\underline{\omega}}\overline{\mathbf{J}}_s = \begin{bmatrix} \left\| \left[jkp\omega_m \right] \right\|_{1:2:m_s-2}^{k} & \mathbf{0} \\ \mathbf{0} & 0 \end{bmatrix} $$

where $L_{sk} = L_{s0} + a_{Mk}\frac{m_s}{2}M_{s0}$ and $L_{s0} = L_s - M_{s0}$. These last equations show that the k-th component $a_{Mk}M_{s0}$ of the mutual inductance is related only to the subspace $\overline{\Sigma}_{\omega k}$. The transformed current and voltage vectors ${}^{\underline{\omega}}\overline{\mathbf{I}}_s = {}^t\overline{\mathbf{T}}_{\underline{\omega} N}^{*}\,{}^t\mathbf{I}_s$ and ${}^{\underline{\omega}}\overline{\mathbf{V}}_s = {}^t\overline{\mathbf{T}}_{\underline{\omega} N}^{*}\,{}^t\mathbf{V}_s$, which are function of the m_s stator currents and voltages I_{sh} and V_{sh}, have the following structure:

$$ {}^{\underline{\omega}}\overline{\mathbf{I}}_s = \begin{bmatrix} \left\| \left[\frac{e^{-jk\theta}}{\sqrt{m_s}} \sum_{h=1}^{m_s} I_{sh}\, e^{jk(h-1)\gamma_s} \right] \right\|_{1:2:m_s-2}^{k} \\ \sqrt{\frac{1}{m_s}} \sum_{h=1}^{m_s} I_{sh} \end{bmatrix} = \begin{bmatrix} \left\| \left[{}^{\omega}\overline{I}_{sk} \right] \right\|_{1:2:m_s-2}^{k} \\ {}^{\omega}\overline{I}_{sm_s} \end{bmatrix} $$

$$ {}^{\underline{\omega}}\overline{\mathbf{V}}_s = \begin{bmatrix} \left\|\left[\frac{e^{-jk\theta}}{\sqrt{m_s}} \sum_{h=1}^{m_s} V_{sh}\, e^{jk(h-1)\gamma_s} \right]\right\|_{1:2:m_s-2}^{k} \\ \sqrt{\frac{1}{m_s}} \sum_{h=1}^{m_s} V_{sh} \end{bmatrix} = \begin{bmatrix} \left\|\left[{}^{\omega}\overline{V}_{sk} \right]\right\|_{1:2:m_s-2}^{k} \\ {}^{\omega}\overline{V}_{sm_s} \end{bmatrix} $$

The complex components ${}^{\omega}\overline{I}_{sk}$ and ${}^{\omega}\overline{V}_{sk}$ of vectors ${}^{\underline{\omega}}\overline{\mathbf{I}}_s$ and ${}^{\underline{\omega}}\overline{\mathbf{V}}_s$ can be expressed as follows:

$$ {}^{\omega}\overline{I}_{sk} = I_{dk} + jI_{qk}, \qquad {}^{\omega}\overline{V}_{sk} = V_{dk} + jV_{qk} $$

where I_{dk}, I_{qk}, V_{dk} and V_{qk} are the direct and quadrature components of the current and voltage vectors. Note that the last components ${}^{\omega}\overline{I}_{sm_s}$ and ${}^{\omega}\overline{V}_{sm_s}$ of vectors ${}^{\underline{\omega}}\mathbf{I}_s$ and ${}^{\underline{\omega}}\mathbf{V}_s$are real and proportional to the sum of the m_s stator currents I_{sh} and the m_s stator voltages V_{sh}, respectively. The transformed torque vector ${}^{\underline{\omega}}\overline{\mathbf{K}}_{\tau N}$ is:

$$ {}^{\underline{\omega}}\overline{\mathbf{K}}_{\tau N} = {}^{t}\overline{\mathbf{T}}^{*}_{\underline{\omega}N}\, {}^{t}\mathbf{K}_{\tau} = \begin{bmatrix} \left\|\left[{}^{\omega}\overline{K}_{\tau k} \right]\right\|_{1:2:m_s-2}^{k} \\ {}^{\omega}\overline{K}_{\tau m_s} \end{bmatrix} = jp\varphi_c \sqrt{\frac{m_s}{2}} \cdot $$

$$ \cdot \begin{bmatrix} \left\|\left[\sum_{n=0:2\,m_s}^{\infty} (n+k)\, a_{n+k}\, e^{j\theta n} - (n-k)\, a_{n-k}\, e^{-j\theta n} \right]\right\|_{1:2:m_s-2}^{k} \\ \frac{1}{\sqrt{2}} \sum_{n=m_s:2m_s}^{\infty} n a_n (e^{jn\theta} - e^{-jn\theta}) \end{bmatrix} $$

The expression of the torque vector ${}^{\underline{\omega}}\mathbf{K}_{\tau N}$ is function of the coefficients a_n of the flux Fourier series (6) and therefore it is suitable for any shape of the normalized rotor flux $\bar{\phi}(\theta)$. Note that each component ${}^{\omega}\overline{K}_{\tau k}$, with $k \in [1 : 2 : m_s - 2]$, depends only to the coefficients a_n of order $n = 2m_s h \pm k$, where $h = 0, 1, 2..$, see Fig. 2. The last component ${}^{\omega}\overline{K}_{\tau m_s}$ depends only to the coefficients a_n of order $n = hm_s$ with odd h. This component is a real function that can be rewritten as:

$$ {}^{\omega}\overline{K}_{\tau m_s} = \sum_{h=1:2}^{\infty} b_h \sin\left(h\, m_s\, p\, \omega_m t\right), \tag{10} $$

where

$$ b_h = -p\varphi_c \sqrt{m_s} h m_s a_{hm_s}. \tag{11} $$

The state space dynamic system $\overline{S}_{\omega}$ given in (9) can be graphically represented, see [6], by the compact vectorial POG scheme shown in Fig. 4. The matrix ${}^{t}\mathbf{T}$ present between the power sections ①-② relates the phase vectors ${}^{t}\mathbf{V}_s$ and ${}^{t}\mathbf{I}_s$ to the terminal vectors ${}^{t}\mathbf{V}_l$ and ${}^{t}\mathbf{I}_l$. The *elaboration blocks* present between the power sections ③-④ represent the *Electrical part* of the system, while the blocks present between sections⑤-⑥ represent the *Mechanical part*

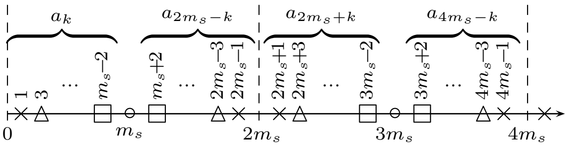


Fig. 2. Harmonic characterization of coefficients ${}^{\omega}\overline{K}_{\tau k}$: $\times \leftrightarrow k = 1$, $\triangle \leftrightarrow k = 3$, $\square \leftrightarrow k = m_s - 2$, $\circ \leftrightarrow k = m_s$.

of the system. The *connection block* present between sections ④-⑤ represents the energy and power conversion (without accumulation nor dissipation) between the electrical and mechanical parts of the motor. The function $\mathfrak{Re}(\cdot)$ present between the sections ②-③ and ④-⑤ is used to convert the transformed complex vector ${}^{\underline{\omega}}\overline{\mathbf{I}}_s$ to real vector ${}^{t}\mathbf{I}_s$:

$$ {}^{t}\mathbf{I}_s = \mathfrak{Re}\left({}^{t}\overline{\mathbf{T}}_{\underline{\omega}N}\, {}^{\underline{\omega}}\overline{\mathbf{I}}_s\right) = \mathfrak{Re}\left({}^{t}\overline{\mathbf{T}}_{\underline{\omega}}\, \mathbf{N}\, {}^{\underline{\omega}}\overline{\mathbf{I}}_s\right) $$

and to real motor torque τ_m:

$$ \tau_m = \mathfrak{Re}\left({}^{\underline{\omega}}\overline{\mathbf{K}}^{*}_{\tau N}\, {}^{\underline{\omega}}\overline{\mathbf{I}}_s\right) = \mathfrak{Re}\left(\sum_{k=1:2}^{m_s} {}^{\omega}\overline{K}^{*}_{\tau k}\, {}^{\omega}\overline{I}_{sk}\right). $$

It can be shown, see [4], that when the normalized rotor flux has the structure:

$$ \bar{\phi}(\theta) = \sum_{i=1:2}^{m_s-2} a_i \cos(i\,\theta) \tag{12} $$

the transformed torque vector ${}^{\underline{\omega}}\overline{\mathbf{K}}_{\tau N}$ is constant (not function of the electric angle θ):

$$ {}^{\underline{\omega}}\overline{\mathbf{K}}_{\tau N}(\theta) = {}^{\underline{\omega}}\overline{\mathbf{K}}_{\tau N} = jp\varphi_c \sqrt{\frac{m_s}{2}} \begin{bmatrix} \left\|\left[k a_k \right]\right\|_{1:2:m_s-2}^{k} \\ 0 \end{bmatrix}. $$

Only in this case it is possible to generate a constant torque. In all the other cases an undesired torque ripple is always present. In the next section the coefficient with order higher than m_s will be neglected. Moreover two different types of the normalized rotor flux for each type of stator connection will be considered: the first one will has the shape (12) while the second one will has also the coefficient a_{m_s} differen from zero.

3 Star and Delta connected motor

The differential equations (9) depend on the star or delta connection of the motor. In particular the m_s-th differential equation of system (9) is:

$$ L_{s0}\, {}^{\omega}\dot{\overline{I}}_{sm_s} = -R_s\, {}^{\omega}\overline{I}_{sm_s} + {}^{\omega}\overline{K}_{sm_s}\omega_m + {}^{\omega}\overline{V}_{sm_s}. \tag{13} $$

1) When the multi-phase motor is ***star connected***, the last component ${}^{\omega}I_{sm_s}$ of vector ${}^{\underline{\omega}}\overline{\mathbf{I}}_s$ is zero and the differential equation (13) becomes:

$$ {}^{\omega}\overline{K}_{sm_s}\,\omega_m + {}^{\omega}\overline{V}_{sm_s} = 0. \tag{14} $$

In this case the dynamic dimension of a star-connected motor is $m_s - 1$. Moreover, from (1) and (14) it follows that:

$$V_{s0} = \frac{1}{m_s}\sum_{h=1}^{m_s} V_h + \frac{1}{\sqrt{m_s}}\,{}^{\omega}\overline{K}_{sm_s}\,\omega_m.$$

This relation shows that the common voltage V_{s0} is zero when the input voltages V_h are balanced and when the normalized rotor flux $\bar{\phi}$ has the shape given in (12), i.e. when $a_{m_s} = 0$ and ${}^{\omega}\overline{K}_{sm_s} = 0$. On the contrary, the voltages V_{s0} is not zero and time-variant when $a_{m_s} \neq 0$. Since ${}^{\omega}\overline{I}_{sm_s} = 0$ the generated torque is constant regardless of ${}^{\omega}\overline{K}_{sm_s}$.

2) When the multi-phase motor is ***delta connected***, the last component ${}^{\omega}\overline{V}_{sm_s}$ of vector ${}^{\underline{\omega}}\overline{\mathbf{V}}_s$ is zero and the differential equation (13) becomes:

$$L_{s0}\,{}^{\omega}\dot{\overline{I}}_{sm_s} = -R_s\,{}^{\omega}\overline{I}_{sm_s} + {}^{\omega}\overline{K}_{sm_s}\,\omega_m. \qquad (15)$$

Substituting (10) in (15) one obtains:

$$L_{s0}{}^{\omega}\dot{\overline{I}}_{sm_s} = -R_s{}^{\omega}\overline{I}_{sm_s} + \omega_m \sum_{h=1:2}^{\infty} b_h \sin\left(h m_s p \omega_m t\right).$$

This is a linear equation with transfer function $G(s) = \frac{\omega_m}{L_{s0}s+R_s}$ excited by a sinusoidal input with frequency $h\,\omega_R$ and $\omega_R = m_s\,p\,\omega_m$. The steady state solution of this equation can be written as follows:

$${}^{\omega}\overline{I}_{sm_s}(t) = \sum_{h=1:2}^{\infty} b_h |G_h| \sin\left(h\,\omega_i t + \angle G_h\right) \qquad (16)$$

where $|G_h| = |G(jh\omega_R)|$, $\angle G_h = \angle G(jh\omega_R)$ and b_h is given in (11). In the case of $b_h = 0$ for $h \geq 2$, the torque τ_{m_s} corresponding to current ${}^{\omega}I_{sm_s}$ can be expressed as:

$$\tau_{m_s}(t) = {}^{\omega}\overline{K}_{sm_s}\,{}^{\omega}\overline{I}_{sm_s} \simeq \overline{\tau_{m_s}}(t) + \widetilde{\tau}_{m_s}(t)$$

where the mean value $\overline{\tau_{m_s}}(t)$ of torque τ_{m_s} is:

$$\overline{\tau_{m_s}}(t) = -\frac{b_1^2 |G(j\omega_R)|}{2}\cos\left(\angle G(j\omega_R)\right) \qquad (17)$$

and the ripple torque $\widetilde{\tau}_{m_s}(t)$ has the structure:

$$\widetilde{\tau}_{m_s}(t) = -\frac{b_1^2 |G(j\omega_R)|}{2}\cos\left(2\omega_i t + \angle G(j\omega_R)\right). \qquad (18)$$

Note that when $a_{m_s} \neq 0$ and ${}^{\omega}\overline{K}_{sm_s} \neq 0$ the torque τ_{m_s} has a negative mean value $\overline{\tau_{m_s}}$, see (17), and a torque ripple $\widetilde{\tau}_{m_s}$ at frequency $2\omega_R$, see (18). When it is present the term τ_{m_s} reduces the amplitude of the total torque τ_m introducing also an undesired torque ripple. Otherwise, when the normalized rotor flux has the shape given in (12) the equation (15) becomes:

$$L_{s0}\,{}^{\omega}\dot{\overline{I}}_{sm_s} = -R_s\,{}^{\omega}\overline{I}_{sm_s}. \qquad (19)$$

The solution of this equation is:

$${}^{\omega}\overline{I}_{sm_s}(t) = {}^{\omega}\overline{I}_{sm_s}(0)\,e^{-t/\tau}, \qquad \tau = \frac{L_{s0}}{Rs} \qquad (20)$$

and therefore in steady state condition the current ${}^{\omega}\overline{I}_{sm_s}$ is zero. In this case it is ${}^{\omega}\overline{K}_{sm_s} = 0$ and the torque τ_{m_s} is zero regardless of ${}^{\omega}\overline{I}_{sm_s}$. When the dynamics of equation (19) is disregarded the dynamic dimension of the motor is $m_s - 1$, equal to the one obtained for the star-connected case.

4 Vectorial Control

Torque τ_m can be controlled by the desired current vector in frame $\overline{\Sigma}_\omega$. The optimal current vector ${}^{\omega}\overline{\mathbf{I}}_d$ which provides the desired torque τ_d minimizing the dissipation is the current vector parallel to the torque vector ${}^{\underline{\omega}}\overline{\mathbf{K}}_{\tau N}$ in frame $\overline{\Sigma}_\omega$ (see [4]):

$${}^{\underline{\omega}}\overline{\mathbf{I}}_d = \frac{\tau_d}{|{}^{\underline{\omega}}\overline{\mathbf{K}}_{\tau N}|}\,{}^{\underline{\omega}}\widehat{\overline{\mathbf{K}}}_{\tau N} = \frac{{}^{\underline{\omega}}\overline{\mathbf{K}}_{\tau N}}{|{}^{\underline{\omega}}\overline{\mathbf{K}}_{\tau N}|^2}\,\tau_d, \qquad (21)$$

where ${}^{\underline{\omega}}\widehat{\overline{\mathbf{K}}}_{\tau N}$ denotes the versor of vector ${}^{\underline{\omega}}\overline{\mathbf{K}}_{\tau N}$. When ${}^{\omega}\overline{\mathbf{I}}_d$ is constant, the condition ${}^{\omega}\overline{\mathbf{I}}_s = {}^{\omega}\overline{\mathbf{I}}_d$ can be achieved using the control law:

$${}^{\underline{\omega}}\overline{\mathbf{V}}_s = {}^{\underline{\omega}}\overline{\mathbf{Z}}_s\,{}^{\underline{\omega}}\overline{\mathbf{I}}_s + {}^{\underline{\omega}}\overline{\mathbf{K}}_{\tau N}\,\omega_m - \mathbf{K}_c({}^{\underline{\omega}}\overline{\mathbf{I}}_s - {}^{\underline{\omega}}\overline{\mathbf{I}}_d) \quad (22)$$

where $\mathbf{K}_c > 0$ is a proper diagonal matrix used for the tuning of the control. The current vector ${}^{\underline{\omega}}\overline{\mathbf{I}}_s$ can be obtained from the terminal current vector ${}^{t}\mathbf{I}_l$ inverting the connection matrix ${}^{t}\mathbf{T}$:

$${}^{\underline{\omega}}\overline{\mathbf{I}}_s = \left({}^{t}\mathbf{T}\,{}^{t}\overline{\mathbf{T}}_{\underline{\omega}N}\right)^{-1}{}^{t}\mathbf{I}_l = {}^{t}\overline{\mathbf{T}}^{*}_{\underline{\omega}N}\,{}^{t}\mathbf{T}^{-1}\,{}^{t}\mathbf{I}_l. \quad (23)$$

Matrix ${}^{t}\mathbf{T}$ is invertible for a star-connected motor and is singular ($det({}^{t}\mathbf{T}_{\Delta}) = 0$) for a delta-connected motors, see (2). This problem can be overcome diagonalizing the connection matrix ${}^{t}\mathbf{T}$ by means of matrix ${}^{t}\overline{\mathbf{T}}_{\underline{\omega}}$: ${}^{\underline{\omega}}\overline{\mathbf{T}} = {}^{t}\overline{\mathbf{T}}^{*}_{\underline{\omega}}\,{}^{t}\mathbf{T}\,{}^{t}\overline{\mathbf{T}}_{\underline{\omega}}$ where:

$${}^{\underline{\omega}}\overline{\mathbf{T}} = \begin{cases} \begin{bmatrix} \left\| \left[1 - e^{jk\gamma_s} \right] \right\|^{k}_{1:2:m_s-2} & \mathbf{0} \\ \mathbf{0} & 0 \end{bmatrix} & \text{if delta-connected} \\ \mathbf{I}_{\frac{m_s+1}{2}} & \text{if star-connected} \end{cases}$$

This equation shows that in the delta-connection the matrix ${}^{\underline{\omega}}\overline{\mathbf{T}}$ is singular because the last eigenvalue is zero. Since ${}^{t}\overline{\mathbf{T}}^{*}_{\underline{\omega}}\,{}^{t}\overline{\mathbf{T}}_{\underline{\omega}} = \mathbf{I}_{\frac{m_s+1}{2}}$, equation (23) can be rewritten as:

$$\begin{aligned} {}^{\underline{\omega}}\overline{\mathbf{I}}_s &= \left({}^{t}\overline{\mathbf{T}}_{\underline{\omega}}\,{}^{t}\overline{\mathbf{T}}^{*}_{\underline{\omega}}\,{}^{t}\mathbf{T}\,{}^{t}\overline{\mathbf{T}}_{\underline{\omega}}\mathbf{N}\right)^{-1}{}^{t}\mathbf{I}_l \\ &= \left({}^{t}\overline{\mathbf{T}}_{\underline{\omega}N}\,{}^{\underline{\omega}}\overline{\mathbf{T}}\right)^{-1}{}^{t}\mathbf{I}_l = {}^{\underline{\omega}}\overline{\mathbf{T}}^{-1}\,{}^{t}\overline{\mathbf{T}}^{*}_{\underline{\omega}N}\,{}^{t}\mathbf{I}_l \end{aligned} \qquad (24)$$

where matrix ${}^{\underline{\omega}}\overline{\mathbf{T}}^{-1}$ is defined as follows:

$${}^{\underline{\omega}}\overline{\mathbf{T}}^{-1} = \begin{cases} \begin{bmatrix} \left\| \left[\dfrac{1}{1 - e^{jk\gamma_s}} \right] \right\|^{k}_{1:2:m_s-2} & \mathbf{0} \\ \mathbf{0} & 0 \end{bmatrix} & \text{if delta-connected} \\ \mathbf{I}_{\frac{m_s+1}{2}} & \text{if star-connected} \end{cases}$$

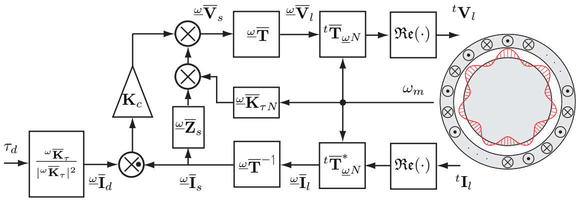


Fig. 3. Multi-phase control motor drive scheme.

Note that in the delta-connection the null eigenvalue is related to the last component ${}^{\omega}\overline{I}_{sm_s}$ of current vector ${}^{\omega}\overline{\mathbf{I}}_s$ therefore it is not possible to control this component. However it is still possible to implement the control law (22) because ${}^{\omega}\overline{I}_{sm_s}$ is independent from the voltage input, see (14), (16) and (20). The equations (21), (22) and (24) are used together in the control block diagram shown in Fig. 3.
The main difference between the two types of stator connection are summarized in Tab.II.

	Star		Delta	
	$a_{m_s}=0$	$a_{m_s}\neq 0$	$a_{m_s}=0$	$a_{m_s}\neq 0$
Dyn. dim.	m_s-1	m_s	m_s-1	m_s
τ_m	constant	constant	constant	ripple
${}^{t}\mathbf{T}$	invertible	invertible	singular	singular
${}^{\omega}\overline{I}_{sm_s}$	0 contr.	0 contr.	0 in ss uncontr.	$\sin(\cdot)$ uncontr.

II. Resume table of the difference between the star and delta connection.

5 Simulation

Both the control diagram in Fig. 3 and the POG model in Fig. 4 have been implemented in Matlab-Simulink. The simulation results shown in this section have been obtained using the following electrical and mechanical parameters: $m_s = 5$, $p = 1$, $R_s = 1.5\,\Omega$, $L_s = 0.02$H, $M_{s0} = 0.01$ H, $a_{M1} = 1$, $a_{M3} = 1/9$, $\varphi_r = 0.02$ Wb, $J_m = 0.6$ kg m^2, $b_m = 0.25$ Nm s/rad, $a_1 = 0.25$, $a_3 = 0.75$, desired torque $\tau_d = 15$ Nm and external torque $\tau_e = 0$ Nm. Three different motors are compared:

1) star-connected motor with $a_5 = 0$,
2) delta-connected motor with $a_5 = 0$,
3) delta-connected motor with $a_5 = 0.05$.

The time behaviors of the motor velocity ω_m, motor torque τ_m and current vectors ${}^{\omega}\overline{\mathbf{I}}_s$ for the three controlled motors are shown in Fig. 5(a) and Fig. 5(b). The blue lines are referred to case 1 and case 2, while the cyan lines are related to case 3. According to (20) the torques generated in case 1 and case 2 are equal (the order of the error is 10^{-14}). The torque τ_m generated in case 3 is smaller then cases 1 and 2 and it is affected by ripple. The reduction of torque and the amplitude of oscillation can be calculated by (17) and (18). The transient of the components ${}^{\omega}\overline{I}_{sk}$ of vector ${}^{\omega}\overline{\mathbf{I}}_s$ are shown in Fig. 5(b). The time behaviors of components ${}^{\omega}\overline{I}_{s1}$ (red line) and ${}^{\omega}\overline{I}_{s3}$ (green line) is the same for the three motors, while it is different for ${}^{\omega}\overline{I}_{s5}$ because the control law (22) can not control this last component. Note that the component ${}^{\omega}\overline{I}_{s5}$ is zero in case 1, see (14), is zero in steady state condition (blue line) in case 2, see (16), and is sinusoidal (cyan line) in case 3, see (20). The time behaviors of the three terminal voltage and current vectors ${}^{t}\mathbf{V}_l$, ${}^{t}\mathbf{I}_l$ for the three motors are shown Fig. 6(a) and Fig. 6(b). Note that the transient of vectors in case 2 and case 3 is the same because the control law (22) can not control the last component ${}^{\omega}\overline{I}_{s5}$. The simulation result are agree with the differences shown in Tab.II

6 Conclusion

In this paper the modeling of multi-phase permanent magnet synchronous motors with an odd number of star and delta connected phases has been studied. The dynamic model of the motor in the case of star-connected phases and delta-connected phases has been investigated and a minimum dissipation control law has been proposed. Simulations results show the effectiveness of the presented control law and put in evidence the different dynamic behavior related to the type of stator connection.

7. References

[1] E. Semail, X. Kestelyn, A. Bouscayrol, “Right Harmonic Spectrum for the Back-Electromotive Force of a n-phase Synchronous Motor”,39th IAS Annual Meeting,2004.

[2] L. Parsa and H.A. Toliyat, “Five-Phase Permanent Magnet Motor Drives”, IEEE Tran. on Ind. Applications, 2005, Vol.41, No.1, pp.30-37.

[3] G. Grandi, G. Serra, A. Tani, “General Analysis of Multi-Phase Systems Based on Space Vector Approach”, EPE-PEMC’06, Portoroz, Slovenia, Sept 2006.

[4] R. Zanasi, F. Grossi, “Multi-phase Synchronous Motors: POG Modeling and Optimal Shaping of the Rotor Flux”, ELECTRIMACS 2008, Québec, Canada, June 2008.

[5] R. Zanasi, F. Grossi, M. Fei, “ Complex Dynamic Models of Multi-phase Permanent Magnet Synchronous Motors”, IFAC 2011 18th, Milano, Italy, Aug 2010.

[6] R. Zanasi, “The Power-Oriented Graphs Technique: system modeling and basic properties”, VPPC’10, Lille, France, Sept 2010.

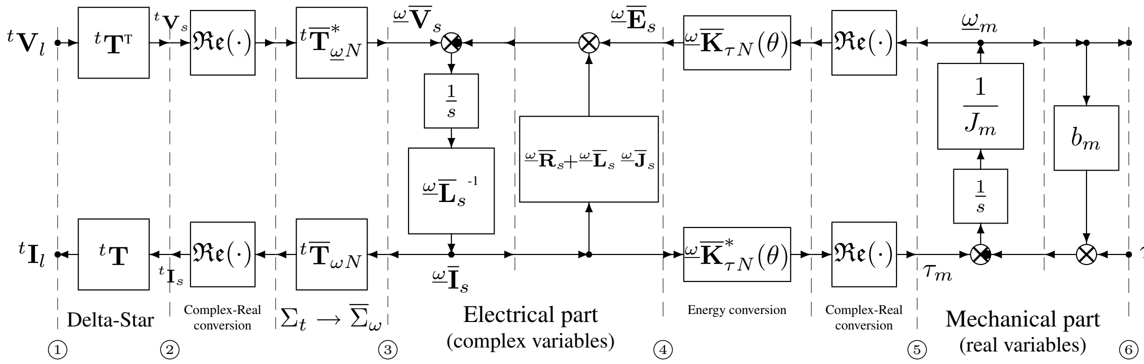
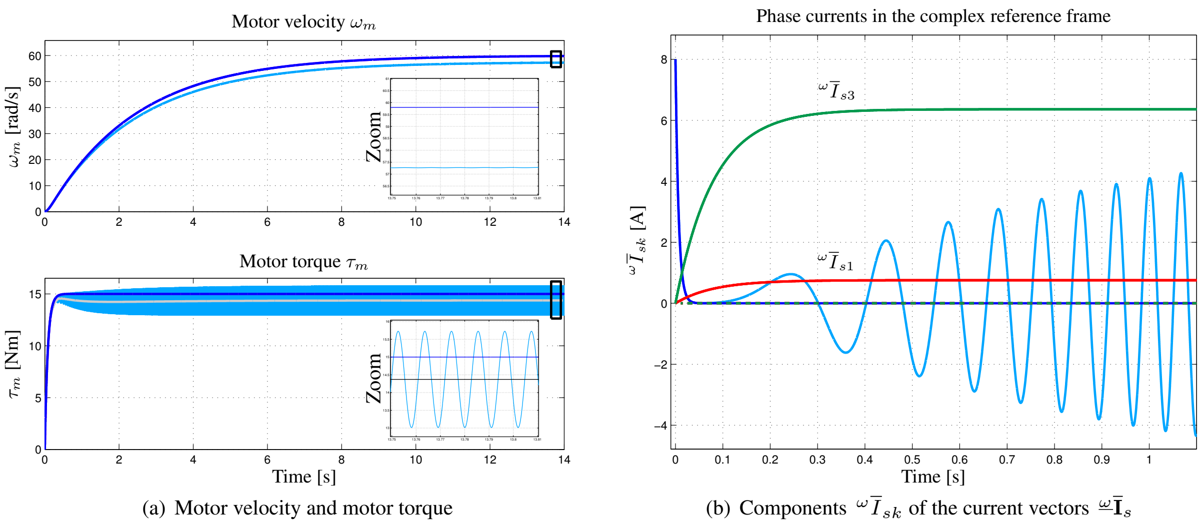


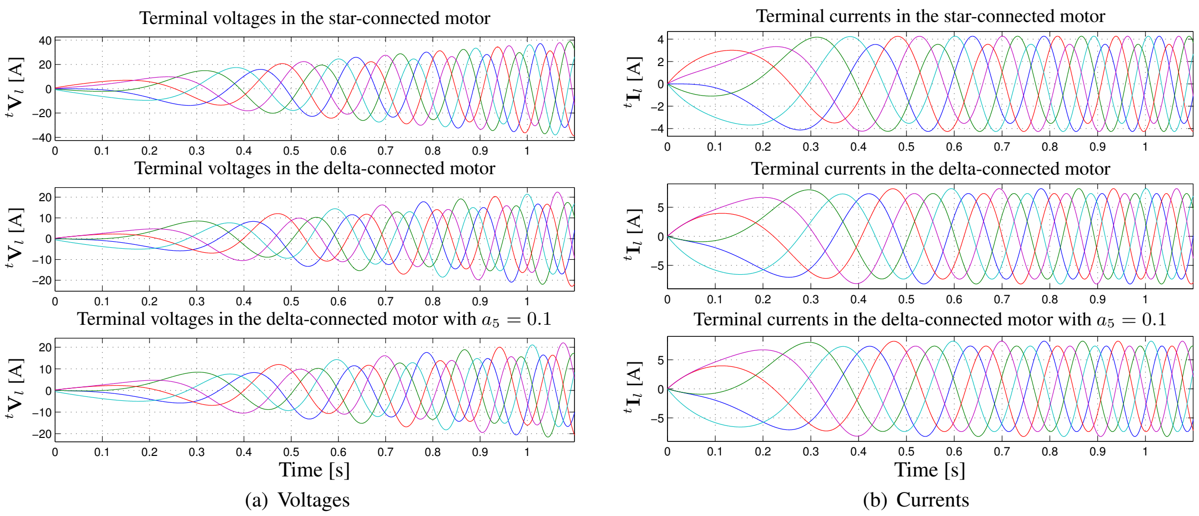
Fig. 4. POG scheme of a multi-phase electrical motor in the reduced complex rotating frame $\overline{\Sigma}_\omega$.



(a) Motor velocity and motor torque

(b) Components ${}^\omega\overline{I}_{sk}$ of the current vectors ${}^\omega\overline{\mathbf{I}}_s$

Fig. 5. Motor velocity ω_m and motor torque τ_m generated controlling the transformed phase current vectors ${}^\omega\overline{\mathbf{I}}_s$ with star and delta-connected motors.



(a) Voltages

(b) Currents

Fig. 6. Star-connected and delta-connected stator phases: terminal voltage vector ${}^t\mathbf{V}_l$, terminal current vector ${}^t\mathbf{I}_l$ in the original reference frame Σ_t.