Analytical Design of Lead-Lag Compensators on Nyquist and Nichols planes


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Abstract: In this paper the dynamic structure and the control properties of a new form of lead-lag compensator with complex zeros and poles are presented. A simple and exact analytical and graphical method on the Nyquist and Nichols planes for the design of lead-lag compensators satisfying design specifications on gain margin, phase margin and crossover frequency is proposed. Simulations results show the good performances of the presented method.

Keywords: Lead-Lag Compensator, Gain margin, Phase margin, Nyquist diagrams, Nichols diagrams.

1. INTRODUCTION

Recent works of Flores et al. (2007) and Messner (2009) show a renew interest in the design of classical controllers. A new form of lead and lag compensators with complex poles and zeros was proposed in Messner et al. (2007). Along this line, in this paper we present a new structure of lead-lag compensator with complex poles and zeros, which encompasses the classical form with real poles and zeros. The gain and phase margins (GPM) are important measures of the robustness of dynamical systems, Ho et al. (1995). Different methods can be found in the literature to satisfy GPM specifications, mostly based on trial-and-error procedures. Some of them on PID design are presented in Fung et al. (1998), Wang et al. (1999) and Lee (2004). A graphical design of lead-lag compensators on GPM specifications was presented in Yeung et al. (1998). In this paper we propose an improved graphical procedure for the design of lead-lag compensators on the Nyquist and Nichols planes. Moreover, a new numerical procedure to exactly satisfy design specifications on GPM and gain/phase crossover frequency is presented. The paper is organized as follows: in Section II the basic properties of a new form of lead-lag compensator are presented. In Section III some basic inversion formulae suitable for the design of lead-lag compensators are described. In Section IV the solutions of three different lead-lag design problems satisfying GPM specifications are presented and their graphical interpretations on Nyquist and Nichols plane are proposed. Numerical examples and conclusions end the paper.

2. LEAD-LAG COMPENSATORS: THE GENERAL STRUCTURE

Consider a lead-lag compensator described by the transfer function

\[ C(s) = \frac{s^2 + \gamma \delta \omega_n s + \omega_n^2}{s^2 + \omega_n^2 s + \omega_n^2}, \]  

where \( \gamma, \delta \) and \( \omega_n \) are real and positive. When \( \gamma \delta < 1 \) and/or \( \delta < 1 \) the zeros and/or the poles of the lead-lag compensator \( C(s) \) are complex conjugate with negative real part. The compensator \( C(s) \) is written in a general form which encompasses the classical lead-lag structure with real poles and real zeros. The compensator \( C(s) \) has a unity static gain which does not change the static behavior of the controlled system. The frequency response of \( C(s) \) is

\[ C(j\omega) = \frac{\omega_n^2 - \omega^2 + j 2 \gamma \delta \omega_n \omega}{\omega_n^2 - \omega^2 + j 2 \delta \omega_n \omega}, \]  

which can also be written, for \( \omega \neq \omega_n \), as

\[ C(j\omega) = \frac{1 + j \gamma Y(\omega)}{1 + j Y(\omega)} = \frac{1 + j X(\omega)}{1 + j Y(\omega)}, \]  

where

\[ X(\omega) = \frac{2 \gamma \delta \omega_n \omega_n}{\omega_n^2 - \omega^2}, \quad Y(\omega) = \frac{2 \delta \omega_n \omega_n}{\omega_n^2 - \omega^2}, \quad \gamma = \frac{X(\omega)}{Y(\omega)}. \]  

Since it is assumed that \( \gamma, \delta \) and \( \omega_n \) are real and positive, \( X(\omega) \) and \( Y(\omega) \) are positive when \( \omega < \omega_n \) and negative when \( \omega > \omega_n \). The parameter \( \gamma \) is the gain of \( C(j\omega) \) at frequency \( \omega = \omega_n \) \( (\gamma = C(j\omega_n)) \) and is the minimum (or maximum) amplitude of \( C(j\omega) \). The Nyquist diagram of \( C(j\omega) \) for \( \omega_n = 1 \) and for different values of parameters \( \gamma \) and \( \delta \) is shown in Fig. 1. The shape of these diagrams are circles, as the following property explains.

Definition 1. Let \( \mathcal{C}(\gamma) \) denote the set of all the lead-lag compensators \( C(s) \) as defined in (1) having the same parameter \( \gamma \), that is

\[ \mathcal{C}(\gamma) = \left\{ C(s) \text{ as in (1)} \left| \delta > 0, \omega_n > 0 \right. \right\}. \]  

Moreover, let \( \mathcal{C}_\gamma(s) \in \mathcal{C}(\gamma) \) denote one element of set \( \mathcal{C}(\gamma) \) chosen arbitrarily.

Property 1. The shape of the frequency response \( \mathcal{C}_\gamma(j\omega) \) of \( \mathcal{C}_\gamma(s) \) on the Nyquist plane, see Fig. 1, is a circle with center \( C_0 \) and radius \( R_0 \)

\[ C(\gamma) = C_0 + R_0 e^{j\theta}, \quad C_0 = \frac{\gamma + 1}{2}, \quad R_0 = \frac{|\gamma - 1|}{2}, \]  

where \( \gamma  \) is the gain of \( C(j\omega) \) at frequency \( \omega = \omega_n \) \( (\gamma = C(j\omega_n)) \).