The Power-Oriented Graphs Technique: system modeling and basic properties

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Abstract—In this paper the Power-Oriented Graphs (POG) technique is introduced and its basic graphical properties are compared with those of the Bond Graph (BG) technique. The two techniques are based on the same concepts, are equivalent from a mathematical point of view and use two different graphical representations for modeling physical systems. The BG schemes are quite compact and more flexible in the exact description of all the power connections within the system. The POG schemes are easier to use for beginners and use vectorial compact forms when the system complexity increases. The paper ends describing some "dissipative" properties of the POG state space models.

I. INTRODUCTION

The Bond Graphs (BG) [1], [2], the Power-Oriented Graphs (POG) [3], [4] and the Energetic Macroscopic Representation (EMR) [5], [6] are graphical modeling techniques that use an energetic approach for modeling physical systems. The basic concepts of the energetic approach were firstly introduces by H.M. Paynter in the early-sixties together with the basic "bond" graphical notation, see [1]. In the last fifty years the BG technique has been deeply studied, largely developed and applied to a very large number of different physical systems, see [2] and the inside references. The key element of the BG is the use of a graphical notation which is quite compact and flexible in representing the power connections, but is not easy to use for beginners and this aspect has somehow "limited" the wide-world diffusion of this good technique. The POG technique, introduced in 1991, is based on the same energetic approach of the BG technique, but it uses a different graphical notation (i.e. simple block diagrams) for modeling the physical systems. The POG block schemes are easy to use, easy to understand and can be directly implemented in Simulink. For these reasons the POG technique can be a useful tool for promoting the use of the energetic approach also between beginners and young researchers. The EMR is a graphical technique introduced in 2000, see [5], which clearly shows the power coupling among elements and the energy flux through the system without showing the mathematical details of the model. This technique is mainly focused on giving rules for the automatic deduction of effective control schemes that can be implemented in real-time. A comparison between the BG, POG and EMR graphical techniques can be found in [8]. In this paper the main concepts of POG modeling are introduced and compared with those of the BG technique. This comparison clearly shows that these two techniques use two different graphical representations, but they are essentially "equivalent" from a mathematical point of view. Examples of application of the POG graphical technique can be found in [3], [4], [7], [8] and the inside references. Examples of hybrid automotive systems modeled using the POG and BG techniques are given in [9] and [11], respectively.

II. THE POWER-ORIENTED GRAPHS TECHNIQUE

The BG, POG and EMR graphical techniques use the *Power* and *Energy* variables as basic concepts for modeling physical systems. This *energetic approach* is useful for modeling because all the dissipative physical systems are characterized by these properties: 1) a system always *stores and/or dissipates energy*; 2) the dynamic model of a physical system describes *how the energy moves* within the system, 3) the energy moves from point to point only by means of two *power variables*.

The two POG basic blocks: the POG technique uses only two basic blocks for modeling the physical systems, see Fig. 1: a) the **elaboration block** (e.b.) is used for modeling all

the physical elements that store and/or dissipate energy (i.e. springs, masses, dampers, capacities, inductances, resistances, etc.). With this block it is possible to model all the 1-port elements (capacitors C, inertias I and resistor R) used in the BG technique. The summation element at the top of the block is suitable for modeling all the 3-port connection elements (0-junctions and 1-junctions) of the BG technique. The black spot within the summation element represents, when is present, a minus sign that multiplies the entering variable. The e.b. can be scalar or vectorial and for linear systems matrix $\mathbf{G}(s)$ is always a square matrix of positive real transfer functions.

b) the *connection block* (c.b.) is used for modeling all the physical elements that "transform the power without losses" (i.e. *neutral elements* such as gear reductions, transformers, etc.). This block models all the 2-port elements (transformers TR, gyrators GY, modulated transformers MTR and modulated gyrators MGY) of the BG technique. Also the c.b. can be scalar or vectorial. In the vectorial case matrix **K** can also be rectangular, time varying or function of other state variables.

Power sections: the dashed lines in Fig. 1 represent the power sections which connect the two POG basic blocks with the external world. There are no restrictions on the choice of the vectors \mathbf{x} and \mathbf{y} involved in each dashed line except the fact that the inner product $\langle \mathbf{x}, \mathbf{y} \rangle = \mathbf{x}^T \mathbf{y}$ must have the physical meaning of power flowing through the section. In the following sections the basic concepts of how to model physical systems using power variables are introduced and the POG elaboration and connection blocks will be described in

		I	Electrical	I	Mech. Tras.		Mech. Rot.		Hydraulic
	\mathcal{D}_e	C	Capacitor	M	Mass	J	Inertia	C_I	Hyd. Capacitor
	q_e	Q	Charge	p	Momentum	p	Ang. Momentum	V	Volume
Across-Var.	v_e	V	Voltage	v	Velocity	ω	Ang. Velocity	P	Pressure
	\mathcal{D}_f	L	Inductor	E	Spring	E	Spring	L_I	Hyd. Inductor
	q_f	ϕ	Flux	\boldsymbol{x}	Displacement	θ	Ang. Displacement	ϕ_I	Hyd. Flux
Trough-Var.	v_f	I	Current	F	Force	au	Torque	Q	Volume flow rate
	\mathcal{R}	R	Resistor	b	Friction	b	Ang. Friction	R_I	Hyd. Resistor

Figure 2. The main energetic domains: the physical elements \mathcal{D}_e , \mathcal{D}_f and \mathcal{R} ; the energy variables q_e , q_f ; the power variables v_e , v_f .

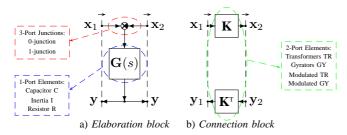


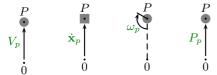
Figure 1. POG basic blocks: elaboration block and connection block.

details and compared with the corresponding symbols used in the Bond Graphs technique.

III. SYSTEM MODELING USING POWER VARIABLES

Energetic domains: the main energetic domains encountered in modeling physical systems are: electrical, mechanical (translational and rotational) and hydraulic. Each energetic domain has its own couple of power variables, see Fig. 2.

Power variables: they can be divided in two groups: 1) the "across-variables" (i.e. voltage V_p , velocity $\dot{\mathbf{x}}_p$, angular velocity ω_p and pressure P_p) which are defined "between two points P and 0" of the space:



2) The "through-variables" (i.e. current I_p , force F_p , torque τ_p and volume flow rate Q_p) which are defined "in each point P" of the space:

$$\stackrel{P}{\bullet}_{\stackrel{}{I_p}} \qquad \stackrel{P}{\stackrel{}{\bullet}_{\stackrel{}{I_p}}} \qquad \stackrel{P}{\stackrel{}{\bullet}_{\stackrel{}{Q_p}}} \qquad \stackrel{P}{\stackrel{}{\bullet}_{\stackrel{}{Q_p}}}$$

Dynamic structure of the Energetic Domains: each *energetic domains* is characterized by only 3 different types of physical elements:

- 2 dynamic elements D_e and D_f which store the energy (i.e. capacitors, inductors, masses, springs, etc.);
- 1 static element R which dissipates (or generates) the energy (i.e. resistors, frictions, etc.);

The system dynamics can be described using 4 variables:

- 2 energy variables q_e and q_f which define how much energy is stored within the dynamic elements;
- 2 power variables v_e and v_f which describe how the energy moves within the system.

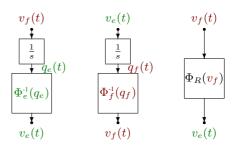


Figure 3. Structure of Dynamic elements D_e and D_f and Static element R.

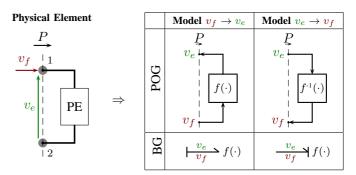


Figure 4. The two POG block diagrams used for graphically describe the mathematical model of the physical element PE. Two different orientations correspond (depending on PE) to the integral and derivative causality models.

The dynamic/static elements and the energy/power variables for the considered energetic domains are shown in Fig. 2.

Mathematical structure of the physical elements: the dynamic element D_e is characterized by:

- 1) an internal energy variable $q_e(t)$;
- 2) a through-variable $v_f(t)$ as input variable;
- 3) an across-variable $v_e(t)$ as output variable;
- 4) a constitutive relation $q_e = \Phi_e(v_e)$ which links the internal variable $q_e(t)$ to the output variable $v_e(t)$;
- 5) a differential equation $\dot{q}_e(t) = v_f(t)$ which links the internal variable $q_e(t)$ to the input variable $v_f(t)$;

The energy E_e stored in the dynamic element D_e is function only of the internal energy variable q_e :

$$E_e = \int_0^t v_e(t) \, v_f(t) \, dt = \int_0^{q_e} \Phi_1^{-1}(q_e) \, dq_e = E_e(q_e).$$

The dynamic element D_f has a structure which is "dual" respect to the structure of dynamic element D_e , see Fig. 3. The dual structure can be easily obtained performing the following substitutions: $q_e(t) \rightarrow q_f(t)$, $v_f(t) \leftrightarrow v_e(t)$ and $\Phi_e(v_e) \rightarrow \Phi_f(v_f)$. The static element R is completely characterized by

	a) Power P	flows from A to B	b) Power P flows from B to A		
Power sections	$ \begin{array}{c} P \\ v_e \\ A-B \end{array} $	v_e V_e $A-B$	v_e v_e $A-B$	v_e $A-B$	
POG	$\begin{array}{c} \stackrel{P}{\underset{\mid}{\bigvee}} \\ v_{e+} \\ \stackrel{\mid}{\underset{\mid}{\bigvee}} \\ v_{f+} \end{array}$	v_{e+}	v_{e+}	$v_{e dagger}$	
BG	$\frac{e}{f}$	$\frac{e}{f}$	<u>e</u> _ f	<u>e</u> _ f	

Figure 5. Conventions for the correct definition of the sign of the power P.

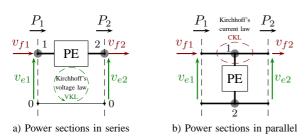


Figure 6. Two different power connections of the PE with the external world.

a static function $v_e = \Phi_R(v_f)$ which links the input variable v_f to the output variable v_e , see Fig. 3.

Conventions on the positive power flows: the physical systems are composed by basic Physical Elements (PE) (i.e. dynamic elements D_e and D_f or static element R) which interact with the external world by means of two terminals (denoted 1 and 2 in Fig. 4), each one characterized by two power variables (v_{e1}, v_{f1}) and (v_{e2}, v_{f2}) . Choosing $v_e = v_{e1} - v_{e2}$ and $v_f = v_{f1} = v_{f2}$ as new power variables, it follows that the power interaction of the PE with the external world can be described using only the power section P shown in the left part of Fig. 4. The value of the power P flowing through the section is the product of the two power variables $v_e(t)$ and $v_f(t)$: $P(t) = v_e(t) v_f(t)$. The sign and the direction of power P(t) depend on the sign and the reference positive direction chosen for the variables $v_e(t)$ and $v_f(t)$. The POG conventions used for a correct definition of the sign of the power P flowing through a physical section A-B are shown Fig. 5. The POG technique uses a small arrow "→" drawn in the vicinity of power section (i.e. the dashed line) to indicate (when it is necessary) the positive direction of power P. In Fig. 5 are also reported the graphical symbols used by BG technique for describing the same conventions.

Models with integral and derivative causality: the dynamic model of each physical element PE can always be graphically described by using the block diagrams shown in the right part of Fig. 4. These diagrams correspond to the two possible orientations of the dynamic model of the PE: v_f as input and v_e as output, v_e as input and v_f as output. The function $f(\cdot)$ in Fig. 4

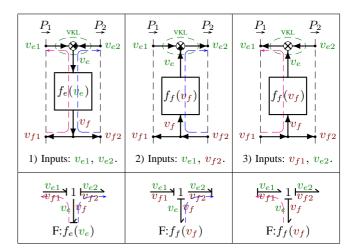


Figure 7. POG and BG graphical representations of a physical element PE with its power sections connected *in series*, see Fig. 6.a.

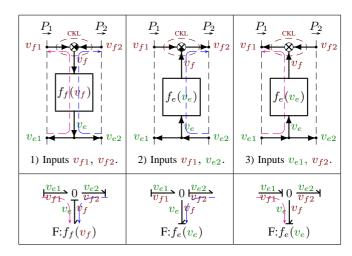


Figure 8. POG and BG graphical representations of a physical element PE with its power sections connected *in parallel*, see Fig. 6.b.

symbolically represents one of the dynamic or static models shown in Fig. 3. If the PE is a static element, i.e. if PE = \mathcal{R} , both diagrams are suitable for describing the mathematical model of the element. If the PE is a dynamic element, i.e. if PE = D_e or PE = D_f , the two diagrams represent the two possible causality modes (integral and derivative) of the dynamic element: the *integral causality model* is physically realizable, useful in simulation and is the preferred dynamic model in the POG technique. The other *derivative causality model* is still a correct mathematical model of the PE, but it is not used in the POG technique because it is not physically realizable and it is not useful in simulation.

Power sections in series and in parallel: each Physical Element (PE) interacts with the external world through the power sections associated to its terminals. The two basic power connections of the physical element PE with the external world are shown in Fig. 6: a) the connection in series when the two terminals share the same through-variable $v_f = v_{f1} = v_{f2}$; b) the connection in parallel when the two terminals share the same across-variable $v_e = v_{e1} = v_{e2}$. Three different POG and BG graphical representations of a PE connected in series

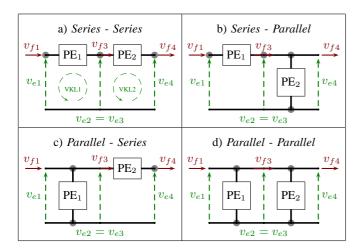


Figure 9. Four types of connections of two physical elements PE₁ and PE₂.

are shown in Fig. 7. The following considerations hold:

- 1) All these graphical representations exactly describe the mathematical model of the PE, but they have different model orientation, i.e. different choices of the input and output variables. In particular, the POG diagrams can be obtained one from the other inverting one of the input-output path. The second POG diagram in Fig. 7, for example, is obtained from the first one inverting the path that goes from v_{e2} to v_{f2} as shown by the dashed blue lines. The third POG diagram is obtained from the first one inverting the path that goes from v_{e1} to v_{f1} as shown by the dashed red lines. The first POG diagram in Fig. 7 is the basic element for POG modeling all the physical elements and is called the POG *elaboration block*.
- 2) The summation elements which are present in the POG block diagrams of Fig. 7 are a mathematical description of the Voltage Kirchhoff's Law (VKL) applied to the *across variables* v_{e1} , v_{e2} and v_e involved in the *closed path* which is always present when the PE is connected in series, i.e. the green closed dashed path shown in Fig. 6.a.
- 3) In Fig. 7 there is a perfect correspondence between the POG and the BG graphical representations: the BG "half-arrow" corresponds to POG power arrow " \rightarrow "; the BG "stroke" corresponds to POG "model orientation"; the BG "1-series" symbol corresponds in POG to the fact that all the power sections share the same through-variable v_f and the adder involves only the across-variables.
- 4) Note that in the POG graphical representations of Fig. 7 the *across variables* are drawn in the upper side of the figure, while the *through variables* are drawn in the lower side of the figure. This is NOT a general rule for the POG graphical representations. In fact, a POG block diagram can always be rewritten upside-down or left-right switched without loosing its meaning and its graphical interpretation.

Similar considerations can also be done for a physical element PE connected *in parallel*, see Fig. 6.b. In this case the three different POG and BG graphical representations of the mathematical model of the PE are shown in Fig. 8. In this figure the BG "0-parallel" symbol corresponds to the fact

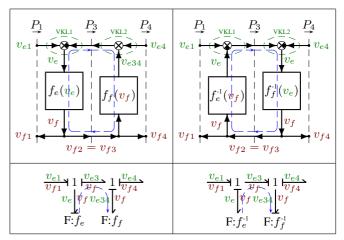


Figure 10. Two POG models (and two BG models) corresponding to the "a) Series - Series" connection shown in Fig. 9

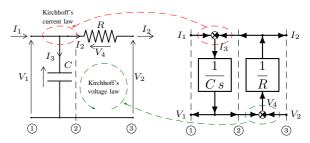


Figure 11. POG modeling of an electrical RC circuit.

that in POG all the power sections share the same across-variable v_e and the adder involves only the through-variables. Moreover, in this case, the summation elements present in the POG block diagrams of Fig. 8 are a mathematical description of the Current Kirchhoff's Law (CKL) applied to the *through variables* v_{f1} , v_{f2} and v_f involved in the "node" corresponding to terminal 1 of the PE connected in parallel, see the red closed dashed line shown in Fig. 6.b.

Power connections of two physical elements: two physical elements PE₁ and PE₂ can be connected only though theirs terminals, i.e. through "the power sections" associated to the terminals. The four possible ways of connecting two physical elements PE₁ and PE₂ are shown in Fig. 9. When two power sections are connected, a feedback loop always arises in the POG graphical representation of the system. In Fig. 10 it is shown, for example, two different POG mathematical models (and two BG models) that can be associated to the "a) Series - Series" connection shown in Fig. 9. Note that the second POG model can be obtained from the first one by inverting the closed loop path (i.e. the dashed blue line) present in the first POG model. One model can be preferred respect to the other depending on the causality orientation desired for the two physical elements PE₁ and PE₂. Also in this case it is evident the perfect correspondence between the POG and the BG models: the POG inversion of the internal path corresponds to the BG stroke inversion of the path which goes from f_e to f_f . A simple example of POG modeling is shown in Fig. 11 where a C-parallel element is connected with an R-series element:

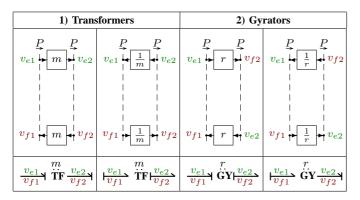


Figure 12. POG and BG graphical symbols used for the Connection blocks.

this is a particular case of the "c) *Parallel - Series*" connection shown in Fig. 9. Note the direct correspondence between the power sections ①, ② and ③ in the system and the dashed power sections ①, ② and ③ in the POG scheme.

Connection blocks: the physical elements can also be connected using real physical elements which redistributes the power without storing nor dissipating energy (i.e. any type of gear reduction, mechanical levers, electrical transformers, etc.). The BG technique divide these neutral elements into two categories: the transformers which link the power variables of the same type (i.e. $v_{e2} = m v_{e1}$, $v_{f1} = m v_{f2}$) and the gyrators which link power variables of different types (i.e. $v_{f2} = r v_{e1}, v_{f1} = r v_{e2}$). The basic property satisfied by the neutral elements is $P_1 = v_{e1} v_{f1} = v_{e2} v_{f2} = P_2$, i.e. the input power flow P_1 is always equal to the output power flow P_2 . The POG and BG graphical representations of the basic neutral elements are shown in Fig. 12. From the figure it is evident that the POG technique uses only one type of block (i.e. the connection block) for representing all the possible types of neutral elements that can be found in real world.

Example of POG modeling: a DC motor connected with an hydraulic pump is shown in Fig. 13. This system involves three different energetic domains: electrical, mechanical and hydraulic. The corresponding POG graphical representation in shown in Fig. 14: the power sections present in the POG scheme have a direct correspondence with the real physical sections. Let $\mathbf{x} = \begin{bmatrix} I_a \ \omega_m \ V_0 \end{bmatrix}^\mathsf{T}$ be the state vector of the system, i.e. the output variables of the dynamic elements. From the POG scheme one directly obtains the following state space dynamic model $\mathbf{L}\dot{\mathbf{x}} = -\mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$:

$$\underbrace{\begin{bmatrix} L_{a} & 0 & 0 \\ 0 & J_{m} & 0 \\ 0 & 0 & C_{0} \end{bmatrix}}_{\mathbf{L}} \underbrace{\begin{bmatrix} \dot{I}_{a} \\ \dot{\omega}_{m} \\ \dot{P}_{0} \end{bmatrix}}_{\dot{\mathbf{x}}} = \underbrace{\begin{bmatrix} -R_{a} - K_{m} & 0 \\ K_{m} & -b_{m} - K_{p} \\ 0 & K_{p} & -\alpha_{p} \end{bmatrix}}_{-\mathbf{A}} \underbrace{\begin{bmatrix} I_{a} \\ \omega_{m} \\ P_{0} \end{bmatrix}}_{\mathbf{X}} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}}_{\mathbf{B}} \underbrace{\begin{bmatrix} V_{a} \\ Q_{0} \end{bmatrix}}_{\mathbf{U}} \tag{1}$$

where ${\bf u}$ is the input vector and ${\bf L}$, ${\bf A}$ and ${\bf B}$ are matrices that can be obtained by direct inspection of the POG scheme. In the considered case matrix ${\bf L}$ is diagonal and its elements are the coefficients of the constitutive relations ($\phi=L_aI_a$, $p=J_m\omega_m,\ V=C_0P_0$) of the dynamic elements of the system. The coefficients of matrices $-{\bf A}$ and ${\bf B}$ are the gains of all the paths that link the state variables ${\bf x}$ and the input variables ${\bf u}$ to the inputs $\dot{\phi}$, $\dot{\omega}_m$, \dot{V} of the integrators in the

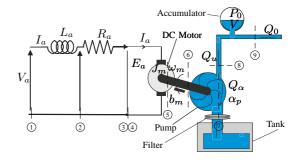


Figure 13. A DC motor connected with an hydraulic pump.

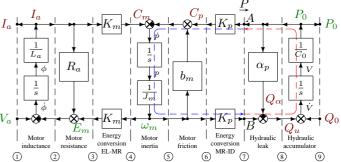


Figure 14. POG scheme of the DC motor with hydraulic pump of Fig. 13.

system. The element $a_{2,3}=-K_p$ of matrix $-{\bf A}$, for example, is the gain of the path that goes from the third state variable P_0 to the input $\dot p$ of the second integrator. The element $b_{3,2}=1$ of matrix ${\bf B}$ is the gain of the path that goes from the second input Q_0 to the input $\dot V$ of the third integrator.

POG state space model: for linear systems, the general state space model obtained from a generic POG scheme is given by eq. (2) in Fig. 15: y is the output vector, L is the energy matrix, A is the power matrix, B is the input power matrix, C is the output power matrix and D is the input-output power matrix. The POG state space model can be graphically represented by the compact vectorial POG scheme shown in the left part of Fig. 15. This "compact graphical form" cannot be obtained by using the Bong Graphs technique because x, u and u are, in the general case, effort-flow mixed vectors. For a POG linear system expressed in the compact form (2) the following properties hold: 1) the energy matrix L is symmetric positive definite: $\mathbf{L} = \mathbf{L}^{\mathsf{T}} > 0$; 2) the energy E_s stored in the system can be expressed as: $E_s = \frac{1}{2} \mathbf{x}^T \mathbf{L} \mathbf{x}$; 3) the dissipating power P_d in the system can be expressed as: $P_d = \mathbf{x}^T \mathbf{A}_s \mathbf{x}$ where $\mathbf{A}_s = \frac{(\mathbf{A} + \mathbf{A}^T)}{2}$ is the symmetric part of the power matrix **A**; **4**) the skew-symmetric part $\mathbf{A}_w = \frac{(\mathbf{A} - \mathbf{A}^T)}{2}$ of the power matrix A represents the power redistribution within the system "without losses", i.e. $P_d = \mathbf{x}^T \mathbf{A}_w \mathbf{x} = 0$. For system (1), for example, we have that: $E_s=\frac{1}{2}L_aI_a^2+\frac{1}{2}J_m\omega_m^2+\frac{1}{2}C_0P_0^2$, $P_d=R_aI_a^2+b_m\omega_m^2+\alpha_pP_0^2$ and matrix ${\bf A}_w$ is function only of the connection parameters K_m and K_p .

Transformed and reduced POG systems: a POG system expressed in the compact form (1) can be transformed in an equivalent POG system, see eq. (3) in Fig. 15, using a "congruent" transformation $\mathbf{x} = \mathbf{T}\mathbf{z}$ where $\overline{\mathbf{L}} = \mathbf{T}^{\mathrm{T}}\mathbf{L}\mathbf{T}$, $\overline{\mathbf{A}} = \mathbf{T}^{\mathrm{T}}\mathbf{A}\mathbf{T}$, $\overline{\mathbf{B}} = \mathbf{T}^{\mathrm{T}}\mathbf{B}$, $\overline{\mathbf{C}} = \mathbf{C}\mathbf{T}$ and $\overline{\mathbf{D}} = \mathbf{D}$. The trans-

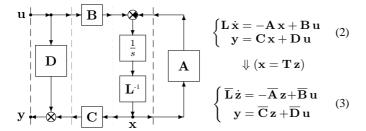


Figure 15. POG block scheme of a generic dynamic system.

formed system conserves the same properties of the original POG system. Note that this "congruent" transformation do not need the inversion of matrix **T**. When an eigenvalue of matrix **L** tends to zero (or to infinity), system (2) degenerates towards a lower dimension dynamic system. In this case the reduced and transformed system can be obtained using a "rectangular" matrix **T**. Details of how the POG systems can be reduced, transformed and "input-output inverted" can be found in [9].

POG graphical rules: the POG schemes always satisfy the following graphical rules: 1) all the loops of a POG scheme contains an "odd" number of minus signs (i.e. of the black spots in the summation elements). This rule is a direct consequence of the fact that in the POG schemes a loop always appears when two physical elements are connected, see Fig.10 and this loop contains at least one "minus sign" for letting the powers flows have the same positive direction; 2) chosen two generic points A and B of a POG scheme, all the paths that go from A to B contain either an "odd" number or an "even" number of minus signs. This rule follows directly from rule 1; 3) the direction of the power flowing through a section is positive if an "even" number of minus signs is present along one of the paths which goes from the input to the output of the section. Let us consider, for example, the power section (7) of Fig. 14. The power flows from left to right because the red dashed path that goes from B to A contains "zero" minus signs (i.e. an even number). The same result is obtained considering the left part of section (7): the power flows from left to right because the blue dashed path that goes from A to B contains "one" minus sign (i.e. an odd number). It is evident that rule 3) makes "optional", in the POG scheme, the use of the power arrow "→" drawn in the vicinity of the power sections: this direction can always be determined by direct inspection of the POG scheme.

POG schemes and dissipative systems: the POG dynamic model (2) represents a "dissipative system" if and only if:

$$\mathbf{L} = \mathbf{L}^{\mathsf{T}} > 0$$
 and $\mathbf{G} = \begin{bmatrix} \mathbf{A} & -\mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \ge 0,$ (4)

i.e. if and only if matrix \mathbf{L} is symmetric definite positive and \mathbf{G} is a semi positive definite matrix. Proof: a POG system is dissipative, see [10], if the power inequality $\int_0^t \mathbf{u}^\mathsf{T} \mathbf{y} \, dt + E_s(0) \geq E_s(\mathbf{x})$ holds for all the trajectories, being E_s the stored energy. Chosen $E_s = \frac{1}{2}\mathbf{x}^\mathsf{T}\mathbf{L}\mathbf{x}$, the time-derivative of the power inequality provides the relation: $\mathbf{u}^\mathsf{T}\mathbf{y} - \mathbf{x}^\mathsf{T}\mathbf{L}\dot{\mathbf{x}} \geq 0$. Using vectors \mathbf{y} and $\mathbf{L}\dot{\mathbf{x}}$ from (2), one obtains the inequality $\mathbf{u}^\mathsf{T}\mathbf{C}\,\mathbf{x} + \mathbf{u}^\mathsf{T}\mathbf{D}\,\mathbf{u} + \mathbf{x}^\mathsf{T}\mathbf{A}\,\mathbf{x} - \mathbf{x}^\mathsf{T}\mathbf{B}\,\mathbf{u} \geq 0$ which can be rewritten

as $\mathbf{z}^T \mathbf{G} \mathbf{z} \ge 0$ where $\mathbf{z} = [\mathbf{x} \ \mathbf{u}]^T$, i.e. $\mathbf{G} \ge 0$. When $\mathbf{D} = 0$, i.e. for strictly proper systems, from (4) follows $\mathbf{C} = \mathbf{B}^T$.

Dissipative dynamic systems: a dynamic system given in the form $\dot{\tilde{\mathbf{x}}} = \mathbf{F}\tilde{\mathbf{x}} + \mathbf{G}\mathbf{u}$, $\mathbf{y} = \mathbf{H}\tilde{\mathbf{x}} + \mathbf{D}\mathbf{u}$ is a dissipative system (and therefore can be described in the POG compact form (2)) if and only if it exists a symmetric positive definite matrix $\mathbf{L} > 0$ such that the following matrix \mathbf{M} (or matrix \mathbf{N}) is a semi positive definite matrix:

$$\mathbf{M} = \begin{bmatrix} \mathbf{LF} & -\mathbf{LG} \\ \mathbf{H} & \mathbf{D} \end{bmatrix} \ge 0 \quad \text{or} \quad \mathbf{N} = \begin{bmatrix} \mathbf{FL} & -\mathbf{G} \\ \mathbf{HL} & \mathbf{D} \end{bmatrix} \ge 0. \quad (5)$$

When the property holds, the POG compact form (2) is obtained choosing $\mathbf{x} = \tilde{\mathbf{x}}$, $\mathbf{A} = \mathbf{LF}$, $\mathbf{B} = \mathbf{LG}$ and $\mathbf{C} = \mathbf{H}$ (or $\tilde{\mathbf{x}} = \mathbf{Lx}$, $\mathbf{A} = \mathbf{FL}$, $\mathbf{B} = \mathbf{G}$ and $\mathbf{C} = \mathbf{HL}$). This property is deeply connected with the Kalman-Yakubovich-Popov lemma applied to dissipative systems. Note that when $\mathbf{D} = 0$ the inequalities (5) reduce to $\mathbf{LF} \geq 0$ and $\mathbf{H} = (\mathbf{LG})^T$ (or $\mathbf{FL} \geq 0$ and $\mathbf{G} = (\mathbf{HL})^T$).

IV. CONCLUSIONS

In this paper the POG technique has been introduced and its basic graphical properties have been compared with those of the BG technique. The BG schemes are quite compact and more flexible in the graphical representation of the system, while the POG schemes are easy to use, easy to understand and can be directly implemented in Simulink. Both the techniques provide a exact graphical definition of the mathematical model of the system. The two techniques can be useful for the synthesis of the control, but not in explicit and direct way. The POG technique is easier for the beginners while the BG technique is more suitable for expert peoples.

REFERENCES

- Paynter, H.M., Analysis and Design of Engineering Systems, MIT-press, Camb., MA, 1961.
- [2] D. C. Karnopp, D.L. Margolis, R. C. Rosemberg, System dynamics -Modeling and Simulation of Mechatronic Systems, Wiley Interscience, ISBN 0-471-33301-8, 3rd ed. 2000.
- [3] R. Zanasi, "Power Oriented Modelling of Dynamical System for Simulation", IMACS Symp. on Modelling and Control of Technological System, Lille, France, May 1991.
- [4] Zanasi R., "Dynamics of a n-links Manipulator by Using Power-Oriented Graph", SYROCO '94, Capri, Italy, 1994.
- [5] A. Bouscayrol, B. Daval, B. de Fornel, B. Franois, J. P. Hautier, F. Meibody-Tabar, M. Pietrzak-David, "Multimachine Multiconverter System: application for electromechanical drives", Eur. Physics Journal Appl. Physics, vol. 10, no. 2, pp. 131-147, May 2000.
- [6] J. C. Mercieca, J. N. Verhille, A. Bouscayrol, "Energetic Macroscopic Representation of a subway traction system for a simulation model", IEEE-ISIE'04, Ajaccio, May 2004, pp. 1519-1524.
- [7] R. Morselli, R. Zanasi, "Modeling of Automotive Control Systems Using Power Oriented Graphs", 32nd Annual Conference of the IEEE Industrial Electronics Society, IECON 2006, Parigi, 7-10 Novembre, 2006.
- [8] R. Zanasi, G. H. Geitner, A. Bouscayrol, W. Lhomme, "Different energetic techniques for modelling traction drives", ELECTRIMACS 2008, Qubec, Canada, June 8-11 2008.
- [9] R. Zanasi, F. Grossi, "The POG Technique for Modeling Planetary Gears and Hybrid Automotive Systems", Vehicular Power and Propulsion Conference VPPC 2009, Dearborn, Michigan, USA 7-11 September 2009.
- [10] B. Brogliato, R. Lozano, B. Maschke and O. Egeland Dissipative, "Systems Analysis and Control. Theory and Applications", Springer-Verlag, London, 2007.
- [11] M. Filippa, C. Mi, J. Shen, R. Stevenson, "Modeling a hybrid electric vechicle powertrain test cell unsing Bond Graphs", IEEE Trans. on Vehicular Technology, May 2005, Vol. 54, pp. 837-845.