Modelling and control of a vehicle with tire-road interaction using energy-based techniques

F. Grossi\textsuperscript{1}, W. Lhomme\textsuperscript{2}, R. Zanasi\textsuperscript{1}, A. Bouscayrol\textsuperscript{2}
\textsuperscript{1}DII, University of Modena and Reggio Emilia, Modena, Italy
\textsuperscript{2}L2EP, University of Lille 1, France
Roberto.Zanasi@unimore.it, Walter.Lhomme@univ-lille1.fr

Abstract – In this paper a vehicle with the modelling of its tire-road interaction is described using two different energy-based graphical techniques: Power-Oriented Graphs (POG) and Energetic Macroscopic Representation (EMR). The main features of both approaches are presented and an integration between them is proposed: they can be used together to model different parts of the same system. Using the two techniques the whole vehicle can be represented with either a mathematical or a macroscopic view. From EMR an inversion-based control is given in order to control the vehicle velocity.

Keywords – modelling; control; tire-road interaction; EMR; POG

I. INTRODUCTION

The strong interactions between the various components of the electromechanical systems require to study them in their global nature. To optimize their behaviour, the modeling and the simulation are an important step. Nowadays, lots of modelling methods are available to describe electromechanical systems such as Bond Graphs [1]-[2], Causal Ordering Graph [3], Power-Oriented Graphs (POG) [4]-[5] and Energetic Macroscopic Representation (EMR) [6]-[8]. All these techniques use power interaction between subsystems as the basic concept for modelling. In this paper POG and EMR are exploited and merged: they can be considered as complementary tools and can be exploited depending on the desired goal.

The main target of the POG modelling technique is the analysis of physical systems, it allows to write the system equations in the state space form, it is suitable for education as it is easily comprehensible. Its main drawbacks are the linear topology structure of the scheme and the absence of a methodology to build the control structure.

EMR is a graphical description exclusively based on physical causality (i.e. integral causality). This property enables an easily deduction of control scheme that can be implemented in real-time, that is not the case for other graphical descriptions. Such a dynamic modelling highlights energy properties of the power components such as energy storage, energy conversion and energy distribution. The representation is planar; it clearly shows the coupling among elements and the energy flux through the system. The structure is easy to be read, however it does not show mathematical details of the model.

The aim of this paper is to give a model of a vehicle involving the tire-road contact law and implement a velocity control by using an integration of both modelling techniques: POG is used to represent an energetic model of the tire-road interaction with a detailed mathematical description, while EMR is used to represent the whole system, including the traction and the inversion-based control structure. Both techniques are merged as EMR is a macroscopic description, while through POG a detailed mathematical model of the EMR blocks inner structure can be given. Both modelling has been previously in [9]. Regarding the previous study this paper highlights the control of the system.

In section II the studied system is presented. In section III the basic principles of POG are explained and the POG representation of the system is given. The main features of EMR and the EMR and its inversion-based control of the considered system are dealt with in section IV. In section V some simulation results are given and discussed. At last, a discussion between the POG and the EMR techniques is given and some conclusions are drawn in sections VI and VII.

II. THE STUDIED SYSTEM

In this paper an classical vehicle is considered. The traction system is a motor put on the front axle. To simplify this study a simplified bicycle-model of a four wheels vehicle, in which left and right front wheels are kept together in one unique wheel as well as rear wheels, is considered (Figure 1). The considered vehicle is without suspensions, it moves only in the plane \((x, y)\) and it interacts with the ground by means of tires.

![Figure 1. Scheme of the considered system and its variables](image)

The three black spots represent the three rigid bodies involved in the system: the front axle (index 1), the rear axle (index 2) and the chassis (index c). In the considered case the number of degrees of freedom is limited because translation wheels are solidly connected to the chassis and rotations are...
possible only around z axis. The system can thus be described with five degrees of freedom: three for the vehicle body and two for the two axles. Moments of inertia of each axle can be kept in the centre of mass of the axles because they are independent from the three degrees of freedom of the body vehicle.

III. REPRESENTATION USING POWER-ORIENTED GRAPHS

A. Power-Oriented Graphs basic principles

The “Power-Oriented Graphs”, are “signal flow graphs” combined with a particular “modular” structure which essentially uses only two blocks (cf. Appendix). The basic characteristic of this modular structure is the direct correspondence between pairs of system variables and real power flows: the product of the two variables involved in each dashed line of the graph has the physical meaning of “power flowing through the section”. The two basic blocks are named “elaboration block” (e.b.) and “connection block” (c.b.). The circle present in the e.b. is a summation element and the black spot represents a minus sign that multiplies the entering variable. There is no restriction on variables x and y other than the fact that their inner product $\langle x, y \rangle = x^T y$ must have the physical meaning of a “power”. In this way the POG schemes always show a direct correspondence between pairs of system variables and real power flows. The e.b. and the c.b. are suitable for representing both scalar and vectorial systems. In the vectorial case, $G(s)$ and $K$ are matrices; $G(s)$ is always square, $K$ can also be rectangular. While the elaboration block can store and dissipate energy (i.e. springs, masses and dampers), the connection block can only “transform” the energy, that is, transform the system variables from one type of energy-field to another (i.e. any type of gear reduction).

B. POG representation of the system

The POG scheme [10]-[11] of the considered system is shown in Figure 2. In the paper matrices and vectors are represented by bold symbols. The input torque is defined as $\tau = [\tau_1 \tau_2]^T$ where $\tau_1$ and $\tau_2$ are the torques applied to the joint between the chassis and front and rear axles respectively. The total input force applied to the system is $F = [0 \tau_m \tau_1 \tau_2]^T$ where $\tau_m$ is the torque applied to the chassis. This is described by $F = T^T \tau$ using the transformation matrix:

$$
T = \begin{bmatrix}
0 & 0 & -1 & 1 & 0 \\
0 & 0 & -1 & 0 & 1
\end{bmatrix}
$$

(1)

This transformation is presented in the POG scheme within connection block between sections ② and ③. The conjugate variables of $\tau$ are the velocities $\Delta \omega$ which are the relative rotational velocities between both axles and the chassis:

$$
\Delta \omega = T^T \omega = \begin{bmatrix}
\omega_1 - \omega_m \\
\omega_2 - \omega_m
\end{bmatrix}
$$

(2)

The inertia $J$ and the mass $M$ matrices and the corresponding velocity vector $V$ are defined as follows:

$$
M = \text{diag} \{m_M, m_M, J\}
$$

(3)

$$
J = \text{diag} \{J_M, J_1, J_2\}
$$

(4)

$$
V = [\dot{x}_m \dot{y}_m \omega_0 \omega_1 \omega_2]^T
$$

(5)

where $m_M = m_1 + m_2$ and $J_M = J_1 + m_1 I_1 + m_2 I_2$ are the mass and moment of inertia of the whole vehicle, $L_1$ and $L_2$ are the distances between the three rigid bodies (cf. Figure 1), $J_1$ and $J_2$ are the moments of inertia of the wheels; $\dot{x}_m$, $\dot{y}_m$ and $\omega_m$ are respectively the longitudinal, vertical and rotational velocities of the chassis, $\omega_1$ and $\omega_2$ are the rotational velocities of the axles. The force vector $F_p = [F_p \ F_{py} \ \tau_{pm} \ \tau_{pm} \ \tau_{p2}]^T$ is the vector of forces and torques given by the environment along the five considered directions. All the components of vector $F_p$ are kept equal to zero except for the longitudinal $F_{px}$ and vertical $F_{py}$ forces:

$$
F_{px} = F_0 + a_1 \dot{x}_m + a_2 \dot{x}_m^2 + m_M g \sin(\alpha)
$$

(6)

$$
F_{py} = m_M g \cos(\alpha)
$$

(7)

with $F_0$ the initial rolling force, $a_1$ the rolling coefficient, $a_2$ the drag coefficient, $\alpha$ the slope rate and $g$ the gravity acceleration.

The elaboration block between power sections ② and ③ describes the mechanical dynamics of the system:

![Figure 2. The POG scheme of the simplified vehicle](image-url)
The vertical position $y_m$ and the angular position $\theta_m$ of the centre of mass of the chassis can be expressed as a function of vertical positions $y_1$ and $y_2$ of the centre of mass of the axles:

$$y_m = y_1 + L_1 \sin (\psi_1 - \theta_m) = y_2 + L_2 \sin (\psi_2 + \theta_m)$$

(9)

$$\theta_m = \arcsin\left(\frac{y_1 - y_2}{L}\right)$$

(10)

The longitudinal $\dot{x}$, vertical $\dot{y}$ and angular $\omega$ velocities of the centre of mass of both axles (with indices 1 and 2) are considered into the velocity vector $\dot{X} = [\dot{x}_1 \ \dot{y}_1 \ \omega_1 \ \dot{x}_2 \ \dot{y}_2 \ \omega_2]^T$.

In order to obtain the velocity vector $\dot{X}$ from the velocity vector $V$, the transformation $\dot{X} = T^T \dot{V}$ can be applied, where the transformation matrix $T^T$ is defined as follows:

$$T^T = \begin{bmatrix}
1 & 0 & L_1 \sin (\psi_1 - \theta_m) & 0 & 0 \\
0 & 1 & L_1 \cos (\psi_1 - \theta_m) & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
1 & 0 & L_2 \sin (\psi_2 + \theta_m) & 0 & 0 \\
0 & 1 & -L_2 \cos (\psi_2 + \theta_m) & 0 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}$$

(11)

This transformation appears in the POG scheme in the connection block between sections $\circ$ and $\otimes$. Afterwards the velocity vector $\dot{X}_c = [\dot{x}_1 \ \dot{y}_1 \ \omega_1 \ \dot{x}_2 \ \dot{y}_2 \ \omega_2]^T$ is needed; whose components are the velocities of the contact points of the tires with the ground $P_1$ and $P_2$ (cf. Figure 1). The vector $\dot{X}_c$ can be obtained using the transformation $\dot{X}_c = C^T \dot{X}$ where the transformation matrix $C^T$ is defined as:

$$C^T = \begin{bmatrix}
1 & 0 & -y_1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

(12)

This matrix is composed by two rotation matrices (one for each wheel) where $y_j$ and $y_j$ are the coordinates along $y$ of the wheels centres of mass. In the POG scheme this transformation matrix is present in the connection block between sections $\circ$ and $\otimes$.

The dynamics of the elastic interactions of the tires with the ground is represented by the POG elaboration block present between sections $\circ$ and $\otimes$. This elastic dynamics is described by the equation:

$$\dot{X}_c - (V_{col} + V_{col}) = \frac{d(EF)}{dt} - \frac{d}{dt}\left(EbC[\dot{X}_c - (V_{col} + V_{col})]\right)$$

(13)

$E^J$ and $b$ are respectively the six-dimensional stiffness and damping matrices of the tire-soil contact area centred in the contact points:

$$E^J = \text{diag} \{K_{x_1}, K_{x_2}, 0, K_{y_2}, K_{y_2}, 0\}$$

(14)

$$b = \text{diag} \{b_{x_1}, b_{x_2}, 0, b_{y_1}, b_{y_2}, 0\}$$

(15)

where $K_{x_1}$, $K_{x_2}$, $K_{y_2}$, $K_{y_2}$ are the translational stiffness coefficients of the tire-soil contact areas and $b_{x_1}$, $b_{x_2}$, $b_{y_1}$, $b_{y_2}$ are the damping coefficients. Note that in (14) and (15) only the particular case of deformations along $x$ and $y$ axis is considered. Matrix $E^J$ relates the elastic displacement to the force $F_e$ generated in the contact points. The elastic element $E^J$ located in the contact point is characterized at an end by the rolling velocity $\dot{X}_c$ of the tire and at the other end by the sum of the “skidding” velocity $V_{sk}$ and the “slipping” velocity $V_{sl}$. Blocks between sections $\circ$ and $\otimes$ represent the slip and skid phenomena [10]. The energetic model proposed here is an alternative to the Pacejka magic formulas [12]. It allows overcoming some drawbacks like the definition of slip ratio that is not valid for all working conditions and the fact that Pacejka characteristics are static curves. More explanations are given in [9] and [10].

IV. REPRESENTATION USING ENERGETIC MACROSCOPIC REPRESENTATION

A. Energetic Macroscopic Representation basic principles

Energetic Macroscopic Representation (EMR) has been developed in 2000. This formalism allows symbolizing explicitly the energy couplings within a system. It defines a synthetic representation for the complex electromechanical systems, respecting a functional description. This tool allows describing a system without having a too heavy graphic reading. It is based in an underlying way on the integral causality which underlies the notion of energy. This concept uses various elements: source, accumulation of energy, conversion and coupling without accumulation of energy. Every pictogram (cf. Appendix) can be described by transfer functions, state models, or any other tools of modelling and representation. These constituents are interconnected according to the principle of the action and the reaction. This description points out the coupling devices which distribute energy. It has been shown that these components are the key of energy management in such systems.

B. EMR of the system

The EMR representation of the considered system is shown in the upper part of Figure 3. The two mechanical sources in the left part of the scheme (MS1 and MS2) stand for input torques applied between the chassis and the front and rear axles. The coupling element (overlapped triangle) represents the coupling between the input torques and it has in the right part the three dimensional vectors $\tau_{tot}$ and $\omega_{tot}$:

$$\tau_{tot} = [-\tau_1 + \tau_2, \tau_1, \tau_2]^T$$

(16)
The topological structure of POG is linear but for EMR the structure is planar. That means it is possible to represent the inertias and the masses separately. In order to do that some EMR couplings are used. The inertias and the masses are represented thanks to the accumulation elements through the inertia matrix $J$ (4) and the mass $m_M$:

$$\omega_{\text{tot}} = [\omega_m \omega_1 \omega_2]^T$$

(18)

$\tau_{\text{tot}} - \tau_r = J \frac{d\omega_{\text{tot}}}{dt}$

(19)

$$-F_{rs} - F_{ps} = m_M \frac{dx_m}{dt}$$

(20)

$$-F_{ry} - F_{py} = m_M \frac{dy_m}{dt}$$

(21)

The three-dimensional coupling element represents the coupling with the tires of the wheels:

$$\begin{bmatrix} \tau_r \ F_{rs} \ F_{ry} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 1 \ 1 & 0 & 0 & 0 \ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} F_r^T \ F_{rs}^T \ F_{ry}^T \end{bmatrix}$$

(22)

$$V = \begin{bmatrix} \dot{x}_m \ \dot{y}_m \ \omega_{\text{tot}}^T \end{bmatrix}^T$$

(23)

The second-dimensional coupling element corresponds to the transformations to have variables referred to the contact point of the tire with the ground:

$$\begin{bmatrix} \tau_{\text{ref}} \ \tau_{\text{ref}} \ \tau_{\text{ref}} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 1 \ 1 & 0 & 0 & 0 \ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} F_{\text{ref}}^T \ F_{\text{ref}}^T \ F_{\text{ref}}^T \end{bmatrix}$$

(24)

The last accumulation element represents the elastic element of the tire, which is described by the relationship (13). The two mechanical sources in the right part of the scheme (Env. x and Env. y) represent the environment that acts on the system by the longitudinal $F_{px}$ (6) and vertical $F_{py}$ (7) forces. Finally the source at the end stands for the slip and skid phenomena (S+S) as they behave as dissipations.

C. Control Structure

EMR allows determining in a systematic way an inversion-based control. This control structure accepts a maximum of control operations and measurements. This Maximum Control Structure (MCS) is obtained by means of a step-by-step inversion of the system decomposed into elementary sub-systems as it is explained in [9]. From the modelling graph a local control can be obtained according to the following inversion principle: any control of a process stems from the inversion of its model. Two basic inversions have been defined: rigid (conversion element) and causal (accumulation element). The rigid relation is timeless; its inversion can be obtained in a direct way (with or without measurement). On the other hand, the causal relation cannot be directly inverted. Indeed this inversion would correspond to a derivative causality, which is not physical. Linked to energy, this kind of relation is not physically achievable. To solve this problem, a controller which reduces the difference between a reference and its measurement is used. This kind of inversion is called an indirect inversion. This inversion requires necessarily the use of a sensor to measure its output.
If the upstream input is chosen as tuning variable, the downstream input becomes so a disturbance. Its effect is then rejected explicitly through a compensation or through a dynamical linearization. Other inversions have been developed such as the inversion of the couplings which require supplementary inputs according to their nature. The following step consists to realize a practical control structure which aims to control the system as it would be made on the real system (estimations, simplifications…).

The inversion rules above have been applied on the system, the target being to control the longitudinal velocity \( \dot{x}_m \) of the vehicle. Before developing the control structure a tuning chain has to be defined. This is the causal chain from the tuning inputs, which act on the system, to the variable to be controlled. Considering the scheme of Figure 3 the tuning chain can be defined starting from the scalar input torque \( \tau_1 \) (torque applied to the joint between the chassis and front axle) and ending at the vehicle speed \( \dot{x}_m \) through the different vectors (\( \tau_1 \) through \( \omega \), \( \omega \) through \( \omega_{\text{rots}} \), \( \omega \) through \( \omega_{\text{rots}} \), \( \dot{x}_c \) through \( X_c \), \( f_{\text{ext}} \) through \( F_x \), and \( F_{\text{ext}} \) through \( F_x \)):

![Figure 4. Tuning chain of the considered system](image)

The inversion rules above have been applied on the system, the target being to control the longitudinal velocity \( \dot{x}_m \) of the vehicle. Before developing the control structure a tuning chain has to be defined. This is the causal chain from the tuning inputs, which act on the system, to the variable to be controlled. Considering the scheme of Figure 3 the tuning chain can be defined starting from the scalar input torque \( \tau_1 \) (torque applied to the joint between the chassis and front axle) and ending at the vehicle speed \( \dot{x}_m \) through the different vectors (\( \tau_1 \) through \( \omega \), \( \omega \) through \( \omega_{\text{rots}} \), \( \omega \) through \( \omega_{\text{rots}} \), \( \dot{x}_c \) through \( X_c \), \( f_{\text{ext}} \) through \( F_x \), and \( F_{\text{ext}} \) through \( F_x \)):

The inversion-based control (bottom part of Figure 3) is then obtained thanks to the control chain (Figure 5) by inversion of the tuning chain. The relationships linked to the control are detailed below:

**Inversion of the masses element via (20):**

\[
F_{\text{ext}} - F_{\text{ix}} = -C_{\text{ix}} (\dot{x}_{\text{ix}} - \dot{x}_{\text{ix-meas}}) - F_{\text{ix-meas}} \tag{25}
\]

**Inversion of the coupling element via (24):**

\[
f_{\text{ext}} = f_{\text{ext-ref}} - f_{\text{ext-meas}} \tag{26}
\]

**Inversion of the elastic element via (13):**

\[
\dot{x}_{\text{c}} = C_{\text{c}} (f_{\text{ext-ref}} - f_{\text{ext-meas}}) + (V_{\text{c-ke}} + V_{\text{c-meas}}) \tag{27}
\]

**Inversion of the coupling element via (24):**

\[
\omega_t = \frac{\dot{x}_{\text{ix-meas}} + L_t \sin(\psi_t - \theta_m) \omega_{\text{ix-meas}} - \dot{x}_{\text{c-ref}}}{y_t} \tag{28}
\]

**Inversion of the inertias element via (19):**

\[
\tau_t = C_{\text{al}} (\omega_t - \omega_{\text{al-meas}}) + \tau_{\text{c-ref}} \tag{29}
\]

This structure allows deducing three cascade controllers because there are three accumulation elements to invert: \( C_{\text{ix}} \) for the vehicle speed \( \dot{x}_m \) (25), \( C_{\text{c}} \) for the longitudinal force generated in the contact point of the front axle \( f_{\text{c}} \) (27), and \( C_{\text{al}} \) for the rotational speed of the front axle \( \omega \) (29). The obtained control contains the measured variables to compensate whose are needed like \( F_{\text{ix-meas}}, f_{\text{c}}-\text{meas}, V_{\text{c-fe}}, V_{\text{c-meas}}, \omega_{\text{c-meas}}, \) and \( \tau_{\text{c-meas}} \).

![Figure 5. Control chain of the considered system](image)

V. SIMULATION

In order to simulate the model of the scheme of the vehicle given in Figure 3 has been implemented in MATLAB-Simulink\textsuperscript{TM}. The main parameters used in simulation are the following: \( m_1 = 1192 \) kg mass of the vehicle body; \( m_{1,2} = 40 \) kg mass of each wheel; \( J_{1,2} = 1.2793 \) kg m\textsuperscript{2} moment of inertia of each wheel; \( R = 0.325 \) m radius of each wheel; \( L = 2.5 \) m distance between front and rear axle shafts; \( [K_1, K_2] = [50000, 636000] \) N/m, stiffness coefficients of the tire. The controller gains have been tuned to achieve good performances using classical design methods. Some simulations have been carried out. Figure 6 shows the reference longitudinal velocity of the vehicle, the vehicle velocity, the control torque and the slip and the skid velocity. Note that when a large torque is requested the tire starts skidding.

VI. DISCUSSION

Two different graphical descriptions have been presented in order to describe the features of each tool for the modelling of a vehicle with tire-road interaction. In this section other aspects are pointed out concerning the targets, the graphical formalism and mathematical formalism of each technique.

The main objective of the POG modelling technique is the analysis of physical systems, while EMR aims mainly at the control of the system. A detailed comparison of EMR and POG graphical techniques is given in Tab. 1. In most cases the generation of the well-known Bond-Graph based on a given POG is easily practicable and vice versa. The POG technique defines only two kinds of blocks and it is unambiguous what is inside the block: it comes from the definition of the block itself. Regarding the teaching and initial training of power flow based modelling, the POG profits by its minimal set of defined elements and the possibility of a quick implementation into a simulation structure. The POG translation into MATLAB-Simulink\textsuperscript{TM} models is straightforward and does not need dedicated libraries. In the EMR technique there are several blocks standing for different kind of elements (dissipations are not explicitly represented), but the structure of what is really inside each block depends on the system, on which model is chosen for the represented subsystem and is not defined a priori by the definition of the block itself. It requires specific MATLAB-Simulink\textsuperscript{TM} libraries [7].
POG admits both integral and derivative causality and uses only integral causality when it is desired to keep the physical meaning. EMR is restricted to integral causality, but provides a better base for the derivation of control structures because of the coupling devices’ graphical definition. EMR formalism uses the definition of action and reaction variables and the instantaneous power exchanged between two subsystems is the product of the action variable and the reaction variable. In the POG formalism there is no explicit reference to the action/reaction principle as it comes out when connecting elements, indeed each connection generates a feedback. Also in POG the instantaneous power exchanged between two subsystems is given by the product of the two power variables involved in each power section of the scheme. The topological structure of POG is linear, for EMR is planar [9].

Although the previous remarks show an implicitly close relationship between POG and EMR, this may not be seen clearly apparent via the diagrams and results from the graphical representations on the one hand, as well as, on the other hand, from the reduced set of elements for POG. A possible scalar POG representation does not influence this statement essentially. Nevertheless it may be recognized that POG connection blocks correspond to EMR converter elements, one or several POG elaboration blocks correspond to energy accumulation elements and POG summation elements serve as EMR coupling devices [14].

VII. CONCLUSION

In this paper, the possibility of make POG and EMR cooperate has been proposed to model a vehicle considering the interaction of the tire with the road. With this approach it is possible to realize the inversion-based control from EMR in a direct way representing the system with a macroscopic view. A Maximum Control Structure has been given in order to control the vehicle longitudinal speed. Moreover, at the same time, a detailed mathematical description from POG can be used to simulate low level aspects. The whole model has been implemented in MATLAB-Simulink and some simulation results have been provided. In particular a discussion between the POG and EMR techniques is given to underline the possibility of cooperation between the two techniques in order to merge their different features for the desired purposes.

REFERENCES

TABLE I. COMPARISON OF EMR AND POG [14]

<table>
<thead>
<tr>
<th>Mnemonic</th>
<th>EMR</th>
<th>POG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Title</td>
<td>Energetic Macroscopic Representation</td>
<td>Power Oriented Graph</td>
</tr>
<tr>
<td>Author</td>
<td>Alain BOUSCAYROL</td>
<td>Roberto ZANASI</td>
</tr>
<tr>
<td>Year</td>
<td>2000</td>
<td>1991</td>
</tr>
<tr>
<td>Symbolism</td>
<td>dependent of the energy domain</td>
<td>independent of the energy domain</td>
</tr>
<tr>
<td>Energy domain</td>
<td>electrically/mechanically, extensible in principle</td>
<td>all known</td>
</tr>
<tr>
<td>Connections</td>
<td>unidirectional</td>
<td>unidirectional</td>
</tr>
<tr>
<td>Power variables</td>
<td>scalar or vectorial (but not obvious in this example)</td>
<td>scalar or vectorial</td>
</tr>
<tr>
<td>Causality</td>
<td>exclusive integral</td>
<td>integral preferably; differential possible</td>
</tr>
<tr>
<td>Basic elements</td>
<td>9 (electrical / mechanical)</td>
<td>4 = 2 (basic elements) + 2 (I/O, mixing point)</td>
</tr>
<tr>
<td>Visibility of both directions</td>
<td>graphically visible</td>
<td>graphically visible</td>
</tr>
<tr>
<td>Assistance for the control</td>
<td>inversion rules, see [3]</td>
<td>none</td>
</tr>
<tr>
<td>Reference direction for power flow</td>
<td>no</td>
<td>it is not explicit, but it is present in the graph</td>
</tr>
<tr>
<td>Displacement / momentum explicitly</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>Special measure element</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Mathematical model from graphical description</td>
<td>partially obtainable</td>
<td>directly obtainable (explicit in the graph)</td>
</tr>
<tr>
<td>MATLAB-SimulinkTM library</td>
<td>icon library</td>
<td>none (not necessary)</td>
</tr>
<tr>
<td>Usage hints</td>
<td>user defined subsystems</td>
<td>standard blocks</td>
</tr>
<tr>
<td>Main objective</td>
<td>simulation and control</td>
<td>simulation and analysis</td>
</tr>
</tbody>
</table>

APPENDIX: SYNOPTIC OF THE GRAPHICAL DESCRIPTIONS

<table>
<thead>
<tr>
<th>Energetic Macroscopic Representation (left) and Control Structure (right) blocks</th>
<th>Power Oriented Graph blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="MS.png" alt="Mechanical source of energy" /></td>
<td><img src="G(s).png" alt="Control block with controller" /></td>
</tr>
<tr>
<td><img src="element.png" alt="Element with energy accumulation" /></td>
<td><img src="K.png" alt="Control block with measure without controller" /></td>
</tr>
<tr>
<td><img src="non-linear.png" alt="Non-linear mechanical converter (without energy accumulation)" /></td>
<td><img src="K%5ET.png" alt="Control block without measure and controller" /></td>
</tr>
<tr>
<td><img src="electromechanical.png" alt="Electromechanical converter (without energy accumulation)" /></td>
<td></td>
</tr>
</tbody>
</table>
