Direct method for digital lead-lag design: analytical and graphical solutions

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Abstract—The paper presents a direct digital method to the design of a discrete lead-lag compensator for a robust control. The design specifications on phase margin, gain margin and crossover frequency of the close-loop system can be obtained by using numerical and graphical solutions. The discrete design method proposed in the paper is also compared with the continuous-time case. Some numerical examples are presented. The proposed method can be useful both on educational and industrial environments.

I. INTRODUCTION

Nowadays digital control of automatic systems is widely used for its benefits over analog control, including reduced parts count and greater flexibility. The discrete-time control design methods can be classified as indirect and direct. The first are widely used because require only limited knowledge of the discrete-time control theory and are based on the vast background of the continuous-time control. Conversely, direct design of classical discrete regulators receives far less attention than indirect design in control textbooks [1]-[3]. However, the discretization of a continuous-time control system creates new phenomena not present in the original continuous-time control system, such as considerable inaccuracies in the the locations of poles and zeros [4]. This paper presents a method for the direct design of lead-lag regulators for the robust control. The recent literature shows a renewed interest in the design of this type of regulators [5]. As known, PID controllers are the most widely used controllers in industry because of their simplicity. However in some cases lead-lag controllers, compared with PID regulators, lead to a better tradeoff between the static accuracy, system stability and insensibility to disturbance in frequency domain [6]. The three parameters of the presented second order lead-lag regulator can be synthesized in order to meet design specifications on the phase, the gain margins and the gain crossover frequency. In classical control design, the gain and the phase margins are important frequency-domain measures used to assess robustness and performance, while the gain crossover frequency affects the rise time and the bandwidth of the close-loop system [7]. A survey of different methods for the continuous-time compensation design based on these frequency domain specifications can be found in [8]. The solution of this robust control design for lead-lag regulators leads to a set of nonlinear and coupled equations difficult to solve. In the literature has been recently proposed a new control technique for the design of these classical compensators. It is based on the so called inversion formulae that express the frequency response of the compensator in polar form. This technique has been applied so far to the continuous and discrete lead and lag regulators [9] and to the continuous lead-lag compensators [10]. This paper presents the extension of this method to the design of discrete lead-lag compensators. An effective graphical method on the Nyquist plane is also presented. The similarity of the continuous and discrete design methods allows on easy use of the discrete direct procedure.

The paper is organized as follows. In Section II, the fundamental characteristics of the continuous-time lead-lag compensator and the design method to meet the given frequency domain specifications are recalled. In Section III the general structure and the properties of the continuous-time lead-lag compensator are described. In Section IV, the direct synthesis of the lead-lag controller in continuous-time domain is presented. Numerical examples and the comparison with other methods end the paper.

II. THE CONTINUOUS-TIME CASE

Consider the block diagram of the continuous-time system shown in Fig. 1 where $G(s)$ denotes the transfer function of the LTI plant to be controlled, which may include the gain and the integration terms required to meet the steady-state accuracy specifications and $C(s)$ denotes the lead-lag compensator to be design. The form of the considered regulator includes real or complex zeros and poles:

$$C(s) = \frac{s^2 + 2\gamma \delta \omega_n s + \omega_n^2}{s^2 + 2\delta \omega_n s + \omega_n^2},$$  \hspace{1cm} (1)

where the parameters $\gamma$, $\delta$ and $\omega_n$ are real and positive. The synthesis of these parameters does not change the static behavior of the controlled system, since the static gain of $C(s)$ is unity. The frequency response $C(j\omega)$ can be written as

$$C(j\omega) = \frac{1 + jX(\omega)}{1 + jY(\omega)},$$  \hspace{1cm} (2)